S.V. Gupta

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Units of Measurement

Past, Present and Future.
International System of Units

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Springer Series in Materials Science ISSN 0933-033X
ISBN 978-3-642-00737-8 e-ISBN 978-3-642-00738-5
DOI 10.1007/978-3-642-00738-5
Springer Heidelberg Dordrecht London New York

Library of Congress Control Number: 2009929023

Cover design: SPi Publisher Services

Printed on acid-free paper

Springer is part of Springer Science+Business Media (www.springer.com)

Dedicated to my wife, Mrs Prem Gupta
and to my Children

Preface

Professor A. R. Verma, former Director National Physical Laboratory, New Delhi, inspired me for writing about the units of measurements as a chapter in my forthcoming book on practical mass measurement. While travelling through India I have found excellent examples of metrology in our historical monuments and old temples. In Tiruchirappally, I visited a temple which was in the centre of the city and had several identical big arch-shaped gates. The pathways were perpendicular to each other and all the gates along the road were exactly in one straight line. Inside the innermost sanctuary where the main deity was placed there was a small opening in the roof. The opening was positioned in such a way that every morning when the sun rose its first ray would pass through this opening and fall on the deity round the year. This made me think about the metrology in ancient times. So I wrote a chapter on metrology in olden days and its development in brief up to the present.

The International System of Units of measurement adopted in 1962 has seven base units as well as a host of derived and dimensionless units. The International Bureau of Weights and Measures occasionally publishes a booklet, which is an authentic document. Most of the National Measurement Laboratories like those in the USA and the UK strictly copy it and, in some cases, translate the document in their national languages. To make it clear that the number of base units need not be seven all the time, I discussed various three- and four-dimensional measuring systems. I established that a minimum of four base units are required in terms of which all other units of measurements can be expressed.

I have also attempted to provide a brief history of CGS and FPS systems. It has been found that the FPS system is a few hundred years older than the CGS or metric systems. Most of the national laboratories have strictly followed the latest available BIPM document. I have also followed the BIPM document on SI units 8th Edition of 2006. In this edition, a chapter on quantities, units and dimensions along with units used in specialized fields of health, biology and human health have been included. I have included them as such with all their notes and explanations. The new elements that I have added are the reasoning to arrive at the derived units, the explanation of the base units of ampere and the intensity of illumination, and the unification of electrostatic and electromagnetic units. Chapter 8 of the book deals with the future definitions of base units and their effects. One of the chapters also gives the brief life history of scientists who have been honoured by assigning their name to a unit.

The book is written in such a way that it caters to the need of one and all. Students of class X and above can profitably use Chapters 1 to 8 barring certain portions of Chapters 1, 2, 3 and 5. Biographies of the scientists associated with units of measurements will definitely be inspiring to young students and metrologists. The last two chapters are for specialists who are interested in redefining the units of measurements or in the evolution of a new measurement system based on fundamental constants. Metrologists at all levels will be delighted to know the origin of the names for base units and derived units.

I acknowledge the great help which I received from Dr. R. S. Davis, Head of Mass BIPM, Professor A. J. Wallard, Director BIPM, and Dr. Claudine Thomas, Secretary Consultative Committee of Units (CCU) at BIPM. They explained to me the meaning of the redefinition of the unit, keeping the same name and effect as the old unit. I wish to thank Dr. Vikram Kumar, Director National Physical Laboratory and President of the Metrology Society of India, New Delhi, who agreed to bring out this document. I will fail in my duty if I do not express my most sincere thanks to the referees to whom this manuscript was sent. Each of them has gone into minute details and offered editorial suggestions. My thanks are also due to my daughter Mrs. Reeta Gupta, Scientist, National Physical Laboratory, New Delhi.

Delhi *S. V. Gupta*
June 2009

Contents

Acronyms

1 Acronyms for International Organisations

BAAS	British Association for the Advancement of Science
BIH	Bureau International de l'Heure
CARICOM	Carribean Community
CIE	International Commission on Illumination/Commission Internationale de l'Éclairage
IAU	International Astronomical Union
ICRP	International Commission on Radiological Protection
ICRU	International Commission on Radiation Units and Measurements
IEC	International Electro-technical Commission/Commission Électrotechnique Internationale
IERS	International Earth Rotation and Reference Systems Service
ISO	International Organisation for Standardisation
IUPAC	International Union of Pure and Applied Chemistry
IUPAP	International Union of Pure and Applied Physics
OIML	International Organisation of Legal Metrology/ Organisation Internationale de Métrologie Légale
SUNAMCO	Commission for Symbols, Units, Nomenclature, Atomic Masses and Fundamental Constants, IUPAP
TAI	International Atomic Time/Temps Atomique International
WHO	World Health Organisation

2 Acronyms for Metre Convention and Associated Organsiations

BIPM	International Bureau of Weights and Measures/Bureau International des Poids et Mesures
CCAUV	Consultative Committee for Acoustics, Ultrasound and Vibration/ Comité Consultatif de l'Acoustique, des Ultrasons et des Vibrations

CCDS* Consultative Committee for the Definition of the Second/ Comité Consultatif pour la définition de la Seconde, see CCTF
CCE* Consultative Committee for Electricity/Comité Consultatif d'Électricité, see CCEM
CCEM (formerly the CCE) Consultative Committee for Electricity and Magnetism/Comité Consultatif d'Électricité et Magnétisme
CCL Consultative Committee for Length/Comité Consultatif des Longueurs
CCM Consultative Committee for Mass and Related Quantities/ Comité Consultatif pour la Masse et les Grandeurs Apparentées
CCPR Consultative Committee for Photometry and Radiometry/ Comité Consultatif de Photométrie et Radiométrie
CCQM Consultative Committee for Amount of Substance: Metrology in Chemistry/Comité Consultatif pour la Quantité de Matière : Métrologie en Chimie
CCRI Consultative Committee for Ionising Radiation/Comité Consultatif des Rayonnements Ionisants
CCT Consultative Committee for Thermometry/Comité Consultatif de Thermométrie
CCTF (formerly the CCDS) Consultative Committee for Time and Frequency/ Comité Consultatif du Temps et des Fréquences
CCU Consultative Committee for Units/Comité Consultatif des Unités
CGPM General Conference on Weights and Measures/Conférence Générale des Poids et Mesures
CIPM International Committee for Weights and Measures/ Comité International des Poids et Mesures
CODATA Committee on Data for Science and Technology IAU
CR Comptes Rendus of the Conférence Générale des Poids et Mesures, CGPM
PV Procès-Verbaux of the Comité International des Poids et Mesures, CIPM

Note: *Organisations marked with an asterisk either no longer exist or operate under a different acronym.

3 Acronyms for Scientific Terms

CGS Three-dimensional coherent system of units based on the three mechanical units centimetre, gram and second
EPT-76 Provisional low temperature scale of 1976/Échelle provisoire de température de 1976
IPTS-68 International practical temperature scale of 1968
ITS-90 International temperature scale of 1990
MKS System of units based on the three mechanical units metre, kilogram, and second

MKSA	Four-dimensional system of units based on the metre, kilogram, second, and the ampere
SI	International system of units/Système International d'Unités
TCG	Geocentric coordinated TIME/Temps-coordonnée Géocentrique
TT	Terrestrial time
UTC	Coordinated universal time
VSMOW	Vienna standard mean ocean water

Chapter 1
Metrology Through Ages

1.1 Introduction

Since time immemorial human beings have been weighing and measuring objects in one form or the other, and have been trying to define units of measurements. Normally one compares a quantity of a particular body with that of another body, whose quantity, under question, is known. In earlier days, one might have compared similar quantities, one of which could have been a part of the human body, for example the foot of a person. The problem with such definitions was encountered when this unit was to be implemented throughout the country. In this case, persons with different foot lengths would assign different numbers for a given distance. To obviate this difficulty they could have decided to estimate all distances in terms of the king's foot length. However, the problem with this system would be that the king could not be expected to go wherever a distance needed to be measured, and the question of what would happen after his death too would arise.

The next bright idea was to use an artefact like a piece of wood, metal or some other material as a unit of length. One of the earlier records of such an artefact was found in the form of an "Egyptian Cubit" around 2,500 BC. This first well-documented example was derived from the length of the arm from the elbow down to the outstretched tip of the middle finger. By 2,500 BC this had been standardised as a royal master cubit made of black marble (measuring about 52 cm). This cubit was divided into 28 equal parts (each roughly the width of a finger), which could be further divided into fractional parts, the smallest of these being just over a millimetre. About 500 years later a royal decree about length, weight and capacity measures was issued in Babylon. This was a piece of granite whose length was equal to the length of the forearm plus the width of the palm of the Pharaoh ruling at that time.

1.2 History of Metrology in India

1.2.1 Legal Metrology

Since ancient times India has been following good trade practices. One of the most ancient texts of India is the *Manusmriti*. *Manusmriti* via *ashloka* 403 in the 8th chapter mandates the King as follows:

> The king should examine the weights and balances every 6 months to ensure true measurements and to mark them with the royal stamp.

1.2.2 Town Planning

The next record of the history of measurements in India is from the Indus Valley Civilization around 4,000–1,500 BC.

In the *Harappa* era, which is nearly 5,000 years old, one finds excellent examples of town planning and architecture. Strong evidence exists that in the *Mohenjodaro* period all bricks used were of the same relative dimensions, that is the length, breadth and thickness are in the ratio of 4:2:1. The significance of the decimal was already known, even though a Hindu mathematician demonstrated the counting of a decimal at a much later date.

1.2.3 Length Measurements

In the *Mauryan* period during the reign of *Chandragupta Maurya*, around 500 years BC, there was a well-defined system of weights and measures. The government of that time ensured that everybody used the same system. According to this system, the smallest unit of length was *Parmanu*. Lengths measurements as multiples of *Parmanu* in steps of 8 were defined in the *Kautilya Arthshastra* and are given by Professor A. R. Verma [1] as follows:

8 *Parmanu* = 1 *Rajahkan* (dust particle coming from the wheel of a chariot)
8 *Rajahkan* = 1 *Liksha* (egg of lice)
8 *Liksha* = 1 *Yookamadhya*
8 *Yookamadhya* = 1 *Yavamadhya*
8 *Yavamadhya* = 1 *Angul* (width of a finger)
8 *Angul* = 1 *Dhanurmushti*

Taking one *Angul* – the width of a finger – as 1 cm, one can see that even in 500 BC, Indians were measuring lengths as small as 0.3 μm roughly. This also shows the measurement capability and need of the industry in the Mauryan period.

This process of updating units of measurements continued from one period to another. In the *Mughal* period at the time of the *Mughal* emperor *Akbar*, Abul Fazl-i-Allami in his *Ain-i-Akabari* described the *Gaz* as the unit of length. The *Gaz* was divided into 24 equal parts and each part was called *Tassuj*. Some historians called this unit *Ilahigaz* instead of *Gaz*. The system was extensively used for land records, and for the construction of buildings, houses, wells, gardens and roads. The term Gaz continued to be used till the introduction of the metric system in India in 1956. The actual length of one Gaz and the number of its subdivisions underwent several changes. In the British period, the length of the *Gaz* was made equal to the imperial yard and it was subdivided into 36 equal parts, each of which was called an inch. The inch was divided into eight equal parts (commonly called *soot*). For engineering purposes, 1/8 of an inch was divided into two or four equal parts.

1.2.4 Time Measurements

The time measurement system in ancient India was excellent, and it covered a range from microseconds to trillions of years, including the cycles of the universe. In the *Yjur Veda*, chapter VII, *ashloka* 30, there is a description of the months and the year. From the language it is clear that there are 12 months, each consisting of two *paksh* (14 days) according to the orbiting of the moon around the earth. The actual number of days in a month may vary by a day according to the position of the moon and the sun. It is a very practical method for measuring time and does not require any equipment. Everybody can see the moon at night and can tell which day it is. However, it appears that even in those days they knew that the earth is revolving around the sun and that the seasons depend upon the relative position of the earth and the sun. To adjust to this observation the seasons start almost simultaneously with the months. There was a provision for an additional 13th month, which may occur periodically to compensate for the revolution of the earth around the sun. The Vedic year almost coincides with the time of one revolution of the earth around the sun. This practice still continues in Hindu religious calendars used in astrology and for festivals. The seasons and the festivals fall almost in the same lunar month.

Time Intervals

Time intervals (24-h day) used in astrology are even now subdivided into the following submultiples:

1 (24 h) day = 30 *Mahurat*
1 *Mahurat* = 2 *Ghadi*
1 *Ghadi* = 60 *Pal* = 24 min
1 *Pal* = 6 *Asho*
1 *Asho* = 10 *Vipul*
1 *Vipul* = 0.4 s

Much before the present era, in Vedic times, more than 5,000 years BC, Indians had separate names for much smaller time intervals. The term for the smallest time interval as given in the holy book *Shrimadbhagwat Puran* [2] was *permanu*, and its multiples are as follows:

2 *permanu* = 1 *anu*
3 *anu* = 1 *trisrenu*
3 *trisrenu* = 1 *truti*
100 *truti* = 1 *vedh*
3 *vedh* = 1 *love*
3 *love* = 1 *nimesh*
3 *nimesh* = 1 *chhun*
5 *chhun* = 1 *kashta*
15 *kashta* = 1 *laghu*
15 *laghu* = 1 *nadika*
2 *nadika* = 1 *mahurat*
30 *mahurat* = 1 day and 1 night (one sunrise to the next)
7 days and 7 nights = 1 *saptah*
2 *saptah* = 1 *paksh*
2 *paksh* = 1 lunar month
2 months = 1 *ritu*
3 *ritu* = 1 *ayan*
2 *ayan* = one lunar year

From the above data we get:

3,280,500,000 *permanu* = 24-h day = 86,400 s
37,968.75 *permanu* = 1 s
1 *permanu* = 2.6 μs

However, for day-to-day life in the holy book *Mahabharat*, *ashloka* 231 [3], the smallest time interval was considered as *nimesh*, and its multiples, which are used till today, are as follows:

15 *nimesh* = 1 *kashta*
30 *kashta* = 1 *kala*
30 *kala* = 1 *mahurat*
30 *mahurat* = 1 day and 1 night
4.6875 *nimesh* = 1 s or
1 *nimesh* = 0.21333 ≈ 0.2 s

The existence of a separate name for the time interval known as *permanu*, which is equivalent to 2.6 μs, indicates two things: there was a frequent need to have such a small time interval and there existed methods of detecting such small intervals with reasonable precision.

In addition to the above, we get quite a few sets of time intervals and counting of time in various literatures, which is inevitable when we are looking at a history of 10 million years. Some of these systems are mentioned below.

Detailed information on time measurement is given in the *Vishnu Puran*, Book I, Chap. III.

Sidereal Metrics

1 *paramanu* = the normal interval of blinking in humans, or approximately 4 s
1 *vighati* = 6 *paramanu*, or approximately 24 s
1 *ghadiya* = s 60 *vighatis*, or approximately 24 min
1 *muhurta* = 2 *ghadiyas*, or approximately 48 min
1 *nakshatra ahoratram* or sidereal day = *exactly* 30 muhurtas [4]

(Note: A day is considered to begin and end at sunrise, not at midnight.)

Quite a few other systems of time measurement in ancient India have been cited on websites [5]; I have randomly selected a few, just to give a feel about the various systems of time measurement in different time periods. The names of some of the units cited may be the same but they are of different magnitudes, and the spelling of some terms may be slightly different because Sanskrit words have been written in roman script as they sound to different authors.

Smaller Units of Time Used in the Vedas [5]

1 *trasarenu* = the combination of 6 celestial *atoms*
1 *truti* = the time needed to integrate 3 *trasarenu*, or 1/1,687.5 of a second
1 *vedha* = 100 *truti*
1 *lava* = 3 *vedha*
1 *nimesha* = 3 *lava*, or a blink
1 *kshana* = 3 *nimesha*
1 *kashtha* = 5 *kshana*, or about 8 s
1 *laghu* = 15 *kashtha*, or about 2 min
15 *laghu* = 1 *nadika*, which is also called a *danda*

This equals the time before water overflows in a six-*pala* [14 ounce] pot of copper, in which a hole is bored with a gold probe weighing four *masha* and measuring four fingers long. The pot is then placed on a tripod.

2 *dandas* = 1 *muhurta*
6 or 7 *dandas* = 1 *yamah*, or 1/4 of a day or night
There are 4 *prahara* or 4 *yama* in each *day* or each *night*

Lunar Metrics [5]

A *tithi* (also spelled *thithi*) or lunar day is defined as the time it takes for the longitudinal angle between the moon and the sun to increase by 12°. A tithi is variable in time intervals varying approximately from 19 to 26 h.

A *paksa* (also paksha) or lunar fortnight consists of 15 *tithis*.
A *masa* or lunar month (approximately 29.5 days) is divided into 2 *pakshas*: the one between new moon and full moon is called *gaura* (bright) or *shukla paksha*; and the one between full moon and new moon is called *krishna* (dark) *paksha* [5].
A *ritu* comprises of 2 *masas*.
An *ayanam* is made up of 3 *ritus*.
A *varsha* or year consists of 2 *ayanams*.

Tropical Metrics

1 *yaama* = 7.5 *ghati*s
8 *yaama*s = 1 half of the day (either day or night)
ahoratram = a tropical day [5]

In this case also, a day is considered to begin and end at sunrise, not at midnight.

Reckoning of Time Among Other Entities

Reckoning of Time Among the Pitras [5]

Pitra is a Sanskrit word which means forefathers, which may also be considered as history.
1 day of the *pitras* = 1 solar *masa* (month)
30 days of the *pitras* = 1 month of the *pitras*
12 months of the *pitras* = 1 year of the *pitras*
The lifespan of the *pitras* is 100 years of the *pitras* (= 3,000 solar years)

Taking the lifespan of a human being as 100 years, 3,000 years appears to be a reasonable period spanning the history of a particular sect to which the human being belonged.

Reckoning of Time Among the Devas

1 day of the *Devas* = 1 solar year
30 days of the *Devas* = 1 month of the *Devas*
12 months of the *Devas* = 1 year of the *Devas*
The lifespan of the *Devas* is 100 years of the *Devas* (= 36000 solar years)

Deva is again a Sanskrit word meaning a demigod or demigoddess. This may be considered as a specific religion with the specific name of a deity as its head. Taking the lifespan of a religion as 36000 years appears to be a reasonable period for an existence of a specific religion.

The use of the number 10 and its integral powers as a multiplication factor to obtain higher units may be appreciated. The decimal system was deep-rooted in ancient India.

The word *Deva* is also used for the divine. So instead of saying *Deva* day, *Deva* month and *Deva* year we can call them divine day, divine month and divine year, respectively.

Reckoning of Time for Brahma

12000 years of the *Devas* = 1 *Mahayuga* = 4320000 solar years

One *Mahayuga* is divided into 10 *charnas* consisting of four *Yugas*, namely *Satya Yuga*, *Treta Yuga*, *Dwapar Yuga*, and *Kali Yuga.* The duration of each *Yuga* in terms of *charnas* and solar years is tabulated below.

The Four *Yugas*	
4 *charnas* (1728000 solar years)	*Satya Yuga*
3 *charnas* (1296000 solar years)	*Treta Yuga*
2 *charnas* (864000 solar years)	*Dwapar Yuga*
1 *charnas* (432000 solar years)	*Kali Yuga*

As this is a repetitive cycle there are altogether 1,000 cycles of *Mahyugas* in one day of Brahma.

One cycle of the above four *Yugas* makes one *Mahayuga* (4.32 million solar years)

A *Manvantara* consists of 71 *Mahayugas* (306,720,000 solar years). Each *Manvantara* is ruled by a *Manu*.

After each *Manvantara* follows one *Sandhiya Kala* of the same duration as the *Satya Yuga* (1,728,000 solar years). It is said that during a *Sandhiya Kala*, the entire earth is submerged in water.

A *Kalpa* consists of a period of 1,728,000 solar years, and is called *Adi Sandhiya*. This is followed by 14 *Manvantaras* and 14 *Sandhiya Kalas*.

A day of *Brahma* is one *Kalpa*, as is also one night of *Brahma*.

One day of *Brahma* equals 14 times 71 *Mahayugas* + (15 times 4 *Kali Yugas*)

= 994 *Mahayugas* + (60 *Kali Yugas*)
= 994 *Mahayugas* + (6 times 10 *Kali Yugas*)
= 994 *Mahayugas* + 6 *Mahayugas*
= 1,000 *Mahayugas*

The duration of a day of *Brahma* is confirmed by a statement of *ashloka* 17 in the 8th chapter of the *Shrimadbhagvad Gita*: “*sahasra-yuga paryantam aharyad brahmano viduh*”, meaning a day of *Brahma* is 1,000 *mahayugas*. Thus, a day of *Brahma* is a *Kalpa* that has a duration of 4.32 billion solar years. Two *Kalpa*s constitute a day and night of *Brahma*. The second part of the aforesaid *ashloka* also confirms that the duration of the night of *Brahma* is also 1,000 *Mahayugas*.

1 month of *Brahma* = 30 days + 30 nights of *Brahma* = (259,200,000,000 solar years)
1 year of *Brahma* = 12 months of *Brahma* = 3,110,400,000,000 solar years
1 *Parardha* = 50 years of *Brahma* = 155,520,000,000,000 solar years
100 years of *Brahma* is the lifespan of *Brahma* = 311,040,000,000,000 solar years or 311 trillion years
After 100 years of *Brahma*, the universe starts with a new *Brahma*. It may be noted that we are in the time of the second *Brahma*.

We are currently in the 28th *Kali yuga* of the first day of the 51st year of the second *Brahma* in the reign of the 7th *Manu – Manu Vaivasvata* . This is the 51st year of the present *Brahma* and so about 155 trillion years have elapsed since he took over as *Brahma*.

It may be interesting to note the following:
1 Solar year = 365 days 6 h and 9 min
1 Lunar year = 354 days 8 h and 48 min
1 Lunar month = 29 days 12 h and 44 min

The current *Kali Yuga* (Iron Age) began at midnight of 17 February/18 February in 3102 BC in the proleptic Julian calendar.

It is very interesting to note that a day of *Brahma*, a *Kalpa*, has a duration of 4.32 billion solar years. And from the latest dating techniques the age of the earth is found to be between 4.5 and 5 billion years. A close match of these two measurements raises very interesting questions.

Counting of Time

According to Saint *Shukdeva* [6], time is eternal and infinite. But for the purpose of recording the happenings of worldly affairs and events, especially for larger intervals of time, the year is taken as the unit and has the following multiples:

1 *Kali Yuga* = 432,000 years
2 *Kali yugas* = 1 *Dwapar Yuga*
3 *Kali yugas* = 1 *Treta Yuga*
4 *Kali yugas* = 1 *Satya Yuga*
1 *Chaturyugee* = 4,320,000 years (sum total of all the four *Yugas*)
71 *Chaturyugees* = 1 *Manvantara* = 306,720,000 years

Each *Manvantara* is preceded and followed by a lull period of 1,728,000 years, which is equivalent to one *Satya Yuga* and is called *Sandhiya*.

14 *Manvantras* + 15 *Satya Yugas* or 15 *Sandhiyas* = 1 *Kalpa* = 4,320,000,000 years
1 *Kalpa* = 1 day of *Brahma* or 1 night of *Brahma*
2 *Kalpas* = 1 day and 1 night of *Brahma*

100 years of *Brahma* is the period during which a universe is born and meets its end. After this period a new universe is born again and a new civilization starts, and this is a never-ending process. Therefore Saint *Shukdeva* said that time is eternal and infinite.

At the beginning of any Hindu rite, an *ashloka* in Sanskrit is read out, which means that this ritual is being performed for a certain purpose (name of the purpose) by this particular person (name of the person including father's and family names) at this place (full address with country name) at this time of day (*Ghadi* and *Pala* or in hours and minutes) in 5109 of *Kalyugi* year (for AD 2007, or Hindu calendar year 2064 *Sambat*) of the 28th *Chaturyugee* of the 7th *Manvantra* on the first day of the 51st year of the 2nd *Brahma*. This is a wonderful example of counting time from the start of the universe to the present time.

Time Scale in Seconds

From the description of the names of different time intervals, one can easily deduce that our forefathers had a range of time measurement from 10^{-7}s on the smaller side to 10^{22} s on the extremely large side. Giving specific names to the vast range of time measurements suggests that they were in need of such small and large time intervals and also that measurement capabilities did exist for such small and big time intervals. The logarithmic scale of time is shown in Fig. 1.1 along with specific names of some units in various systems of time counting in seconds [4]. I would like to draw the attention of readers to some names of time intervals that might have been the same in different systems, but differing largely in values. One such example is the term *truti*.

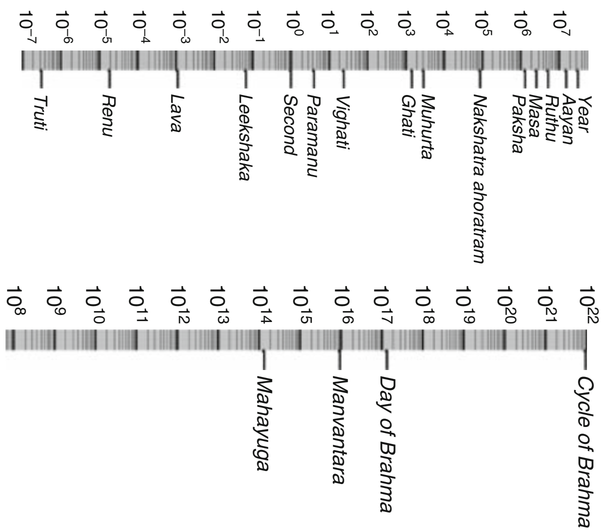

Fig. 1.1 Logarithmic time scale in ancient India

1.2.5 Units of Time and Angle

The division of the circle into 360° and the day into hours, minutes and seconds can be traced back to the Babylonians, who had a sexagesimal system of numbers. The 360° may have been related to a year of 360 days.

1.2.6 Mass Measurement

In Vedic times, equal care was taken for trade and commerce and there was a well-established system of weights and volume measures. Similar to the present system, they had different weights and sometimes even different nomenclatures for the use of different merchandise.

For Trade in Food Grains and Similar Items

For units of mass and volume please refer to [7]. The *drone* is the unit of weight used for food, grains and similar items. Its submultiples were:

1 *Drone* = 4 *Adaka* = 16 *Prastha* = 64 *Kudava* = 256 *Pala* = 1024 *Karsa* = 16,384 *Masa*

In other words:

1 *Drone* = 4 *Adaka*
1 *Adaka* = 4 *Prastha*
1 *Prastha* = 4 *Kudava*
1 *Kudava* = 4 *Pala*
1 *Pala* = 4 *Karsa*
1 *Karsa* = 16 *Masa*

One may appreciate that each successive unit is in submultiples of four of its predecessor.

Assuming 1 *Masa* is about 1.1016 g, we have:

1 *Drone* = 18,048.6 g = 18 kg

For Gold Trade [7]

1 *Aksa* = 1 *Karsa* (gold) = 16 *Masa* (gold),

It may be noted that here also *karsa* and *masa* have the same ratio of 16:1 as in the grain trade.

1 *Masa* (gold) = 5 *krsnala* = 1/10 *suvarn* (gold)
Suvarn (gold) = 10 *Masa*

1 *Masa* = 5 *Krsnala*
1 *Suvarn* (gold) will be nearly equal to 11.016 g in present SI unit of mass.

In fact, a number of many other systems of weights were used in different periods of the Vedic era.

For Silver Trade

For silver there appears to be different sets of weights and denominations:

1 *Dharna* = 10 *Pala* = 16 *Masaka* (silver) = 1 *Purana* (silver)
1 *Dharna* = 19 *Sispava* = 2/5 *Kara* = 10 *Pala* = 24 *Rakitika*

1.2.7 Volume Measurements

1 *Drone* (bucket) = 16 *Puskala*
1 *Puskala* = 8 *Kunici*

In another system:

1 *Drone* = 1,024 *Musti* = 200 *Pala* = 1/20 *Kumbha*
1 *Kumbha* = 20*Drone*
1 *Drone* = 200 *Pala*
1 *Pala* = 5.12 *Musti*
1 *Khri* = 16 *Drone* or 64 *Haka*
1 *Drone* = 2 *Adhaka* = 32 *Seer*
1 *Surpa* = 2 *Drone* = 64 *Seer*

Arranging the units in ascending order, we get:

1*Haka* = 1/4 *Drone*
1 *Adhaka* = 1/2 *Drone*
1 *Drone* = 1 *Drone*
2 *Drone* = 1 *Supra*
16 *Drobe* = 1 *Khri*

Here the binary system has been followed.

During the British period, efforts were made to achieve uniformity in mass, length and area measurements. The British rulers connected the Indian weights and measures to those being used in Great Britain at that time, which were inch, foot and yard for length, and grain, ounce and pound for weight. A compromise was reached in mass measurement, as follows:

8 *Ratti* = 1 *Masa*
12 *Masa* = 1 *Tola*
5 *Tola* = 1 *Chhatank*

16 *Chhatank* = 1 *Seer* (very roughly equal to 1 kg)
40 *Seer* = 1 *Maund*
1 *Maund* = 100 pounds troy (exact)

This system of mass measurement continued in the British-occupied India till 1947, the year of our independence. However, in the Princely States, the term *seer* was used but its nominal value in terms of the British standard pound was different from one state to another. At the time of metrication there were as many as 120 types of *seers*. Each *seer* was different from the other, both in shape and value.

Similarly in area measurements the units were different from one state to another. It is sad that even after about 60 years of adoption of the metric system, the old units in land measurements are still being used. In addition to these there are different units by names and values that are used for land area measurements.

1.2.8 Numeration

In Vedic times [8] we had a very good counting system and every numeral from zero to 100 was given a separate name. Thereafter, names were assigned to each numeral in steps of 10. This method continues to a numeral as high as 10^{18}.

Table 1.1 Nomenclature of numerals in Hindi and English

Numeral	Vedic kaal	Sanctioned by the standards of weights and measures numeration rules 1987 as amended in 2002	
		Hindi	English
10^0	*Aik*	One can use the words in any local language recognized by the Indian Constitution, Hindi or English	
10^1	*Das*		
10^2	*Shat*		
10^3	*Hazar*	*Hazar*	Thousand
10^4	*Das hazar*	*Das hazar*	Ten thousand
10^5	*Lakh*	*Lakh*	Hundred thousand
10^6	*Niyut*	*Das lakh*	Million
10^7	*Crore*	*Crore*	Ten million
10^8	*Riburdh*	*Das crore*	Hundred million
10^9	*Vrand*	*Arab*	Billion
10^{10}	*Kharab*	*Das arab*	Ten billion
10^{11}	*Ni-Kharab*	*Kharab*	Hundred billion
10^{12}	*Shankh*	*Das kharab*	Trillion
10^{13}	*Padam*	*Neel*	Ten trillion
10^{14}	*Sagar*	*Das neel*	Hundred trillion
10^{15}	*Rintya*	*Padam*	Thousand trillion
10^{16}	*Madhya*	*Das padam*	Ten thousand trillion
10^{17}	*Pradharya*	*Shankh*	Hundred thousand trillion
10^{18}	*Das pradharya*	*Das shankh*	Thousand thousand trillion

One of the functions of the Standards of Weights and Measures Act of 1976 is to officially prescribe the names of various numerals and the use of digits to write a numeral, and to express numerals having more than three digits. As India is a member of the Metric Treaty as well as the International Organization of Legal Metrology, we tried to initially follow the French system of assigning names to larger numerals. But later, visualizing the practice followed by countries with whom we have the bulk of our trade, slight changes were adopted for terms higher than million, as suggested by the author. The names assigned to various numerals larger than one are given in Table 1.1.

One may notice that although some Hindi words are common in the second and third columns, their numerical values are different.

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Chapter 2
System of Quantities and Units

2.1 Quantities

There are a large number of quantities like length, volume, acceleration, force, momentum, electric charge, current, potential, inductance, etc., which require measurement. It must be clear from the very beginning that there is no fundamental difference in the basic principles of measurement, whether they are made in physics, chemistry, laboratory medicine, biology, or engineering. An attempt, therefore, has been made to meet conceptual needs of measurements in fields such as biochemistry, food science, forensic science, and molecular biology.

However, most of the quantities are either connected with each other by definition or through a physical phenomenon. A system of quantities, therefore, is a set of quantities together with a set of non-contradictory equations relating to those quantities.

For example:

1. Velocity and acceleration are connected with length and time by definition.
2. All mechanical quantities can be expressed in terms of mass, length and time.
3. Gravitational force and interaction between the charges are connected through a physical phenomenon.

So we need not have to define every quantity. We take a system containing only a few quantities such that all other quantities are expressed in terms of those quantities.

A subset of quantities, in terms of which every other quantity may be expressed, is known as a system of base quantities. In the following paragraphs, we will discuss quantity, base and derived quantities and their algebra.

2.2 System of Quantities

2.2.1 Quantity

Quantity may be a property of a phenomenon, body or substance, to which a number can be assigned with respect to a reference.

The reference can be a measurement unit, a measurement procedure, or a reference material.

All quantities together with their defined relations form a set, known as a system of quantities

2.2.2 Base Quantity

Base quantity is a conventionally chosen quantity. No base quantity can be expressed as a product of powers of the other base quantities. Hence it is said that base quantities are mutually independent.

Note: A fair amount of material has been taken from the document titled "The International System of Units" 2006 8th Edition, issued by the International Bureau of Weights and Measures (BIPM)

2.2.3 System of Base Quantities

A subset of base quantities is known as a system of base quantities. No member of the subset can be expressed in terms of the others. The system of base quantities should be such that every other quantity can be conveniently expressed in terms of base quantities. Theoretically speaking, a minimum number of four base quantities is needed to express any other quantity. However, it may not be convenient from the point of understanding and realizing every quantity. For example, the International System of Quantities (ISQ) consists of seven base quantities, namely mass, length, time, temperature, electric current, luminous intensity and mole. Base quantities, symbols used for the base quantities and symbols used to denote their dimension, as adopted in SI, are given in Table 2.1. Normally the symbol of a quantity is written in italics and that of its dimension in capital letters in upright form.

2.2.4 Derived Quantity

A quantity in a system of quantities, which is defined in terms of its base quantities, is known as a derived quantity.

Table 2.1 Base quantity symbols and their units with respective symbols

Quantity	Symbol	Name of unit	Symbol of unit	Symbol of dimension
Length	*L, x, r,* etc.	metre	m	L
Mass	*M*	kilogram	kg	M
Time	*T*	second	s	T
Electric current	I, i	ampere	A	I
Intensity of illumination	I_v	candela	cd	J
Temperature	*T*	kelvin	K	Θ
Mole	*N*	mole	mol	N

For example: In a system of quantities having length and mass as base quantities, mass density is a derived quantity defined as the quotient of mass and volume. Volume is length raised to the power three.

2.2.5 Quantity Equation

Quantity equation is a mathematical relationship between quantities in a given system of quantities, independent of measurement units.

If a quantity Q_1 is a product of two quantities Q_2 and Q_3 and a number n, then its quantity equation is

$$Q_1 = nQ_2Q_3.$$

For example, the kinetic energy E of a moving particle of mass m and velocity v is given as

$$E = (1/2)mv^2.$$

Another example is the expression for the mass m of the amount of substance of a univalent component deposited on an electrode, when a constant current I is passing in a voltameter for a time t, then

$$m = It/F.$$

Here F is the Faraday constant.

2.3 Measurement Unit

A unit is a scalar quantity defined and adopted by convention, with which any other quantity of the same kind can be compared. The ratio of two quantities of the same kind is a pure number.

2.3.1 System of Measurement Units

A set of measurement units corresponding to every quantity in the system of quantities is a system of measurements. The set consists of base units, derived units and dimensionless quantities or quantities of dimension 1.

2.3.2 System of Base Units

Corresponding to a system of base quantities, there is the system of units. For every base quantity there exists a unit. Every other unit can be expressed in terms of base units. Such a subset of units, in terms of which all other units are expressed, is called a system of base units. In this way, we can express the magnitude of any given quantity by a number equal to the ratio of that quantity to its unit.

Properties of Base Units

1. In every system of units there is only one base unit for each base quantity. For example, in the SI, the metre is the base unit of length. The centimetre and the kilometre are also units of length, but they are not base units in the SI. However, in the CGS systems, the centimetre is the base unit of length.
2. A base unit may also serve for a derived quantity of the same dimension. For example, rainfall, when defined as volume per unit area, uses the metre as a coherent derived unit in the SI.
3. For number of entities, the number one, symbol 1, can be regarded as a base unit in any system of units.

Minimum Number of Base Units

A system of three base units, consisting of units of mass, length and time, is good enough to express the units of all other mechanical quantities. For example, volume is length cube; speed is length divided by time taken to travel that length; magnitude of linear acceleration is rate of change in speed with respect to time; force is mass multiplied by acceleration; energy is force multiplied by length; and so on. In the initial stages of measurement, length, mass and time were taken to form a system of base quantities.

You will see in the next chapter that a minimum of four base quantities is required to express each and every other quantity. However, for the sake of convenience and easy understanding, seven base units, corresponding to seven base quantities enumerated in Table 2.1, have been adopted to make the International System of Units.

2.3.3 Derived Unit

A measurement unit for a derived quantity is the derived unit. It is expressible as a product of base units and integral exponents.

For example, the metre per second with the symbol m/s, or the centimetre per second with the symbol cm/s, is a derived unit of speed in the SI. The kilometre per hour with the symbol km/h is a unit of speed outside the SI but is accepted for use by the SI. The knot, equal to one nautical mile per hour, is also a unit of speed outside the SI.

The derived unit of force in SI is $\mathrm{kg\,m\,s^{-2}}$. In expressing derived units in terms of base units, the multiplication sign is optional, and a space may be left between the symbols of the base units instead.

2.3.4 Unit Equation

Unit equation is a mathematical relationship of base units, coherent derived units or other measurement units. The symbol of the unit of a quantity Q is $[Q]$.

Considering the example of the quantity equation in section 2.2.5, the unit equation is given as

$$[Q_1] = [Q_2][Q_3],$$

Where $[Q_1]$, $[Q_2]$, and $[Q_3]$ are the measurement units of Q_1, Q_2, and Q_3, respectively, provided that these measurement units are in a coherent system of units.

The quantity equation of energy with force and displacement is

$$\text{Energy} = \text{Force} \times \text{displacement}.$$

This gives the unit equation for energy as

$$\text{joule} = \text{newton} \times \text{metre}.$$

But

$$\text{newton} = \text{kilogram} \times \text{metre}/(\text{second})^2.$$

Hence, taking J as the symbol of joule, we get

$$\mathrm{J} = \mathrm{kg\,m^2/s^2}, \quad \text{or} \quad \mathrm{kg\,m^2\,s^{-2}},$$

where, kg, m, and s are the symbols for kilogram, metre and second, respectively in SI.

Above is example of a coherent system of units. The unit of speed in kilometre per hour is not a coherent unit as

$$1\,\mathrm{km/h} = 1{,}000\,\mathrm{m}/3{,}600\,\mathrm{s} = (1/3.6)\,\mathrm{m/s}.$$

2.3.5 Properties of Units of Measurement

A unit of measurement is a scalar quantity that is defined and adopted by convention. Any quantity can be compared to its corresponding unit and their ratio is always a pure number. Units of measurement have the following properties:

1. They are designated by conventionally assigned names and symbols.
2. Measurement units of quantities of the same dimension may be designated by the same name and symbol even when the quantities are not of the same kind. For example, joule per kelvin (J/K) is a measurement unit of heat capacity and a unit of entropy. But heat capacity and entropy are generally not considered to be quantities of the same kind. However, in some special cases, the use of names is restricted to specific kinds of quantities only. For example, the unit 1/s is called hertz when used for frequencies and becquerel when used for activities of radionuclide.
3. Measurement units of quantities of dimension one are numbers. In some cases, these units are given special names, e.g. radian, steradian and decibel, or are expressed by quotients such as millimole per mole equal to 10^{-3} and microgram per kilogram equal to 10^{-9}.
4. For a given quantity, the short term "unit" is often combined with the quantity name, such as "mass unit" or "unit of mass" [9].

2.3.6 Coherent Derived Unit

The derived unit for a given system of quantities and for a chosen set of base units is a product of integral exponents of base units with no other proportionality factor than one.

1. Coherence can be determined only with respect to a particular system of quantities and a given set of base units. For example, if the metre, the second, and the mole are base units, the metre per second is the coherent derived unit of velocity when velocity is defined by the quantity equation $v = \mathrm{d}r/\mathrm{d}t$. Similarly, the mole per cubic metre is the coherent derived unit of the amount-of-substance concentration, when amount of substance concentration is defined by the quantity equation c/V. But the kilometre per hour and the knot, as given in examples of derived units, are not coherent derived units in SI.

2. A derived unit can be coherent with respect to one system of quantities, but not to another.

For example, as said earlier, the centimetre per second is the coherent derived unit of speed in the CGS system of units but not in the SI.

3. The coherent derived unit for every derived quantity of dimension one, in a given system of units, is the number one, symbol 1. The name and symbol of the measurement unit one are generally not indicated.

2.4 Quantity of Dimension 1 or Dimensionless Quantity

Before we discuss these quantities, we may like to know a little more about dimensions.

2.4.1 Dimension of a Quantity

A quantity Q is expressed as a product of several base quantities with symbols as given in column five of Table 2.1 as

$$Q = L^{\alpha} M^{\beta} T^{\gamma} I^{\delta} \Theta^{\varepsilon} N^{\zeta} J^{\eta},$$

where α, β, γ, δ, ε, ζ and η are exponents of the base quantities which may be any integer or zero. These are called the dimensional exponents.

There are two versions of the definition of dimension, each equally prevalent. According to one version, dimensions of a quantity are the powers to which base quantities must be raised to represent that quantity. That is, powers (exponents) of base quantities like α, β, γ, δ, etc. are the dimensions of the quantity Q. So a quantity having each exponent as zero is called dimensionless.

According to the second version, as given in [8], the dimension of a quantity is the expression representing the quantity in terms of base quantities raised to the integral powers. That is, dimension Q is expressed as

$$\dim Q = L^{\alpha} M^{\beta} T^{\gamma} I^{\delta} \Theta^{\varepsilon} N^{\zeta} J^{\eta}. \tag{2.1}$$

If each of the exponents α, β, γ, δ, etc. is zero, then the dimension of the quantity will be

$$\dim Q = L^0 M^0 T^0 I^0 H^0 N^0 J^0 = 1.$$

Thus a quantity having each of the exponents of the base quantity equal to zero has the dimension 1.

We see here that a quantity having each of the exponents equal to zero will be called dimensionless or of dimension 1, respectively, if the dimensions of the quantity are defined as exponents or as an expression consisting of the products of base units raised to an integral power. In literature, therefore, we may come across both words, namely "dimensionless" and "of dimension 1", for one and the same quantity.

2.4.2 Quantities of Dimension 1 or Dimensionless Quantities

As we have seen, there are certain quantities that may be called dimensionless or have a dimension 1. Roughly, the aforesaid quantities fall into three categories:

1. Many quantities are the ratios of two quantities of the same kind. Examples of such quantities are angle, solid angle, refractive index, relative permeability, friction factor, Neper and decibel.
2. Some quantities are defined as a complex product of simpler quantities in such a way that when each simpler quantity is expressed in terms of base units, the algebraic sum of exponents of each base unit becomes zero; hence the quantity is dimensionless or of dimension 1 depending upon the convention followed. For example, Reynolds number (R_e) is defined as

$$R_e = \rho v l/\eta,$$
$$[R_e] = ML^{-3}LT^{-1}L/ML^{-1}T^{-1} = M^0L^0T^0 = 1.$$

3. There is yet another class of quantities which represent a count, such as a number of molecules, degeneracy (number of energy levels) and partition function in statistical thermodynamics, etc., that are also called quantities of dimension 1 or dimensionless quantities.

All of these quantities describe a dimension one or dimensionless, and have the coherent SI unit 1. Their values are simply expressed as numbers and, in general, the unit 1 is not explicitly shown. In a few cases, however, special names are given to such units, mainly to avoid confusion between some compound derived units involving such quantities, for example radian, steradian and refractive index.

To summarize, we may state the following:

1. The term "dimensionless quantity" is commonly used for historical reasons. It stems from the fact that exponents are zero in the symbolic representation of the dimension for such quantities.

The term "quantity of dimension one" reflects the convention in which the symbolic representation of the dimension is as indicated in (2.1) above. The symbol for such quantities is 1 [7].

2. The measurement units and values of quantities of dimension one are numbers, but such quantities convey more information than a number.

3. Some quantities of dimension one are defined as the ratios of two quantities of the same kind.

Examples are plane angle, solid angle, refractive index, relative permeability, mass fraction, friction factor and Mach number.

4. Quantities of dimension one can also be numbers of entities.

Examples are number of turns in a coil, number of molecules in a given sample and degeneracy (number of energy levels) in quantum mechanics.

2.4.3 Ordinal Quantity

A quantity defined by a conventional measurement procedure, for which a total ordering relation, according to magnitude, with other quantities of the same kind is defined, but for which no algebraic operations among those quantities are defined, is called an ordinal quantity. Examples are:

(a) Rockwell C hardness
(b) Octane number for petroleum fuel
(c) Earthquake strength on the Richter scale

Notes

1. Ordinal quantities can enter into empirical relations only and have neither measurement units nor quantity dimensions.
2. Ordinal quantities are arranged according to ordinal quantity scales.

2.4.4 Quantity Scale, Measurement Scale

An ordered set of values of quantities of a given kind used in ranking, according to their magnitude, is a quantity scale.

Examples are:

1. Celsius temperature scale
2. Time scale
3. Rockwell C hardness scale

2.4.5 Ordinal Quantity Scale, Ordinal Scale

A conventional reference scale or a quantity scale, defined by formal agreement, on which only comparison of magnitude applies is known as ordinal quantity scale.

Examples are:

(a) Rockwell C hardness scale
(b) Scale of octane numbers for petroleum fuel

Notes

1. An ordinal quantity scale may be established by measurements according to a given measurement procedure.
2. Ordinal quantities are ordered on ordinal quantity scales.

2.4.6 Nominal Property

A property of a phenomenon, body or substance that can be identical but not a comparable property and cannot be ordered with it according to magnitude is a nominal property.

Examples are:

1. Sex of a human being
2. Colour of a paint sample
3. Colour of a spot test in chemistry
4. ISO two-letter country code
5. Sequence of amino acids in a polypeptide

2.5 Conversion Factor Between Units

The ratio of two measurement units of quantities of the same kind is known as the conversion factor.

For example:

$$\mathrm{km/m} = 1{,}000 \text{ and thus } 1\,\mathrm{km} = 1{,}000\,\mathrm{m}.$$

Note that the measurement units may belong to different systems of units but represent the same quantity.

Examples are:

1. 1 pound (avoirdupois) = 0.45359237 kg
2. h/s = 3,600 and thus 1 h = 3,600 s
3. (km/h)/(m/s) = (1/3.6) and thus 1 km/h = (1/3.6) m/s

2.6 Quantity Relations

2.6.1 Quantity Value

Quantity value is a number multiplied by its unit or its reference. To be more specific, we may say that a quantity value is the product of:

- A number and a measurement unit (the unit one is generally not indicated for a quantity of dimension one)

- A number and a reference to a measurement procedure
- A number and a reference material

Notes

1. The number can be real or complex.
2. A quantity value can be presented in more than one way.
3. In the case of vector or tensor quantities, each component has a value as defined above.

Examples are:

(a) Length of a given rod is 5.34 m or 534 cm
(b) Mass of a given body is 0.152 kg or 152 g
(c) Curvature of a given arc is 112 m^{-1}
(d) Celsius temperature of a given sample is −5°C
(e) Electric impedance of a given circuit element at a given frequency is $(7 + 3j)$ Ω, here j is the imaginary unit
(f) Refractive index of a given sample of glass is 1.32
(g) Rockwell C hardness of a given sample (150 kg load) HRC (150 kg) is 43.5
(h) Mass fraction of cadmium in a given sample of copper is 3 μg kg^{-1} or 3×10^{-9}
(i) Molality of Pb_2^+ in a given sample of water is 1.76 mmol kg^{-1}
(j) Force acting on a given particle is (−31.5; 43.2; 17.0) N

2.6.2 Numerical Quantity Value

A number in the expression of a quantity value, other than any number serving as the reference, is a numerical value of the quantity.

Notes

1. For quantities of dimension one, the reference is a measurement unit which is a number and this is not considered a part of the numerical quantity value.

   For example:

   In an amount-of-substance fraction equal to 3 mmol/mol, the numerical value is 3 and the unit is mmol/mol. The unit mmol/mol is numerically equal to 0.001, but this number 0.001 is not part of the numerical quantity value that remains 3.

2. For quantities that have a measurement unit (i.e. those other than ordinal quantities), the numerical value Q_n of a quantity Q is frequently denoted as $Q_n = Q/[\mathrm{Q}]$, where [Q] denotes the measurement unit of Q.

   For example:

   For a mass value of 5 kg, the numerical value in kilograms is $\{m\} = (5\,\mathrm{kg})/\mathrm{kg} = 5$. The numerical value of the same quantity depends upon the unit chosen for expressing the quantity. For example, for a body of 5.123 kg, the numerical value is 5.123 if the unit is kg; it will become 5,123 if unit is g.

2.6.3 Quantity Calculus

A set of mathematical rules and operations applied to quantities other than ordinal quantities is a quantity calculus. In quantity calculus, quantity equations are preferred to numerical value equations because quantity equations are independent of the choice of measurement units.

2.7 Units Used in Biology Biochemistry, Molecular Biology, Forensic Science for Biological Effects

Units for quantities that describe biological effects are often difficult to relate to units of SI because they typically involve weighting factors that may not be precisely known or defined, and which may be both energy- and frequency-dependent. These units, which are not SI units, are described briefly in this section.

There is a class of units for quantifying the biological activity of certain substances used in medical diagnosis and therapy that cannot yet be defined in terms of the units of SI. The mechanism of the specific biological effect that gives these substances their medical use is not yet sufficiently well understood. Hence it is difficult to quantify such units in terms of physico-chemical parameters. In view of their importance for human health and safety, the World Health Organization (WHO) has taken the responsibility for defining WHO International Units (IU) for the biological activity of such substances.

2.7.1 Photochemical or Photo-biological Quantities and Their Units

The photometric quantities and photometric units that are used at present for vision are well established and have been widely used for a long time. They are not affected by the rules in the following paragraphs. For all other photochemical and photobiological quantities the following rules shall be applied for defining the units to be used.

A photochemical or photo-biological quantity is defined in purely physical terms as the quantity derived from the corresponding radiant quantity by evaluating the radiation according to its action upon a selective receptor, the spectral sensitivity of which is defined by the actinic action spectrum of the photochemical or photo-biological effect considered. The quantity is given by the integral over the wavelength of the spectral distribution of the radiant quantity weighted by the appropriate actinic action spectrum. The use of integrals implicitly assumes a law of arithmetic addition for actinic quantities, although such a law is not perfectly obeyed by actual actinic effects. The action spectrum is a relative quantity; it is a quantity of dimension one, with the SI unit one. The radiant quantity has the radiometric unit corresponding to that quantity. Thus, following the rule for obtaining the SI unit for a derived quantity, the unit of the photochemical or photo-biological quantity is the radiometric unit of the corresponding radiant quantity. When giving a quantitative value, it is essential to specify whether a radiometric or actinic quantity is intended as the unit is the same. If an actinic effect exists in several action spectra, the action spectrum used for measurement has to be clearly specified.

This method of defining the unit should be used for photochemical or photo-biological quantities. The Consultative Committee for Photometry and Radiometry in its 9th meeting in 1977 recommended this method.

As an example, the erythemal effective irradiance E_{er} from a source of ultraviolet radiation is obtained by weighting the spectral irradiance of the radiation at wavelength λ by the effectiveness of radiation at this wavelength to cause an erythema, and summing over all wavelengths present in the source spectrum. This can be expressed mathematically as:

$$E_{\sigma} = \int E_{\lambda} s_{\mathrm{er}}(\lambda)\mathrm{d}\lambda,$$

where E_{λ} is the spectral irradiance at wavelength λ (usually reported using the SI unit $\mathrm{Wm^{-2}nm^{-1}}$), and $s_{er}(\lambda)$ is the actinic spectrum normalized to 1 at its maximum spectral value. The erythemal irradiance E_{er} determined in this way is usually quoted in the SI unit $\mathrm{Wm^{-2}}$.

2.8 Units Used in Photometry

2.8.1 Photometry

Optical radiation has the potential to cause chemical changes in certain living or non-living materials: this property is called activism, and radiation capable of causing such changes is referred to as actinic radiation. Actinic radiation has the fundamental characteristic that, at the molecular level, one photon interacts with one molecule to alter or break the molecule into a new molecular species. It is therefore possible to define specific photochemical or photo-biological quantities in terms of the result of optical radiation on the associated chemical or biological receptors. In some cases, the results of measurements of photochemical and photo-biological quantities of this kind can be expressed in terms of SI units. This is discussed briefly here.

In the field of metrology, the only photo-biological quantity that has been formally defined for measurement in the SI is for the interaction of light with the human eye in vision. An SI base unit, the candela, has been defined for this important photo-biological quantity. Several other photometric quantities with units derived from the candela have also been defined (such as the lumen and the lux defined and described in the following chapter).

2.8.2 Actinic Action Spectrum

Optical radiation can be characterized by its spectral power distribution. The mechanisms by which optical radiation is absorbed by chemical or biological systems are usually complicated, and are always dependent on the wavelength (or frequency). For metrological purposes, however, the complexities of the absorption mechanisms can be ignored, and the actinic effect is characterized simply by an actinic action spectrum linking the photochemical or the photo-biological response to the incident radiation. This actinic action spectrum describes the relative effectiveness of monochromatic optical radiation at wavelength λ to elicit a given actinic response. It is given in relative values, normalized to 1 for the maximum of efficacy. Usually actinic action spectra are defined and recommended by international scientific or standardizing organizations.

2.8.3 Types of Visions

For vision, two action spectra have been defined by the International Commission on Illumination (CIE) and endorsed by the CIPM: $\mathrm{V}(\lambda)$ for photopic vision and $\mathrm{V}'(\lambda)$ for scotopic vision. These are used in the measurement of photometric quantities and are an implicit part of the definition of the SI unit for photometry, the candela. Photopic vision is detected by the cones on the retina of the eye, which are sensitive to a high level of luminance L for $\mathrm{L} > \text{ca.}10\,\mathrm{cd\,m^{-2}}$ and are used in daytime vision. Scotopic vision is detected by the rods of the retina, which are sensitive to low-level luminance for $L < (\text{ca.}10^{-3}\mathrm{cd\,m^{-2}})$, used in night vision. In the domain between these levels of luminance both cones and rods are used, and this is described as mesopic vision.

Other action spectra for other actinic effects have also been defined by the CIE, such as the erythemal (skin-reddening) action spectrum for ultraviolet radiation, but these have not been given any special status within the SI.

For further details and definitions of photometric quantities and units, one may refer CIE publication [3], the IEC publication [4] or BIPM monograph [5].

2.9 Unit in the Field of Sound

Sound causes small pressure fluctuations in the air, superimposed on the normal atmospheric pressure, that are sensed by the human ear. The sensitivity of the ear depends on the frequency of the sound, but is not a simple function of either the pressure changes or the frequency. Therefore frequency-weighted quantities are used in acoustics to approximate the way in which sound is perceived. Such quantities with frequency as weight factors are employed, for example, in work to protect against hearing damage. The effects of ultrasonic acoustic waves pose similar concerns in medical diagnosis and therapy.

2.10 Units in the Field of Ionizing Radiations

Ionizing radiation deposits energy in irradiated matter. The ratio of deposited energy to mass is termed absorbed dose. High doses of ionizing radiation kill cells, and this is used in radiation therapy. Appropriate biological weighting functions are used to compare therapeutic effects of different radiation treatments. Low sub-lethal doses can cause damage to living organisms, for instance, by inducing cancer. Appropriate risk-weighted functions are used at low doses as the basis of radiation protection regulations.

2.11 SI Units in the Framework of General Relativity

The question of proper units is addressed in Resolution A4 adopted by the 21st General Assembly of the International Astronomical Union (IAU) in 1991 and by the report of the CCDS Working Group on the Application of General Relativity to Metrology [6].

The definitions of the base units of SI were adopted in a context that takes no account of relativistic effects. When such account is taken, however, it is clear that the definitions apply only in a small spatial domain sharing the motion of the standards that realize them. These units are known as proper units; they are realized from local experiments in which the relativistic effects that need to be taken into account are those of special relativity. The constants of physics are local quantities and their values are expressed in proper units.

Physical realizations of the definition of a unit are usually compared locally. For frequency standards, however, it is possible to make such comparisons at a distance by means of electromagnetic signals. To interpret the results the theory of general relativity is required since it predicts, among other things, a relative frequency shift between standards of about 1 part in 10^{16} per m of altitude difference at the surface of the earth. Effects of this magnitude cannot be neglected when comparing the best frequency standards.

References

1. International Standard ISO 31, Quantities and units, subsequently replaced by ISO/IEC 80000,
2. International Standards ISO 704, ISO 1087–1, and ISO 10241.
3. International Lighting Vocabulary, CIE publication 17.4 (1987)
4. International Electro-technical Vocabulary, IEC publication 50, chapter 845: lighting

5. BIPM, *Principles Governing Photometry*, 32 pp (1983)
6. B. Guinot, Metrologia 34, 261 (1997)
7. International Standards ISO 31–0:1992, sub-clause 2.2.6
8. BIPM, *Le Systeme Internatinal d'Unites*, 8th edn. (BIPM, Sevres, France, 2006)
9. BIPM, *International Vocabulary of metrological terms*, (VIM 3rd Edition), 2006

Chapter 3
Various Systems of Units

Notations used in the chapter

Dimension of base quantities in upper case letters
Dimension of any other quantity Q by symbol [Q]
Unit of any other quantity Q by $\{Q\}$
SI means International System of Units
For symbols of SI base units, symbols as prescribed in the BIPM brochure on SI (International System of units)

3.1 Introduction

We have seen in the previous chapter that a proper system of base quantities and their respective units, agreed by convention with a set of proper algebraic relations, can be used to represent other quantities and their respective units. In the following paragraphs, I first intend to give relations connecting other quantities with base quantities and then various systems of base quantities. A few examples will also be given to make use of relations to arrive at expressions of units of other quantities in terms of units of base quantities. After going through this chapter, you will notice that a set of only four base quantities and their respective base units are essentially required to express all other units of measurements.

3.2 Relations Between the Quantities

Taking length, mass, time and electric current as the base quantities of the system, some relations expressing quantities in terms of these base quantities are given below. There are two types of quantities in any measuring system: some are directly defined in terms of base quantities and others through some phenomena.

3.2.1 Derived Quantities by Definition

Normally a quantity and its unit are expressed in terms of base quantities and their respective units without giving any consideration to the directional properties. Vector quantities and their units simply represent the scalar product of base quantities and their respective units.

1. Area is length square.
2. Volume is length cube.
3. Density is mass divided by volume.
4. Concentration is amount of substance in unit volume of solution.
5. Velocity is rate of change of displacement with respect to time.
6. Acceleration is rate of change in velocity with respect to time.
7. Force is product of mass and acceleration and is in the direction of acceleration.
8. Surface tension is force per unit length.
9. Pressure is force per unit area.
10. Work or energy of any kind is force multiplied by displacement in the direction of force.
11. Moment of force is force multiplied by the shortest distance from the point about which moment is taken to the line of action of force.

Note that both energy and moment of force are essentially the product of force and distance. Hence, the unit of each is the product of units of distance and force.

12. Power is rate of change of energy with respect to time.
13. Wave number is number of waves per unit length.
14. Vergency or power of an optical system or lens is the reciprocal of its focal length.
15. Frequency is number of vibration per unit time.
16. Temperature is average kinetic energy of a molecule of a gas.

However, for convenience, we may choose a separate unit for temperature. For example, kelvin is chosen as the unit of temperature in SI.

17. Heat capacity is energy required to change its temperature through 1 K. (Earlier this term used to be known as thermal capacity.)
18. Specific heat capacity is energy exchanged by unit mass of homogeneous substance to change its temperature through one kelvin. (Earlier this used to be called specific heat.)
19. Latent heat is heat contained in unit mass of a homogeneous substance.
20. Specific energy is heat exchanged by unit mass of the substance for transition from one phase to another at the temperature of its changing phase. No change in temperature occurs during the phase transition. (Earlier this used to be called latent heat).
21. Entropy of a system is ratio of quantity of heat contained and thermodynamic temperature.
22. Specific entropy is ratio of entropy of a system of a homogeneous mass to the mass of the system.

23. Radiant intensity of a point source is power radiated per unit solid angle.
24. Irradiance is ratio of power irradiated per unit area.
25. Radiance of a source is power radiated in a unit solid angle per unit area.
26. Luminance is intensity of illumination per unit area.
27. Luminous flux of a point source is product of luminous intensity and the area of illumination.
28. Illuminance is luminous flux per unit area.

Note: Definitions from 23 to 28 are concerned with radiometry and photometry. However, all radiometric units are in terms of base units of length, mass, time and solid angle. But photometric units involve the process of vision, so they depend upon the sensitivity of the eye to radiations of different frequencies in the visible region. But radiometry is for the entire range of electromagnetic radiations except the visible region.

For photometry, the base quantity is the luminous intensity with SI unit candela, which is a certain fraction of the power of a monochromatic source of a certain frequency in a unit solid angle. The candela is defined at the frequency at which the eye has the maximum response. For radiometry the starting quantity is power with unit watt. Radiant intensity is the power emitted in a unit solid angle. Its counterpart in photometry is the intensity of illumination with unit candela. Other units are shown in Table 3.1.

29. Activity in relation to radioactive substances is number of transformations or transitions per second.
30. Absorbed dose is energy received per unit mass of the absorber.
31. Molar energy is energy per mole of substance.
32. Molar entropy is ratio of molar energy and temperature of the substance.
33. Molar heat capacity is energy required to change 1 K of temperature of one mole of substance.

Table 3.1 Units in photometry and radiometry

Quantity	Radiometry	Photometry		
		Point source		
Power	Radiant power	watts (W)	Luminous power	Lumens $\text{lm} = \text{cd} \cdot \text{sr}$
Directionality Power per unit solid angle	Radiant intensity	W/sr	Intensity of illumination	cd
		Extended source		
Power per unit area	Radiance	W/m^2	Luminance	lm/m^2
Directionality	Power per unit area and solid angle	$\text{W/m}^2\text{sr}$	Power per unit area and solid angle	cd/m^2
		Looking at the illuminated object		
Power/area	Irradiance	W/m^2	Illuminance	$\text{lm/m}^2 = \text{lux}$

3.2.2 Derived Quantities by a Phenomenon

1. Moment of inertia of a body is the product of its mass and the square of its radius of gyration, i.e. mass multiplied by square of length (phenomenon of rotation).
2. Electric charge is the product of electric steady current and time during which current has passed.
3. Electric potential or potential difference or electromotive force is the ratio of power and electric current. Electric potential at a point is the work done on a unit positive charge to bring it from infinity to the point. This definition was used in defining the unit of electric field.
4. For other electrical units, Ohm's Law $V = IR$ is used.
5. Capacitance of a condenser is the ratio of charge and the potential difference across it.
6. Rate of change of magnetic flux Φ with time is the induced electromotive force (emf), so total magnetic flux is the product of emf produced and duration of time for the change, giving

   $$\Delta\Phi/\mathrm{d}t = \text{voltage}.$$

   Units of $\Delta\Phi$ and Δt are the same as that Φ and time respectively; hence

   $$\Phi = \text{voltage} \cdot \text{time}.$$

7. Magnetic moment is the product of current flowing in a circular loop of wire and its area, or pole strength multiplied by distance between the two poles.
8. Magnetic pole strength is the magnetic moment divided by the distance between the poles. Pole strength can also be derived from the formula corresponding to Coulomb's Law, namely

   $$F = \mu_0 N_1 N_2 / 4r^2.$$

   Here F is force, μ_0 is permeability of free space, N_1 and N_2 are pole strengths, and r is the distance between the two poles.
9. Magnetic field strength $= P \cdot I/r$, magnetic field strength from a thin wire of uniform cross-section and of infinite length along the surface of a cylinder of radius r and having a wire as its axis $P \cdot I/r$. P is constant of proportionality $\mu_0/4\pi$ in SI.
10. Permeability of free space is calculated with the formula for force between the two parallel conductors of infinite length and negligible cross section and from the given definition of electric current.
11. Electrical field strength $= \mathrm{dV}/\mathrm{d}\mathbf{r}$ vector $\mathbf{r}$ is in the direction of the electric field.
12. Inductance $\cdot$ current $=$ flux(Φ).
13. Inductance $= \Phi/$Current
14. Permittivity is derived by using the relation for capacity of a parallel plate condenser giving permittivity = capacitance multiplied by the distance between the two plates and divided by the common area between the plates, i.e. Permittivity = Capacity/Length.

15. Permeability is related to permittivity and velocity of light by the following relation

$$\text{Permeability} \times \text{permittivity} = 1/(\text{velocity of light})^2$$

Giving

$$(\text{permeability}) = 1/(\text{permittivity}) \cdot (\text{velocity of light})^2.$$

16. Thermodynamic temperature: As per kinetic theory of gasses, temperature is the average kinetic energy of a molecule. That is, a unit of temperature is the same as a unit of energy.
17. Thermal conductivity k is given by

$$\frac{Energy}{Time} = \frac{k \cdot difference\ in\ temperature \cdot area\ of\ cross - section}{length\ of\ the\ conducting\ rod}$$

giving the dimension of k in SI units as

$$[k] = \frac{ML^2T^{-3} \cdot L}{(ML^2T^{-2} \cdot L^2)} = T^{-1}L^{-1}.$$

However, in SI units, temperature is a base unit called as kelvin with the symbol (K). Replacing $\mathrm{ML^2T^{-2}}$ by K,
we get
SI unit of thermal conductivity as watt per metre kelvin and in terms of base units as

$$\mathrm{m\ kg\ s^{-3}\ K^{-1}}.$$

18. Inductance L is defined by the formula:

$$V = L\ \mathrm{dI/dt},\ \text{i.e.}$$
$$\{L\} = \mathrm{V/(A/s)} = \mathrm{VA^{-1} \cdot s^{-1}}.$$

19. Gravitational constant G is obtained by the formula: $\mathrm{F = G\ m_1 m_2/r^2}$.
20. Dynamic viscosity is the force divided by the product of area of cross section of the flowing fluid and the velocity gradient normal to the direction of flow. The force is in the direction of fluid flow and tangential to the layers having a relative motion. Poiseulle's formula may also be used to drive the units of dynamic viscosity. $Q = \pi pa^4/8\eta\,\mathrm{l}$, where Q is volume flow rate, p is pressure, a is the radius of the capillary tube of length l, η is the dynamic viscosity of the liquid flowing. Giving us $\{\eta\} = \mathrm{Pa \cdot m^4/m \cdot m^3 \cdot s^{-1} = Pa \cdot s}$.
21. Kinematic viscosity ν is the ratio of dynamic viscosity to density of the fluid, giving Us $\{\nu\} = \{\eta\}/\{\text{density}\} = \mathrm{Pas = kg \cdot m^{-1}s^{-2}s/kg\,m^{-3} = m^2s^{-1}}$.

3.3 Three-Dimensional Systems of Units

In general, there are two types of systems of units, namely three-dimensional systems of units and four-dimensional system of units. Using the 3-dimensional system of units, one can express all the mechanical quantities in a unique way but there are some ambiguities in the case of electrical, magnetic and electromagnetic quantities. One example is that of the CGS system described in Sect. 3.3.2.

3.3.1 Gauss System

The need for a coherent system of units which could be accepted internationally was realized sometime before the French revolution. The creation of the decimal and metric systems at the time of the French revolution and the subsequent deposition of two platinum standards representing "metre" and "kilogram" on 22 June 1799 in Paris can be seen as the first steps in the development of the present international system of units. Carl Friedrich Gauss, in 1832, proposed an absolute system of quantities of mass, length and time with their respective units of kilogram, metre and second. Units of the system were based upon the metric standards established at that time. Gauss advocated for the unit of time as defined in astronomy and strongly promoted the application of the 'metric system'. The three base quantities of mass, length and time can represent all mechanical quantities, but this gives rise to ambiguous systems of units in the field of electrostatics, electromagnetic and practical units.

3.3.2 CGS System

W. E. Weber, in 1851, proposed a three-dimensional coherent system of units in which units of length, mass and time, respectively, were centimetre, gram and second. By applying separately the inverse square law of force between electric charges and magnetic poles, he arrived at two sets of electrostatic and electromagnetic units. Prefixes ranging from micro to mega were part of the system. For the purpose of electrical measurements, the British Association for Advancement of Science (BAAS), in 1863, defined ohm, ampere and volt as, respectively, 10^9, 10^{-1} and 10^8 times that of the CGS electromagnetic units.

In 1881 the BASS along with International Electrical Congress (IEC) approved a mutually coherent set of practical units. Besides having centimetre, gram and second as units, the system also included ohm for electrical resistance, volt for electromotive force and ampere for electric current. The unit of ohm was taken as the resistance offered by a column of mercury of uniform cross section having the length of 106.300 cm and mass 14.521 g. The ampere was defined as the steady electric current, which when passed through a silver nitrate solution deposits silver at the rate of 0.001 11,800 gs^{-1}. The volt was taken as the pressure to produce a current of one ampere through a resistance of one ohm.

3.3.3 FPS System

The FPS system is one of the oldest systems of measurement of modern times, which is still being used in countries like Great Britain and the United States of America. Like the CGS system it is also based on three quantities, namely, length, mass and time. Only the units of length and mass are different from those in the CGS system. The unit of length and mass are, respectively, imperial yard and pound avoirdupois. The standard of length in the British System of measurement is the yard, a length that has been preserved almost unchanged from the time of the King Edward I. The Ulna was the predecessor of the yard, which gave its name to the yard of Edward I.

The actual standard bar created by Edward I was lost and the earliest authentic standard of a yard in brass which was available dates back to 1496. This yard differs from the imperial yard by 0.037 in. The yard constructed by the parliamentary Committee in 1758 was given the name imperial yard and legalized by the Act of 1824. It was also made in brass and was within 0.01 in. of the present imperial yard. The present legal standards were constructed after the destruction due to a fire in 1834. To avoid the problem of destruction of the standards due to any reason, the copies of this yard have been deposited at several places like the House of Parliament, Greenwich Royal Observatory, Royal Society of London, Royal Mint and Board of Trade.

The present imperial yard is a solid square bar, 38 in. long and 1^2 in. in transverse section. The bar is made from gunmetal (bronze). Near each end is a cylindrical hole half an inch deep. The distance between the centres of both the holes is 36 in. At the bottom of each hole, a smaller gold plug of 0.1 in. diameter is inserted. Upon the polished surface of this gold plug there are three transverse lines separated by 0.01 in. and two lines with almost the same separation but parallel to the axis of the bar (Fig. 3.1). The measure of the length of the yard is the distance between the two central transverse lines on each end. The midpoints of the transverse central lines lying in between the two axial lines are to be always used to define the yard.

For further information on old length measures, one may consult [1].

The history of the standard of mass is similar to that of the yard. At different times "pounds" of various kinds have been used for different purposes. The Royal Mint of the United Kingdom also used mass standards for minting coins. The term "sterling" is the earlier name of penny, which had a pennyweight of 24 grains. One grain is taken as 1/7000 part of the present avoirdupois pound. It took very much longer for the standard of mass to be coordinated and regulated than that of length. At one time, five different pounds were in use, which were then reduced to two, namely Troy pound and Avoirdupois pound. The Act of 1855 legalized these standards and at the same time reversed the relative positions of avoirdupois and troy weight. Avoirdupois pound was made the legal standard for all general purposes and troy pound was reserved for weighing gold, silver and precious stones as well as the

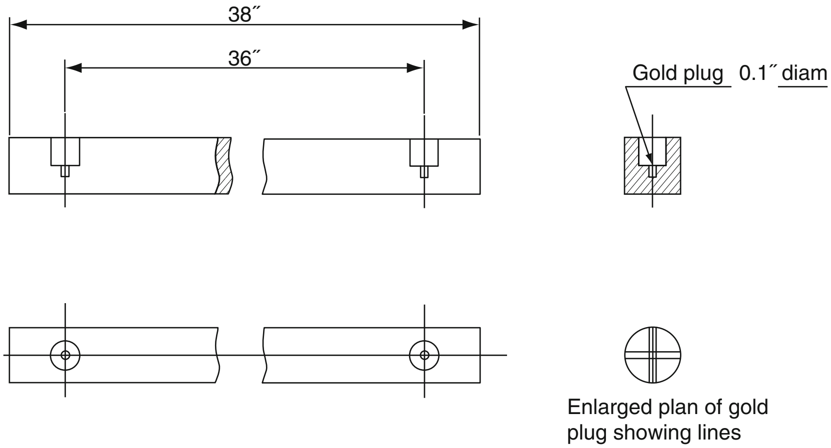

Fig. 3.1 Imperial yard

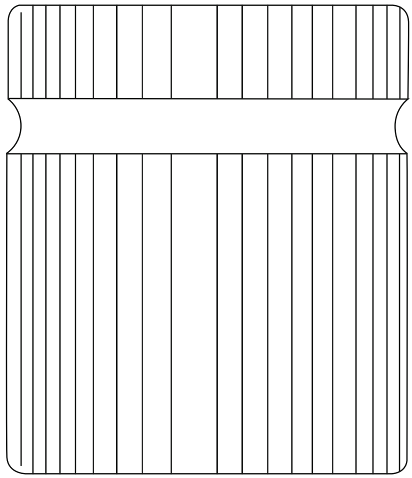

Fig. 3.2 Imperial pound

retail sale of drugs. The Act of 1866 transferred custody of the imperial standards from the Comptroller-General of the Exchequer to the Board of Trade. The Weights and Measures Act of 1878 retained the part pertaining to standards of the Act of 1855 but abolished the troy pound, though the troy ounce of 480 grains remained the legal unit for bullion trade.

The imperial pound avoirdupois is the mass of a certain cylinder of pure platinum. The cylinder is about 1.35 in. high and 1.15 in. in diameter. A groove around the cylinder has been cut about 0.34 in. from the top (Fig. 3.2) so that the prongs of an ivory fork fit into it while lifting the cylinder. This standard was prepared not from the previously existing avoirdupois standard but from certain authenticated copies of the old brass troy pound of 1,758, which, up to the fire in 1834, occupied the position of principle standard [2].

3.4 Four-Dimensional Systems of Units

The minimum number of base quantities in a measuring system which can take care of all the measurable quantities is four. This four-dimensional system is capable of expressing all quantities in all fields of measurements in terms of four base units. The units of all quantities defined in such a system are unique.

3.4.1 Giorgi System

We will now discuss the systems of units having four base quantities. The first to suggest such a system was G. L. T. Giorgi. In 1902, some 30 years after the Metric Treaty was signed, he suggested that in addition to metre, kilogram and second, ampere – the unit of current – may be taken as the fourth unit, and he named it fourth dimension. The system started to be known as MKSA system.

3.4.2 Maxwell System

Clarks James Maxwell [4] proposed a system of units in which permeability of free space was taken as unity and other units of length, mass and time were as follows:

$$\text{Length} = \text{Earth quadrant} = 10^7\ \text{m},$$
$$\text{Mass} = 10^{-11}\ \text{g},$$
$$\text{Time} = 1\ \text{s}.$$

3.4.3 Hartree System

D. R. Hartree [5], in 1927, proposed a system of units based upon physical constants. The charge and rest mass of an electron were taken as units for electric charge and mass respectively. The units of length and time were taken, respectively, as the Bohr radius and the reciprocal of angular velocity of the electron. The aforesaid units in terms of SI units are equivalent to:

Unit of charge = e (charge on an electron) = 160×10^{-21}C

Unit of mass = rest mass of electron = 900×10^{-33}kg

Unit of length = a (Bohr radius) = 53×10^{-12}m

Unit of time = Reciprocal of angular velocity of electron = $1/4\pi Rc = 24 \times 10^{-8} \times s$

The basic problem with this system is that the units are too small to express quantities encountered in daily life.

3.4.4 Units for Atomic and Molecular Measurements

Around the 1950s, it was felt that comparison of results in quantum mechanical calculations was difficult because of the multiplicity of units and symbols used. For example, energy used to be measured in kcal/mol (kilocalories per mol) or in eV (electron volts). Shull and Hall [6] suggested three base units from which other units could be derived: mass of electron (symbol m), charge of electron (symbol e) and rationalized Planck's constant (symbol $\hbar$). Other units like those of length and energy were taken as derived units. The unit of length (symbol b) was taken as the Bohr radius and expressed as $b = \hbar^2/me^2$. The unit of energy was named Hartree with the symbol H, given as $H = me^4/\hbar^2$.

Another most commonly used unit in atomic physics was electric moment, which was the product of Bohr's radius b and charge of the electron e given by $eb = \hbar^2/me$.

3.4.5 McWeeny System

In 1973, McWeeny [7], in addition to e, m and h as base units, introduced permittivity as another base quantity with the symbol κ_o related to permittivity of free space ϵ_o by the relation: $\kappa_o = 4\pi\epsilon_o$.

The values of each base unit and other important derived units in terms of SI units are given in Table 3.2. The numerical values given in column 3 of Table 3.2 are based on the values of constants as were available in 1973.

3.4.6 Ohm, Ampere, Second and Metre System

M. Tarbouriech [8], in 1945, suggested a system of units comprising ohm, ampere, second and metre. The units of ohm and ampere chosen were the same as defined by BASS in 1881. The second – the unit of time – was taken as 1/86,400 of the mean solar day.

3.4.7 Force, Length and Time System

M. Loren Bullock [9] discussed the possibility of replacing the concept of mass with that of weight. Instead of taking the unit of mass, he suggested that gravitational pull on one unit of mass may be taken as one unit of force; other units of length and time remained unchanged in his system. The unit of force was defined as the force experienced by a body of mass equal to one kilogram at a specific location where the value of g – the acceleration due to gravity – was $9.80665\ \mathrm{ms^{-2}}$. Any location at sea level having a latitude close to 45° North will have the required value of g. The system became quite prevalent in the engineering industry and gave birth to units

Table 3.2 Equivalents of McWeeny base and other units in SI

	Base units	
Quantity	Natural units	Value in SI units
Mass M	m = mass of one electron	9.1091×10^{-31} kg
Charge Q	e = charge on the electron	1.60210×10^{-19} C
Action	$\hbar$ = rationalised Planck's constant $\hbar/2\pi$	1.05450×10^{-34} Js
Permittivity	$\kappa_o = 4\pi\varepsilon_o$	$4\pi \times 8.8542 \times 10^{-12}$ Fm^{-1}
	Derived units (mechanical)	
Quantity	In terms of base units	Value in SI units
Length	1 bohr = $\kappa_o\hbar^2/me^2$	5.29167×10^{-11} m
Time	$\kappa_o^2\hbar^3/me^4 = \hbar/E_o$	2.41889×10^{-17} s
Velocity	$e^2/\kappa_o\hbar$	2.18764×10^{6} m · s^{-1}
Force	$m^2e^6/\kappa_o^3\hbar^4$	8.23831×10^{-8} N
Energy	$me^4/\kappa_o^2\hbar^2$	4.35944×10^{18} J
Power	$m^2e^8/\kappa_o^4\hbar^5$	1.80225×10^{-1} W
	Derived units (electrical)	
Current	$me^5/\kappa_o^2\hbar^3$	6.62329×10^{-3} A
Potential	$me^3/\kappa_o^2\hbar^2$	27.2108 V
Capacitance	$\kappa_o^2\hbar^2/me^2$	5.88774×10^{-21} F
Field strength	$m^2e^5/\kappa_o^3\hbar^4$	5.14220×10^{11} V · m^{-1}
Dipole moment	$\kappa_o\hbar^2/me$	8.47778×10^{-30} C · m
Displacement	$m^2e^5/\kappa_o^2\hbar^4$	57.2142C · m^{-2}

like kilogram force, gram force, abbreviated respectively as kgf and gf. In Britain the unit of force was pound force as the unit of mass in the country was pound.

3.4.8 System in Terms of Universal Constants (G, H, E and Q)

Bruno F. Ludovici [10] proposed a system of units which had four quantities: G, gravitational constant; E, permittivity of free space; H, permeability of free space; and Q, electric charge on one electron. The great advantage, according to him, was that defined unit values for each of the four standards were absolutely constant and could be regarded as true standards representing infinite accuracy (zero uncertainty) at all times. The unit values of G, E, H and Q in terms of SI units will approximately be given as:

$$1\,\{G\} = 6.670 \times 10^{-11} m^3 kg^{-1} s^{-2}$$
$$1\,\{E\} = 8.855 \times 10^{-12} m^{-3} kg^{-1} s^2 C^2$$
$$1\,\{H\} = 1.256 \times 10^{-6} m\,kg\,C^{-2}$$
$$1\,\{Q\} = 1.601 \times 10^{-19} C$$

One may see that from the above equations that dimension of

$$\text{Length is } [G]^{1/2}[E]^{1/2}[H][Q]$$
$$\text{Mass is } [G]^{-1/2}[E]^{-1/2}[Q]$$
$$\text{Time is } [G]^{1/2}[E][H]^{3/2}[Q]$$

To distinguish his units of length, mass, time and charge from the SI system he used the prefix basic for each unit. Thus, the units of length, mass, time and charge in the proposed system of measurements were called basic metre, basic kilogram, basic second and basic coulomb. With these equations, one can express the SI units in terms of the proposed units of length, mass, time and charge.

$$1\text{ m} = 2.05 \times 10^{35}\text{ basic metres}$$
$$1\text{ kg} = 1.515 \times 10^{8}\text{ basic kilograms}$$
$$1\text{ s} = 6.15 \times 10^{43}\text{ basic seconds}$$
$$1\text{ C} = 6.25 \times 10^{18}\text{ basic coulombs}$$

Inversion of the above equations will express the proposed units of length, mass, time and charge in terms of SI units as

$$1\text{ basic metre} = 4.88 \times 10^{-36}\text{ m}$$
$$1\text{ basic kilogram} = 6.60 \times 10^{-9}\text{ kg}$$
$$1\text{ basic second} = 1.628 \times 10^{-44}\text{ s}$$
$$1\text{ basic coulomb} = 1.60 \times 110^{-19}\text{ C}$$

3.4.9 System in Terms of Electric Charge, Flux, Length and Time

Professor P. Kalantaroff [11], in 1929, proposed a system of quantities comprising length, time, charge and flux. However, as the proposal was in German it could not get worldwide publicity. Professor Kinitsky [12] again described his work. The good thing was that they adopted the then existing units for their proposed base quantities. That is, metre was taken for length, second for time, coulomb for charge and weber for flux. Taking L, T, Q and Φ as the respective symbols of length, time, charge and flux, the other quantities may be expressed as given in column 3 of Table 3.3.

3.4.10 System in Terms of L, M, T and R

A system of units, similar to that of Giogi, given in Sect. 3.4.1, may be based on four base quantities of length with symbol L, mass with symbol M, time with symbol T and resistance with symbol R. Here the quantity of electric current has been replaced by resistance.

Table 3.3 Derived units expressed in terms of two sets of base units

Quantity	Relation used	In terms of {L}, {T}, {Φ}, {Q}	In terms of {M}, {L}, {T}, {R}
Length	Base quantity	{L}	{L}
Time	Base quantity	{T}	{T}
Electric charge	Base quantity	{Q}	$\{M\}^{1/2}\{L\}\{T\}^{-1/2}R\}^{-1/2}$
Magnetic flux	Base quantity (column 3)	{Φ}	$\{M\}^{1/2}\{L\}\{T\}^{-1/2}\{R\}^{1/2}$
Resistance	Base quantity (column 4)	–	{R}
Displacement	Charge/Area	$\{Q\}\{L\}^{-2}$	$\{M\}^{1/2}\{L\}^{-1}\{T\}^{-1/2}\{R\}^{-1/2}$
Current	Charge/time	$\{Q\}\{T\}^{-1}$	$\{M\}^{1/2}\{L\}\{T\}^{-3/2}\{R\}^{-1/2}$
Magnetic field Intensity	$B = \mu_o I/4r$	$\{Q\}\{L\}^{-1}\{T\}^{-1}$	$\{M\}^{1/2}\{R\}^{-1/2}\{T\}^{-3/2}$
Induction	Magnetic flux/ area	$\{\Phi\}\{L\}^{-2}$	$\{M\}^{1/2}\{L\}^{-1}\{T\}^{-1/2}\{R\}^{1/2}$
Electric voltage	$d\Phi/dt$	$\{\Phi\}\{T\}^{-1}$	$\{M\}^{1/2}\{L\}\{T\}^{-3/2}\{R\}^{1/2}$
Electric field intensity	dV/dr	$\{\Phi\}\{L\}^{-1}\{T\}^{-1}$	$\{M\}^{1/2}\{T\}^{-3/2}\{R\}^{1/2}$
Conductance	1/resistance	$\{Q\}\{\Phi\}^{-1}$	$\{R\}^{-1}$
Resistance	Voltage/current	$\{\Phi\}\{Q\}^{-1}$	–
Inductance	Magnetic flux/current	$\{Q\}^{-1}\{\Phi\}\{T\}$	{R}{T}
Capacitance	Charge/voltage	$\{Q\}\{\Phi\}^{-1}T^{-1}\}$	$\{T\}\{R\}^{-1}$
Permittivity or Dielectric constant	Capacity of a parallel plate condenser	$\{Q\}\{\Phi\}^{-1} \cdot \{L\}^{-1}\{T\}$	$\{R\}^{-1}\{L\}^{-1}\{T\}$
Permeability	$\mu_o \varepsilon o = 1/c^2$	$\{Q\}^{-1} \quad \Phi\{L\}^{-1}\{T\}$	$\{R\}\{L\}^{-1}\{T\}$
Plank's constant	Action = Energy · time	{Q}{Φ}	$\{M\}\{L\}^2\{T\}^{-1}$
Energy	Energy = hν	$\{Q\}\{\Phi\}\{T\}^{-1}$	$\{M\}\{L\}^2\{T\}^{-2}$
Power	Energy/Time	$\{Q\{\Phi\}\{T\}^{-2}$	$\{M\}\{L\}^2\{T\}^{-3}$
Force	Energy/distance	$\{Q\}\{\Phi\{T\}^{-1}\{L\}^{-1}$	$\{M\}\{L\}\{T\}^{-2}$
Mass	Force/Acceleration	$\{Q\}\{\Phi\}\{T\}\{L\}^{-2}$	{M}
Momentum	Mass · Velocity	$\{Q\}\{\Phi\}\{L\}^{-1}$	$\{M\}\{L\}\{T\}^{-1}$
Moment of inertia	Mass · Length^2	{Q}{Φ}{T}	$\{M\}\{L\}^2$
Angular momentum	Moment of inertia · Angular velocity	{Q}{Φ} = h	$\{M\}\{L\}^2\{T\}^{-1}$
Pressure	Force/Area	$\{Q\}\{\Phi\}\{T\}^{-1}\{L\}^{-3}$	$\{M\}\{L\}^{-1}\{T\}^{-2}$
Torque	Moment of inertia · Angular acceleration	$\{Q\}\{\Phi\}\{T\}^{-1}$	$\{M\}\{L\}^2\{T\}^{-2}$
Mass density	Mass/volume	$\{Q\}\{\Phi\}\{T\}\{L\}^{-5}$	$\{M\}\{L\}^{-3}$
Gravitational Constant	G = Force · Distance^2/Mass^2	$\{Q\}^{-1}\{\Phi\}^{-1}\{T\}^{-3}\{L\}^5$	$\{M\}^{-1}\{L\}^3\{T\}^{-2}$

(continued)

Table 3.3 (continued)

Quantity	Relation used	In terms of {L}, {T}, {Φ}, {Q}	In terms of {M}, {L}, {T}, {R}
Specific weight		$\{Q\}\{\Phi\}\{L\}^{-4}\{T\}^{-1}$	$\{M\}\{L\}^{-2}\{T\}^{-2}$
Magnetic moment	Magnetic flux · Distance	$\{\Phi\}\{L\}$	$\{M\}^{1/2}\{L\}^{2}\{T\}^{-1/2}\{R\}^{1/2}$
Electric dipole moment	Charge · Distance	$\{Q\}\{L\}$	$\{M\}^{1/2}\{L\}^{2}\{T\}^{-1/2}\{R\}^{-1/2}$
Quadrupole moment	Charge · Distance^2	$\{Q\}\{L\}^{2}$	$\{M\}^{1/2}\{L\}^{3}\{T\}^{-1/2}\{R\}^{-1/2}$
Temperature	Energy per molecule	$\{Q\}\{\Phi\}\{T\}^{-1}$	$\{M\}\{L\}^{2}\{T\}^{-2}$
Specific heat	Heat per unit mass and temperature	$\{Q\}^{-1}\{\Phi\}^{-1}\cdot\{L\}^{2}\cdot\{T^{-1}\}$	$\{M\}^{-1}$
Thermal conductivity		$\{L\}^{-1}\{T^{v1}\}$	$\{L\}^{-1}\{T\}^{-1}$

3.5 Derived Quantities in Terms of L, M, T and R: An Example

If a system of units comprises units of quantities of length L, mass M, time T and the electrical resistance R, then the other quantities may be expressed dimensionally in terms of M, L, T and R as follows:

1. Velocity = length/time; $\{\text{Velocity}\} = \{L\}\{T\}^{-1}$
2. Acceleration = change in velocity/time; $\{\text{Acceleration}\} = \{L\}\{T\}^{-2}$
3. Force = mass · acceleration; $\{\text{Force}\} = \{M\}\{L\}\{T\}^{-2}$
4. Moment of inertia = mass of a body · square of radius of gyration; $\{\text{Moment of inertia}\} = \{M\}\{L\}^{2}$
5. Work done = force · distance; $\{\text{Work done}\} = \{M\}\{L\}^{2}\{T\}^{-2}$
6. Power = rate of doing work; $\{\text{Power}\} = \{M\}\{L\}^{2}\{T\}^{-3}$
7. Gravitational constant G = force · (distance)^2 / (mass)^2, giving $\{G\} = \{M\}\{L\}\{T\}^{-2}\{L\}^{2}/\{M\}^{2} = \{M\}^{-1}\{L\}^{3}\{T\}^{-2}$
8. Voltage.Power = voltage · current = (voltage)^2/resistance
   Representing voltage by V and electrical resistance by R, we get Power = $\left(V^2/R\right)$; $\{\text{Power}\} = \{M\}\{L\}^{2}\{T\}^{-3}$, giving $\{V\} = \{M\}^{1/2}\{L\}\{T\}^{-3/2}\{R\}^{1/2}$
9. Once we know the voltage, we can determine the units of the current by $I = V/R$, giving $\{I\} = \{M\}^{1/2}\{L\}\{T\}^{-3/2}\{R\}^{-1/2}$
10. Charge Q = Current · Time, so $\{\text{Charge Q}\} = \{M\}^{1/2}\{L\}\{T\}^{-3/2}\{R\}^{-1/2}\{T\} = \{M\}^{1/2}\{L\}\{T\}^{-1/2}\{R\}^{-1/2}$
11. Rate of change of flux Φ is voltage V, so we get $\Phi/T = V$ or $\Phi = V\cdot T$, giving $\{\Phi\} = \{M\}^{1/2}\{L\}\{T\}^{-1/2}\{R\}^{1/2}$
12. Magnetic induction is flux per unit area, so $\{\text{Induction}\} = \{M\}^{1/2}\{L\}^{-1}\{T\}^{-3/2}\{R\}^{1/2}$
13. Electric field strength = dV/dr, giving $\{\text{Electric field strength}\} = \{M\}^{1/2}\{T\}^{-3/2}\{R\}^{1/2}$

14. Capacitance = charge/voltage, giving {Capacitance} = $\{M\}^{1/2}$ {L} $\{T\}^{-1/2}$ $\{R\}^{-1/2}/\left[\{M\}^{1/2}\{L\}\{T\}^{-3/2}\{R\}^{1/2}\right] = \{T\}\{R\}^{-1}$
15. Inductance = magnetic flux/current {Inductance} = $\{M\}^{1/2}\{L\}\{T\}^{-1/2}\{R\}^{1/2}/$ $\{M\}^{1/2}\{L\}\{T\}^{-3/2}\{R\}^{-1/2} = \{T\}\{R\}$
16. Permittivity in free space ε_o = square of charge/force · length^2 (Coulombs Law)

    $\{\text{permittivity}\} = \left(\{M\}^{1/2}\{L\}\{T\}^{-1/2}\{R\}^{-1/2}\right)^2/\{M\}\{L\}\{T\}^{-2}\cdot\{L\}^2 = (\{M\}$ $\{L\}^2\{T\}^{-1}\{R\}^{-1}\big)/\{M\}\{L\}^3\{T\}^{-2} = \{L\}^{-1}\{T\}\{R\}^{-1}$
17. Permeability = $1/\{\text{Permitivity}\cdot(\text{Velocity})^2\}$, giving {permeability} = 1/ $\{L\}^{-1}$ $\{T\}\{R\}^{-1}\{L\}^2\{T\}^{-2} = 1/\{L\}\{T\}^{-1}\{R\}^{-1} = \{L\}^{-1}\{T\}\{R\}$

We have seen that any quantity in any field of measurement can be expressed in terms of any four arbitrarily chosen units. For mechanical measurements, people have been using length, mass and time as base units for a very long time. Quite often these units are called fundamental units. However, to express the quantities in simpler forms in all the fields of measurements, two more units, one each in the thermal and optical fields, are taken.

The example of expressing other units in terms of a set of units of length, time, flux and charge are given in Table 3.3. Column 1 of the table gives the quantity for which units are expressed in columns 3 and 4. Column 2 gives the expression of the quantity in column 1, in terms of units of base quantities.

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Chapter 4
Metre Convention and Evolution of Base Units

4.1 BIPM and Metre Convention

On 20 May 1875, the "Metre Convention", a diplomatic treaty between 17 nations, was signed in Paris. The Metre Convention gave authority to the General Conference on Weights and Measures (Conférence Générale des Poids et des Mesures [CGPM]) and the International Committee for Weights and Measures (Comité International des Poids et des Mesures [CIPM]) to set up the International Bureau of Weights and Measures (Bureau International des Poids et des Mesures [BIPM]) and to act in the matters related to units of measurement. Apart from these 17 countries, Italy in 1876, Norway in 1882, Sweden in 1889 and Denmark in 1912 were among the first few countries that joined the treaty later. This convention was amended in 1921.

India signed the "Metre Convention Treaty" in 1957 after the Standards of Weights and Measures Act of 1956 was enacted by the Parliament. At present there are 51 member countries of the Treaty. In addition, 20 countries are associates of the CGPM.

It may be noted that the foundation of the metric system in relation to mass and length was established by the deposition of two platinum standards representing "metre" and "kilogram" on 22 June 1799 in Paris with the French Archives. Following this, some European countries like the Netherlands in 1820, France in 1837 and Spain in 1860 started using metre and kilogram even before signing the "Metre Convention".

4.1.1 General Conference on Weights and Measures (CGPM)

The CGPM is composed of representatives of all the member countries and it meets every four years. The main objectives of the CGPM are:

1. Discussing and instigating the arrangements required to ensure the propagation and improvement of the International System of Units (SI), which is the modern form of the metric system

2. Confirming the results of new fundamental metrological determinations and the various scientific resolutions of international scope
3. Adopting the important decisions concerning the organization and development of the BIPM

The CGPM adopts all resolutions/decisions through voting with no dissent vote. All proposals for final decisions are mooted through the CIPM.

4.1.2 International Committee for Weights and Measures (CIPM)

The CIPM, a committee of select representatives of member countries, has 18 members each from a different country, who are elected by the CGPM on the basis of their contribution to metrology. The CIPM officially governs the BIPM.

4.1.3 Consultative Committees

With the extension of work in diverse areas of measurements entrusted to the BIPM, the CIPM set up, since 1927, various consultative Committees in specific areas of measurement. At present there are ten Consultative Committees whose function is to study, advice and provide information on matters that are referred to them. These Consultative Committees are responsible for coordinating the international work carried out in their respective fields and for proposing recommendations to the CIPM concerning units. The committees may form temporary or permanent working groups to study special topics.

The Consultative Committees have common regulations [1]. They meet at irregular intervals. The president of each consultative Committee is designated by the CIPM and is normally a member of the CIPM. The members of the Consultative Committees are metrology laboratories and specialized institutes, agreed by the CIPM, which send delegates of their choice.

In addition, there are individual members appointed by the CIPM, and a representative of the BIPM. The criteria for membership of Consultative Committees are according to [2]. At present, there are ten such committees, which will be described in the following subsections.

Consultative Committee for Electricity and Magnetism (CCEM)

Initially a Consultative Committee for Electricity was set up in 1927 to look after the units in the area of electricity. On the inclusion of work in the area of magnetism in 1997, the present name, Consultative Committee for Electricity and Magnetism (CCEM), was assigned.

Consultative Committee for Photometry and Radiometry (CCPR)

To look after the units of photometry the Consultative Committee for Photometry was established in 1930. After the inclusion of work pertaining to radiometry it was given its present name Consultative Committee for Photometry and Radiometry (CCPR) in 1971.

Consultative Committee for Thermometry (CCT)

This committee was set up in 1937.

Consultative Committee for Length (CCL)

Initially the name of this committee was Consultative Committee for the Definition of Metre and it was set up in 1952. To include a wider perspective of length measurement, it was given this new name in 1997.

Consultative Committee for Time and Frequency (CCTF)

This name was given to the old Consultative Committee for Definition of the Second (CCDS), which was set up in 1956.

Consultative Committee for Ionizing Radiation (CCRI)

A new name was given in 1997 to the old Consultative Committee for Standards of Ionising Radiation (CCEMRI). The CCEMRI was set up in 1958. In 1969 the Committee established four sections, namely Section I: X-rays and γ rays, electrons; Section II: Measurement of radionuclides; Section III: Neutron measurements; and Section IV: α-energy standards. In 1975, this last section was dissolved and Section II was made responsible for its field of activity [3].

Consultative Committee for Units (CCU)

This committee was set up in 1964 and replaced the Commission for the System of Units set up by CIPM in 1954.

Consultative Committee for Mass and Related Quantities (CCM)

This committee was set up in 1980.

Consultative Committee for Amount of Substance: Metrology in Chemistry (CCQM)

This committee was set up in 1993.

Consultative Committee for Acoustics, Ultrasound and Vibration (CCAUV)

This is the youngest committee as it was set up in 1999.

Each of these committees may meet as many times in a year as it wishes to. The committees give their recommendations to the CIPM for approval, which in turn submits them to the CGPM for ratification.

4.1.4 International Bureau of Weights and Measures (BIPM)

The headquarters of BIPM are in Sevres near Paris, on the Pavillon de Breteuil (Parc de Saint-Cloud), with a ground area of 43,520 m^2. The French Government placed the whole area at the disposal of BIPM. Its upkeep is financed jointly by the Member States of the Metre Convention.

Scientific Activities

The activities of the BIPM, which in the beginning were limited to measurements of length and mass, and to metrological studies in relation to these quantities, have been extended to standards of measurement of electricity (1927), photometry and radiometry (1937), ionizing radiation (1960), time scales (1988) and chemistry (2000). The original laboratories were built in 1876–1878 and were enlarged in 1929. Further new buildings were constructed in 1963–1964 for the ionizing radiation laboratories, for the laser work in 1984, and for a library and offices in 1988. In 2001, a new building for the workshop, offices and meeting rooms was inaugurated.

Objects of BIPM

The BIPM is the custodian of all international standards of units and is involved in the following activities:

1. Establishing the fundamental standards and scales of measurement of the principal physical quantities and maintaining the international prototypes
2. Carrying out comparisons of national standards and international standards
3. Ensuring the coordination of corresponding measuring techniques

4. Carrying out and coordinating determinations relating to the fundamental physical constants that are involved in the above-mentioned activities
5. Mutual Recognition Arrangement with National Measurements Institutes

Staff at BIPM

Some 45 physicists and technicians work in the BIPM laboratories. They mainly conduct metrological research, carry out international comparisons for realizations of units and do calibrations of standards. An annual report and the Director's Report on the activity and management of the International Bureau of Weights and Measures give details of the work in progress.

Publications

The proceedings of the General Conference and the CIPM are published by the BIPM in the following series:

- Report of the meeting of the General Conference on Weights and Measures
- Report of the meeting of the International Committee for Weights and Measures

One can get all the reports of meetings of the Consultative Committees held from 2003 onward in their original language on the BIPM website.

The BIPM also publishes monographs on special metrological subjects, and the brochure "The International System of Units (SI)" is periodically updated. All decisions and recommendations concerning units are also given in this brochure.

The scientific work of the BIPM is published in the open scientific literature and an annual list of publications appears in the Director's Report on the activity and management of the International Bureau of Weights and Measures.

Metrologia

Since 1965, *Metrologia*, an international journal published under the auspices of the CIPM, has printed articles dealing with scientific metrology, improvements in methods of measurement, work on standards and units, as well as reports concerning the activities, decisions and recommendations of the various bodies created under the Metre Convention.

4.1.5 Linkages of Various Organs of Metre Convention

Linkages of various organs of the Metre Convention, functions of the BIPM and various Consultative Committees are shown in Fig. 4.1.

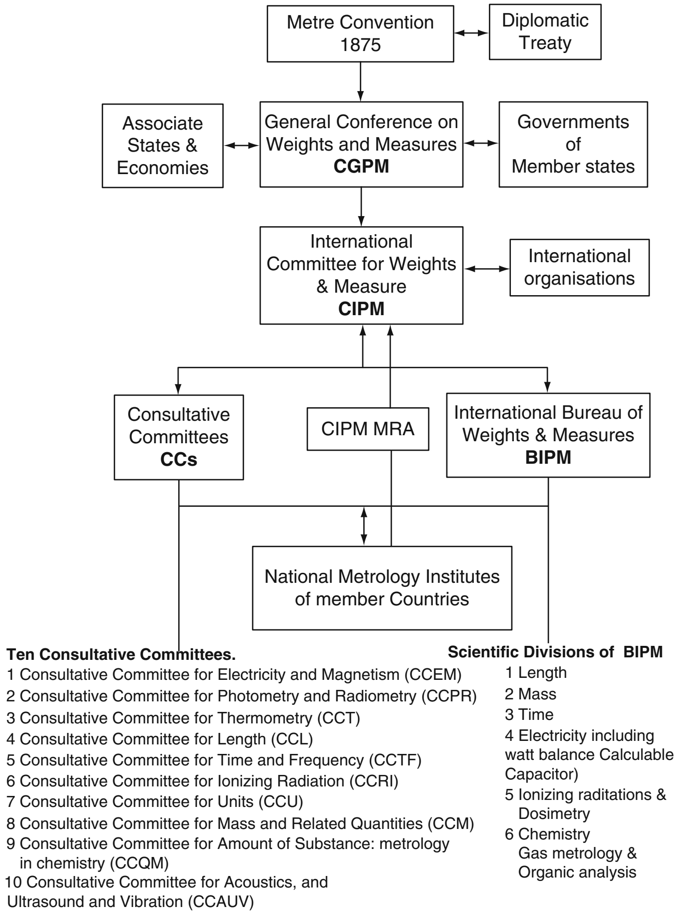


Fig. 4.1 Linkages of organs of the Metre Convention and the CGPM

4.2 International System of Units (SI)

Starting with metre and kilogram as base units in 1875, the CGPM with the help of the CIPM and the consultative committees kept on adding units of measurement in various other areas; for example, photometric, temperature and electric units were added in 1948. The practical international system that has four units, namely metre, kilogram, second and ampere (MKSA), was amalgamated with other units by the 11th CGPM in 1960, and called the International System of Units, abbreviated as SI. The system is in use since then not only in the signatory countries of the Metre Convention but in many other countries as well. To start with, the International System of Units had only six base units. The seventh base unit, mole for amount of substance, was added in 1971. Hence the present International System of Units is built on seven base quantities, namely length, mass, time, electric current, temperature, luminous intensity and amount of substance. The system includes units of base quantities, their respective symbols and a host of units for other quantities. The well-defined relations connect all quantities with base quantities. The units form a coherent system of units. It also covers some units that are not within SI but are permitted for the use with SI units. The units cover practically all fields of measurements including human health and safety.

4.2.1 Base Units

We have seen in the previous chapter that only four base units are sufficient for describing any system of units. The units of luminous intensity, temperature and amount of substance, though not necessary, have been included for the matter of convenience and for the ease in expressing and understanding the units of measurement in these specific areas. The base units of the present International System of Units are:

1. Metre for length, with the symbol m
2. Kilogram for mass, with the symbol kg
3. Second for time, with the symbol s
4. Ampere for electric current, with the symbol A
5. Kelvin for temperature, with the symbol K
6. Candela for intensity of illumination, with the symbol cd
7. Mole for amount of substance, with the symbol mol

The interrelation between the units of length, mass, time and ampere and several derived units is shown in Fig. 4.2. Solid lines show multiplication while dotted lines indicate division by that unit.

Base quantities and base units with their respective names and symbols, years of inclusion, modification and year of final definition are given in Table 4.1 [3].

4.3 Evolution of Base Units

The foundation stone of the metric system was laid on 20 May 1875, when 17 countries signed the famous Metre Convention Treaty (French name: Convention du Mètre). The present International System of Units (French name: Système International d'Unités), with the universally adopted abbreviation SI, was formally announced in 1960. It took around 85 years to evolve a convenient and worldwide acceptable system of units. Worldwide national metrological laboratories spent a lot of time before coming to a consensus for a system of units based on sound scientific principles and supported by experimental results.

It is important to distinguish between the definition of a unit and its realization. The definition of each base unit of the SI is carefully drawn up so that it

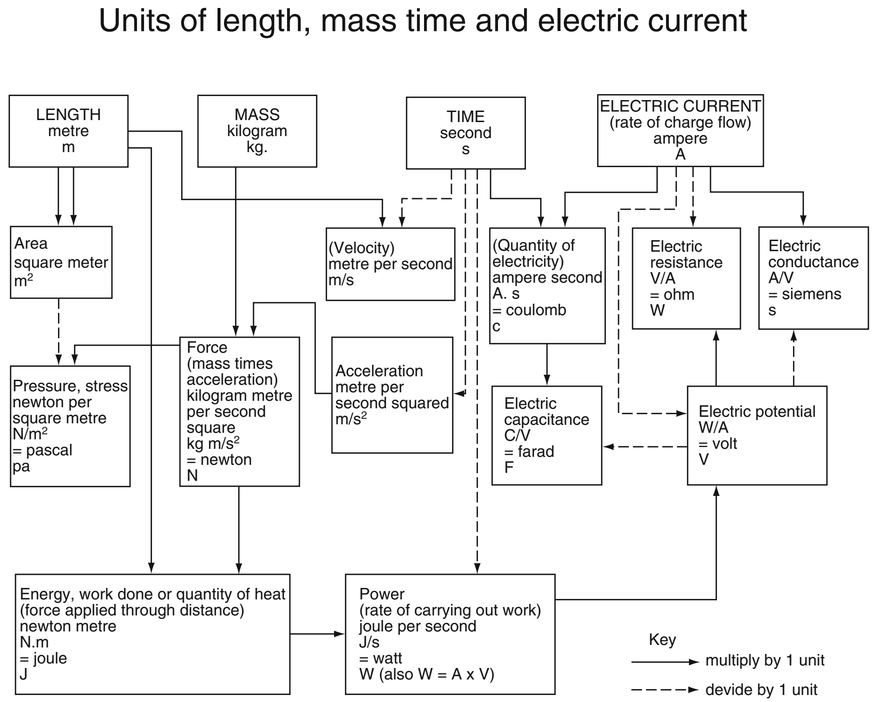


Fig. 4.2 Interrelation of units of mass, length, time and ampere

Table 4.1 Present status of Base units

Quantity	Symbol	Name of unit	Symbol of unit	Introduced year/CGPM	Modified year/CGPM	Final year/CGPM	Symbol of dimension
Length	*L*, *x*, *r etc.*	metre	m	1889 1st	1960 11th	1983 17th	L
Mass	*m*	kilogram	kg	1889 1st	–	1889 1st	M
Time	*T*	second	s	1960 11th	1968 13th	1968 13th	T
Electric current	*I*, *i*	ampere	A	1948 9th	1948 11th	1948 9th	I
Intensity of illumination	I_v	candela	cd	1948 9th	1968 13th	1979 16th	J
Temperature	*T*	kelvin	K	1948 9th	1954 10th	1954 10th	Θ
Mole	*n*	mole	mol	1971 14th	–	–	N

is unique and provides a sound theoretical basis upon which the most accurate and reproducible measurements can be made. The realization of the definition of a unit is the procedure by which the definition may be used to establish the value and associated uncertainty of a quantity of the same kind as the unit. A description of how the definitions of some important units are realized in practice is given on the BIPM website (www.bipm.org/en/si/si_brochure/appendix2/). However, any method consistent with the laws of physics could be used to realize any SI unit. For example, the unit ohm can be realized with high accuracy using the quantum Hall effect and the value of the von Klitzing constant recommended by the CIPM [3, Appendix 1].

A coherent SI derived unit is defined uniquely only in terms of SI base units. For example, the coherent SI derived unit of resistance, the ohm, with the symbol Ω, is uniquely defined by the relation $\Omega = \mathrm{m}^2\mathrm{kgs}^{-3}\mathrm{A}^{-2}$, which follows from the definition of the quantity of electrical resistance.

Finally, it should be recognized that although the seven base quantities (length, mass, time, electric current, thermodynamic temperature, amount of substance and luminous intensity) are by convention regarded as independent, their respective base units (metre, kilogram, second, ampere, kelvin, mole and candela) are in a number of instances interdependent. For example, the definition of metre incorporates the second; the definition of ampere incorporates the metre, kilogram and second; the definition of mole incorporates the kilogram; and the definition of candela incorporates the metre, kilogram and second [3].

4.3.1 Unit of Length

Initially one metre was defined as the 1/40,000,000 part of the earth's meridian passing through Paris. The length of the meridian was measured in terms of the old French unit of length called toise. From the best measurements of the meridian, it was found that the theoretical length of one metre should be 0.513074 toise. A sintered platinum flat bar was adjusted to this length and was deposited with the Archives de France. The section of the bar was 25 mm wide and 4 mm thick. This standard was in the form of an end standard, i.e. the distance between the centres of the end faces of the bar was one metre. This measurement of the metre was officially declared as the "final standard of the metre" on 10 December 1799.

After the Metre Convention was signed, several platinum–iridium bars were fabricated with two transverse marks on each bar so that the distance between the two marks is close to 1 m. The bar on which the distance between the two transverse marks was as close as possible to the distance between the end faces of the "Mètre des Archives" of 1799 was taken as the International Prototype of the Metre. The cross section of the prototype bar is shown in Fig. 4.3.

Engineer G. Tresca worked out the design of the cross section of the bar. The design has two distinct advantages:

- It has a uniform surface in the neutral plane where subdivisions of the metre can be graduated.
- It has greater rigidity for the given mass of metal used.

The latter point is important from the cost point of view of the bar, as platinum–iridium alloy is an expensive material.

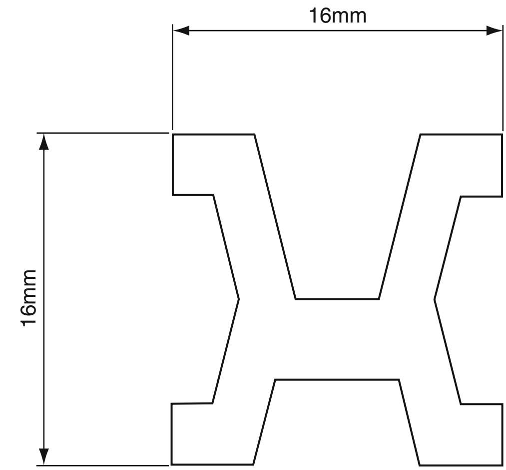


Fig. 4.3 End section of the International Prototype of the Metre bar

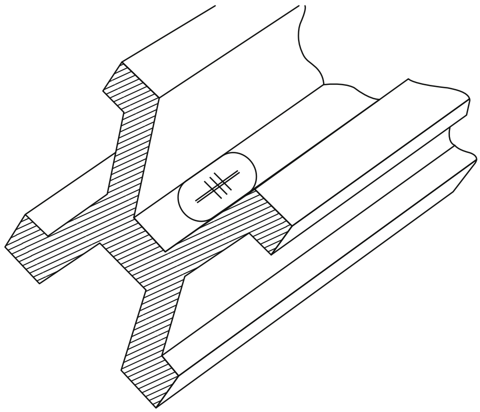


Fig. 4.4 Description of the International Prototype of the Metre bar

The CIPM selected a bar in which the distance between the transverse lines was closest to the distance between the centres of the end faces of the Mètre des Archives of France. The first CGPM, in 1889, adopted that bar as the International Prototype of the Metre. On each end of the bar there is a spot having three transverse lines, each 0.5 mm apart, and two fiducial longitudinal lines separated by 0.2 mm. The end part of the bar having different lines is shown in Fig. 4.4.

Note: This description is based on the information from Dr. R. S. Davis, Head Mass Standards, BIPM, from the book by H. Moreau, who was also the member of staff of the BIPM.

The metre was then defined as the distance between two centres of transverse lines on each end of the bar at the temperature of melting ice. Temperature was measured with a hydrogen thermometer on a centigrade scale. The two longitudinal fiducial lines bound the portion of the transverse lines on each side.

In fact, the 7th CGPM in 1927 formally defined the metre as follows:

> The unit of length is the metre, defined by the distance at 0^oC between the axes of the two central lines marked on the bar of platinum–iridium selected by the 1st CGPM, this bar being subject to standard atmospheric pressure and supported on two cylinders of at least one centimetre diameter, symmetrically placed in the same horizontal plane at a distance of 571 mm from each other.

Later it was felt that the International Prototype of the Metre did not define the metre with the accuracy adequate for the needs of metrology. Moreover, it was desirable to adopt a natural and indestructible standard. So the 11th CGPM, in 1960, defined the metre in terms of wavelengths of visible radiation and declared that the metre was the length equal to 1,650,763.73 wavelengths, in vacuum, of the radiation, due to the transition between the levels $2p_{10}$ and $5d_5$ of the krypton 86 atom. The metre was realized by using an interferometer with a travelling microscope to measure the optical path difference by counting the fringes.

The main problem with radiations from the krypton lamp was its coherence length, which was not more than 50 cm; thus to get interference fringes over a distance of one metre was not possible. The metre, in terms of the specified radiation of krypton 86 used to be scanned in two steps. But lasers made it possible to have interference fringes over distances of more than one metre. In addition, it was also realized that due to their highly monochromatic nature, i.e. narrower bandwidth, less divergence and strong intensity, lasers were found to be better reproducible and easy-to-use sources. Moreover, the following properties were found:

1. The frequency of lasers can be maintained at a constant value.
2. Lesser relative uncertainty is possible in frequency measurement.
3. The measurement of frequency and wavelength of laser radiation has resulted in concordant determination of the speed of light, whose accuracy was limited principally by the realization of the definition of the metre.
4. Wavelengths determined from the frequency measurement and with a given value of the speed of light give a reproducibility superior to the one which could be obtained by comparing the wavelength standard radiations of krypton 86.
5. Scientists working in the field of astronomy and geodesy measure distances in terms of speed of light (electromagnetic waves) and time.

So it was thought prudent to define the metre in terms of the speed of light, which by definition is constant. The 15th CGPM in 1975 accepted 299,792,458 ms^{-1} as the value of the speed of light.

Based on this value, the 17th CGPM, in 1983 [5], defined the metre in the following way:

> The metre is the length of the path travelled by light in vacuum during a time interval of 1/299,792,458 s.

Please note that the effect of this definition is to fix c_o, the speed of light in vacuum, at exactly 299,792,458 ms^{-1}. The original International Prototype of the Metre sanctioned by the 1st CGPM in 1889 [4] is still kept at the BIPM under conditions specified in 1889.

4.3.2 Unit of Mass

Lavovisier, a great scientist of his time, considered water as a natural standard. He proposed that the unit of mass must be kilogram and it must be considered equal to the mass of water of one cubic decimetre. Accordingly a cylinder of pure sintered platinum was fabricated by the French Academy. The mass of the cylinder was made equal to that of water at the temperature of its maximum density and occupying a volume of one cubic decimetre. This cylinder of sintered platinum was kept in the Archives de France on 22 June 1799. The cylinder was given the name "Kilogram de Archives" and was declared the standard of the kilogram on 10th December 1799.

International Prototype of Kilogram

In 1878, three years after BIPM was founded, several cylinders of platinum–iridium of nominal composition 90% platinum and 10% iridium, were prepared. These were compared in 1880, with the Kilogram de Archives. The one whose mass was closest to that of the Kilogram de Archives was chosen as the international prototype by the CIPM in 1883 and a letter K was engraved on it. The same kilogram was approved as such by the 1st General Conference on Weights and Measures in 1889 as the International Prototype of kilogram [4]. The 3rd CGPM, in 1901, declared that the kilogram is the unit of mass rather than weight and is equal to the mass of the International Prototype of kilogram of 1889, kept in the custody of the BIPM.

In 1989, the CIPM took the following decisions in respect of cleaning and washing of the prototypes:

1. That the kilogram as defined in 1889 is the mass of the International Prototype of kilogram just after cleaning and washing.
2. That the BIPM procedure given in document [6–8] should be used for cleaning and washing.
3. That to deduce the mass of the International Prototype of kilogram, at the time of using, the measured change in mass of $+0.0368\ \mu g$ per day must be used.

This gives:

$$\text{Mass of the International Prototype of kilogram} = 1\,\text{kg} + 0.0368\,d\,\mu g$$

where d is the number of days passed since last cleaning and washing.

The BIPM procedure for cleaning platinum–iridium weights has been given in references [6–8].

We know the following about the kilogram:

- The unit of mass is the only unit which is being defined in terms of an artefact.
- All other units have been defined in terms of a physical or atomic phenomenon.

Considering the above facts, the 21st CGPM, in 1999 [4, page 165], passed the following resolution:

The CGPM on considering

1. The need to assure the long-term stability of the SI
2. The intrinsic uncertainty in the long-term stability of the artefact defining the unit of mass
3. The consequent uncertainty in the long-term stability of the other three base units of the SI depend on the kilogram, namely ampere, mole and candela
4. The progress already made in a number of different experiments designed to link the unit of mass to fundamental or atomic constants
5. The desirability of having more than one method of making such a link

recommended that the national laboratories should continue their efforts to refine experiments that link the unit of mass to fundamental or atomic constants with a view to redefine the kilogram.

With regard to attaching a prefix to the unit of mass whose name, for historical reasons, contains a prefix, the 13th CGPM, in 1968 [8], declared that names and symbols for decimal multiples and submultiples of the unit of mass are formed by attaching prefixes to the unit name "gram" and prefix symbols to the unit symbol "g".

It may be noted that it took around 14 years (from 1875 to 1889)

1. To identify the material suitable for the International Prototype of kilogram, its copies and the national prototypes
2. To prepare cylinders with this material
3. To adjust the mass value of these cylinders within ± 1 mg of the mass of the Kilogram de Archives
4. To declare one of them as the International Prototype of kilogram, some as its official copies and others as national prototypes.

The International Prototype of kilogram is kept on a glass plate covered by three bell jars. Levelling screws keep the base horizontal. The kilogram is shown in Fig. 4.5.

It follows that the mass of the International Prototype of kilogram is always 1 kg exactly, $\mathrm{m}(K) = 1$ kg. However, due to the inevitable accumulation of contaminants on surfaces, the international prototype is subject to reversible surface contamination, which approaches 1 μg per year in mass. For this reason, the CIPM declared that, pending further research, the reference mass of the international prototype is that immediately after cleaning and washing by a specified method [7,8]. The reference mass thus defined is used to calibrate national standards of platinum–iridium alloy [9].

Note. The symbol m(K) is used to denote the mass of the International Prototype of kilogram, with the letter "K" engraved on the cylinder.

Fig. 4.5 International Prototype of the Kilogram (Courtesy by BIPM)

4.3.3 Unit of Time

The unit of time, the second, was at one time considered to be the fraction1/86,400 of the mean solar day, which came from the assumption that 24 h is a day, 60 min is 1 h and 60 s make a minute. The exact definition of "mean solar day" was based on astronomical theories. However, measurement showed that irregularities in the rotation of the Earth could not be taken into account by the theory; hence this definition does not allow the required accuracy to be achieved.

In order to define the unit of time more precisely, the 11th CGPM (1960) adopted a definition given by the International Astronomical Union, defining the second with the symbol "s" as 1/315,569,259,747 of the tropical year for 1900, 0 January at 12 h ephemeris time. However, experimental work showed that for a time interval, an atomic clocks, based on a transition between two energy levels of an atom or a molecule, could be realized and reproduced much more precisely.

Considering that a very precise definition of the unit of time is indispensable for the SI units, the 13th CGPM (1967–1968) [10] replaced the above definition of the second with the following:

> The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.

At its 1997 meeting, the CIPM affirmed that this definition refers to a caesium atom in its ground state at a temperature of 0 K and that the splitting in ground state of the caesium 133 atom is exactly 9,192,631,770 Hz.

4.3.4 Unit of Electric Current

Electrical units with the name "international" for current and resistance were introduced by the International Electrical Congress held at Chicago in 1893, and definitions of the "international" ampere and the "international" ohm were confirmed by the International Conference at London in 1908.

Although it was already obvious on the occasion of the 8th CGPM (1933) that there was a unanimous desire to replace those "international" units by "absolute" units, the official decision to abolish them was only taken by the 9th CGPM (1948), which adopted the ampere for the unit of electric current and defined it as:

> The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newtons per metre of length.

The expression "MKS unit of force", which occurs in the original text of 1946, has been replaced here by "newton", a name adopted for this unit by the 9th CGPM (1948).

It will be shown later that the effect of this definition is to fix the permeability of vacuum at exactly $4\pi 10^{-7}\mathrm{Hm}^{-1}$.

It may be noted that ampere has been defined by CGPM such that it is as close as possible to International Ampere and that International Ampere was taken as one-tenth of the electromagnetic unit current (abampere).

4.3.5 Unit of Luminous Intensity

The unit of luminous intensity was initially based on flame or incandescent filament lamps. These standards continued to be used in various countries till 1948. Then these were replaced by the "new candle" based on the luminance of a black body radiator at the freezing point of platinum. Though the definition of new candle was prepared by the International Commission on Illumination (CIE) and by the CIPM before 1937, the CIPM took the decision to promulgate it in 1946. However, it was ratified only in 1948 by the 9th CGPM, which adopted the name "new candle" for this unit and defined it as follows:

> New candle (unit of luminous intensity): the value of the new candle is such that the brightness of the full radiator at the temperature of solidification of platinum is 60 new candles per square centimetre.

Similarly new lumen, the unit of luminous flux, was defined as the luminous flux emitted in unit solid angle (steradian) by a uniform point source having a luminous intensity of 1 new candle.

13th CGPM 1968 not only changed the wordings of the definition of new candle but changed its name also, the name given was candela. The definition of candela given was as follows:

> The candela is the luminous intensity, in the perpendicular direction, of a surface of 1/600000 square metre of a black body at the temperature of freezing platinum under normal standard pressure of 101 325 newtons per square metre.

The definition of "new candle" was abrogated in 1968.

Later on, in 1979, the 16th CGPM considering the following points:

1. Despite the notable efforts of some laboratories there remain excessive divergences between the results of realizations of the candela based upon the black body as primary standard.
2. Radiometric techniques are developing rapidly, allowing precisions that are already equivalent to those of photometry, and these techniques are already in use in national laboratories to realize the candela without having to construct a black body.
3. The relation between luminous quantities of photometry and radiometric quantities, namely the value of 683 lumens per watt for the spectral luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz, has been adopted by the CIPM in 1977.
4. This value has been accepted as being sufficiently accurate for the system of luminous photopic quantities, and it implies a change of only about 3% for the system of luminous scotopic quantities.
5. It therefore ensures satisfactory continuity and applies to both photopic and scotopic photometric quantities and to quantities yet to be defined in the mesopic field.

Therefore, the CGPM decided to give the candela a definition that will allow an improvement in both the ease of realization and the precision of photometric standards.

Hence the candela was redefined by the 16th CGPM, in 1979, in terms of frequency and power [11] instead of the source at the freezing point of platinum:

> The candela is the luminous intensity in a given direction of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian.

It may be mentioned that unlike ampere – the unit of electric current – candela is not named after any scientist, but is the Latin word for "candle".

4.3.6 Unit of Temperature

As temperature is nothing but the average energy of a molecule, it could be expressed in terms of energy; however, for the sake of simplicity and ease it has been separately taken as one of the base units.

In 1948, the 9th CGPM decided the following in respect of measurements of temperature and heat:

1. The zero of the centesimal thermodynamic scale was defined as the temperature 0.01° below that of the triple point of water.

2. The absolute thermodynamic scale with a single fundamental fixed point was provided by the triple point of pure water, the absolute temperature of which will be fixed at a later date.
3. The unit of heat quantity was assigned the name joule.
4. Out of three possible names – degree centigrade, centesimal degree and degree Celsius – degree Celsius was adopted.

Unit of Thermodynamic Temperature (Kelvin)

The definition of the unit of thermodynamic temperature was given in substance by the 10th CGPM (1954), which selected the triple point of water as the fundamental fixed point and assigned to it the temperature 273.16 K, so defining the unit. The 13th CGPM (1967–1968) [11] adopted the name *kelvin* with the symbol K. Earlier it was called "degree Kelvin" with the symbol °K.

The unit of thermodynamic temperature is defined as:

> The kelvin – unit of thermodynamic temperature – is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.

To make the definition more accurate, CIPM in 2005 affirmed that the water in the above definition is Vienna Standard Mean Oceans Water (V-SMOW) having the following isotopic compositions [3]:

Deutron heavy hydrogen ^{2}H is 0.000 155 76 ^{2}H mol per mole of ^{1}H
Oxygen of atomic mass 18 ^{18}O is 0.002 005 2 mole of ^{18}O per mole ^{16}O
Oxygen of atomic mass 17 ^{17}O: 0.000 379 9 mole of ^{17}O per mole of ^{16}O

Following the normal practice of expressing the temperature scales, a thermodynamic temperature with the symbol T is also expressed in terms of its difference from the reference temperature $T_0 = 273.15\,\text{K}$, the ice point. This temperature difference is called the Celsius temperature with the symbol t, and is defined by the quantity equation

$$\mathrm{t}\left(^{\circ}\mathrm{C}\right) = \mathrm{T}_{90}\,(\text{kelvin}) - T_0$$

The unit of Celsius temperature is the degree Celsius with the symbol °C, which by definition is equal in magnitude to the kelvin. A difference or interval of temperature may be expressed in kelvins or in degrees Celsius. The numerical value of a Celsius temperature t expressed in degrees Celsius is given by

$$\mathrm{t}\left(^{\circ}\mathrm{C}\right) = \mathrm{T}_{90}\,(\text{kelvin}) - 273.15$$

Here T is expressed in kelvin.

The kelvin and the degree Celsius are also units of the International Temperature Scale of 1990 (ITS-90) adopted by the CIPM in 1989 [12].

The two names associated with unit of temperature are those of Lord Kelvin and Celsius.

4.3.7 Unit of Amount of Substance (Mole)

On the advice of The International Union of Pure and Applied Physics, The International Union of Pure and Applied Chemistry, and The International Organization for Standardization, the 14th CGPM, in 1971 [3, 13], decided to include mole as the seventh base unit and defined it as follows:

1. The mole with symbol "mol" is the amount of substance of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12.
2. When the mole is used, the elementary entities must be specified which may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.

It follows that the molar mass of carbon 12 is exactly 12 grams per mole:

$$\mathrm{M}\left({}^{12}\mathrm{C}\right) = 12\,\mathrm{g/mol}$$

In 1980 the CIPM approved the report of the CCU (1980) which specified that in this definition, it is understood that unbound atoms of carbon 12, at rest and in their ground state, are referred to.

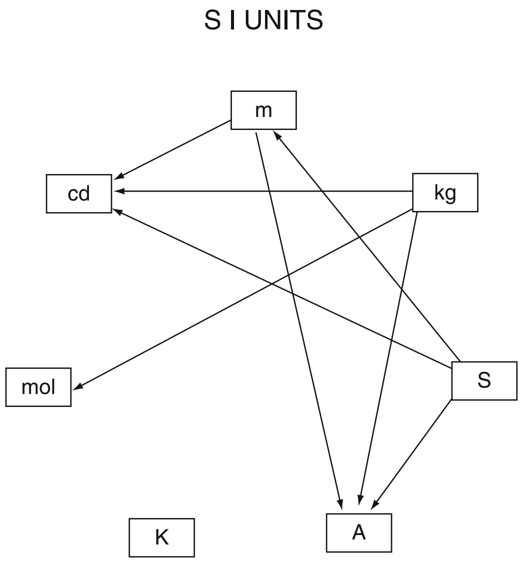
The definition of the mole also determines the value of the constant that relates the number of entities to the amount of substance for any sample. This constant is called the Avogadro constant with the symbol N_A or L. If N(X) denotes the number of entities X in a specified sample, and n(X) denotes the amount of substance of entities X in the same sample, the two are related to N_A as follows:

$$\mathrm{n}\,(\mathrm{X}) = \mathrm{N}\,(\mathrm{X})\,/\mathrm{N_A}$$

One may note that since N(X) is dimensionless, and the unit of n(X) is mole, the coherent SI unit of the Avogadro constant is reciprocal mole with symbol mol^{-1}.

In any particular application, for simplicity and better understanding, the word substance should be replaced by its chemical name. For example, one should say "amount of hydrogen chloride, HCl" or "amount of benzene, C_6H_6". It is important to always give a precise specification of the entity involved (as emphasized in the second sentence of the definition of mole), and the empirical chemical formula of the material involved should be given. Although the word "amount" has a more general dictionary definition, this abbreviation of the full name "amount of substance" may be used for brevity. This also applies to derived quantities such as "amount of substance concentration", which may simply be called "amount

Fig. 4.6 Dependence of base units



concentration". However, in the field of clinical chemistry the name "amount of substance concentration' is generally abbreviated to 'substance concentration".

4.3.8 Dependence of Base Units

It was made clear that four quantities and their units are sufficient for describing a system of units. However, the International System of Units contains seven base quantities with specific units for each. The seven base units describe other units more compactly and conveniently. Obviously all base units are not independent of each other, and their dependence is shown in Fig. 4.6. Except the base units of mass and time all other base units are interrelated.

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Chapter 5
Realization of Base Units

The material used in this chapter has been taken primarily from Appendix 2 of the brochure on the International System of Units (SI) 8th Edition 2006 [1]. But Appendix 2 is not available in the printed form. It is available on the BIPM website and is updated periodically. The latest update is of 20 February 2007 for the realization of the ampere.

5.1 The Metre

The Consultative Committee for Length (CCL) recommended that the metre be realized by one of the following methods:

1. By means of the length l of the path travelled by a plane electromagnetic wave in the time t; this length is obtained from the measured value of time t, using the relation
   a. length $= c_o t$
   b. where $c_o = 299{,}792{,}458\ \text{ms}^{-1}$ exact
2. By means of the wavelength in vacuum λ of a plane electromagnetic wave of frequency f, this wavelength is obtained from the measured frequency f, using the relation
3. $\lambda = c_o/f$
4. By means of any one of the radiations given in Table 5.1 whose stated wavelength in vacuum or whose frequency can be used with uncertainty shown in column 5 of the Table 5.1, provided that the given specifications and accepted good practices are followed.

In all cases, every necessary correction should be applied to take into account actual conditions such as diffraction, gravitation or imperfection in vacuum.

Knowing λ in terms of the metre amounts to the definition that 1 m will contain $1/\lambda$ number of waves of a particular radiation. For example, the wavelength of ^{86}Kr is 605,780,210.3 fm, which gives 1,650,763.73 wavelengths that make a metre – the same definition as that of the 11th CGPM (1960). But the uncertainty of

Table 5.1 Standard radiations recommended by CCL

Element	Absorbing ion	Frequency (kHz)	Wavelength (fm)	Uncertainty
Indium	^{115}In	1,267,402,452,899.92	236,540,853.549 75	3.6×10^{-13}
Hydrogen	^{1}H	1,233,030,706,593.55	243,134,624.626 04	2.0×10^{-13}
Mercury	^{199}Hg	1,064,721,609,899.143	281,568,867.591 969	1.9×10^{-14}
Ytterbium	^{171}Yb	688,358,979,309.312	435,517,610.739 69	2.9×10^{-14}
Ytterbium	^{171}Yb	642,121,496,772.3	466,878,090.060 7	1.6×10^{-12}
Iodine	$^{127}I_2$	582,490,603,442	514,673,466.368	1.8×10^{-11}
Iodine	$^{127}I_2$	563,260,223,513	532,245,036.104	8.9×10^{-12}
Iodine	$^{127}I_2$	551,580,162,400	543,515,663.608	4.5×10^{-11}
Iodine	$^{127}I_2$	520,206,808.4 MHz	576,294,760.4	4×10^{-10}
Krypton	^{86}Kr	–	605,780,210.3	1.3×10^{-9}
Iodine	$^{127}I_2$	489,880,354.9 MHz	611,970,770.0	3×10^{-10}
Iodine	$^{127}I_2$	473,612,353,604	632,991,212.58	2.1×10^{-11}
Iodine	$^{127}I_2$	468,218,332.4 MHz	640,283,468.7	4.5×10^{-10}
Calcium	^{40}Ca	455,986,240,494.150	657,459,439.291 67	1.1×10^{-13}
Strontium	^{88}Sr	444,779,044 095.5	674,025,590.863 1	2.2×10^{-13}
Rubidium	^{85}Rb	385,285,142,375	778,105,421.23	1.3×10^{-11}
C_2H_2	$13C_2H_2$	194,369,569,385	1,542,383,712.37	5×10^{-11}
CH_4	CH_4 molecule	88,376,181,600.18	3,392,231,397.327	$3 \times 10–^{12}$
Osmium tetra-oxide	OsO_4 molecule	29,054,057,446.579	10,318,436,884.460	1.4×10^{-13}

measurement in using the ^{86}Kr definition was only 10^{-9}. Taking the velocity of light as exact and measuring frequency, the uncertainty decreases by several orders of magnitude. For example, by measuring frequency of a particular transition of ^{198}Hg, the wavelength obtained is 281,568,867.591 969 fm, defining one metre as equal to 3,551,529.006 wavelengths of that radiation but with an uncertainty of $1.9 \cdot 10^{-14}$, which is the measurement uncertainty of frequency. This is five orders of magnitude better than the definition of metre in terms of a particular radiation of ^{86}Kr. This is the clear advantage of taking the velocity of light as exact and measuring the frequency and deriving the wavelength from the relation

$$\lambda = c_o/f$$

5.1.1 Standard Radiations

The Consultative Committee on Length (CCL) recommended a number of radiations covering the entire range from ultra violet ($\lambda = 237\,\text{nm}$) to far infrared ($\lambda = 10.3\,\mu\text{m}$). The frequency of these radiations is measured with a very small uncertainty and their wavelengths are calculated by taking the fixed value of the velocity of light.

A list of radiations recommended by the CCL in 2006 is given in Table 5.1 [2]. Column 1 gives the element and column 2 the absorbing ion. Measured frequency of the radiation and measurement uncertainty are respectively given in columns 3 and 5. Calculated wavelength is given in column 4.

5.2 The Kilogram

The unit of mass, the kilogram, is the mass of the International Prototype of the Kilogram kept in air under three bell jars at the BIPM. It is a cylinder made of an alloy of 90% platinum and 10% iridium. Due to the inevitable accumulation of contaminants on its surfaces, the international prototype is subject to reversible surface contamination approaching 1 μg per year in mass. For this reason, the CIPM declared that, pending further research, the reference mass of the international prototype is that immediately after cleaning and washing by a specified method [3–5]. If the International Prototype of the Kilogram is used d days after cleaning, then its mass is calculated by the relation

Mass of the International Prototype of the Kilogram $= 1\text{kg} + 0.0368\, d\,\mu\text{g}$

The reference mass as defined above is used to calibrate national standards of platinum–iridium alloy [6].

5.2.1 Method of Cleaning

The BIPM method of cleaning platinum–iridium standards of mass with extra precautions for developing laboratories is described below in brief. The method essentially consists of two operations: (1) cleaning with chamois leather and (2) cleaning with steam.

Cleaning with Chamois Leather

Dust a piece of chamois leather dry, so that any loose dust particles get detached from it. The piece of Chamois leather should be around 20 cm by 20 cm. Soak it in equal volumes of ethanol and ether in a dish. Keep it soaked for 48 h. During this period, keep the dish covered to avoid dust. The absorbed solvent is wrung out. This step is necessary to clean the leather. To clean the leather sufficiently second and indeed a third soakings are necessary. Keep it in this way until the time of cleaning the standards come. Squeeze out the solvent and rub the standard quite hard with the leather. Estimated pressure applied is about 10 kPa. All surfaces of the standard are properly rubbed. This is to remove all organic dirt adhering to the standard.

Cleaning with Steam

Steam washing is carried out after solvent cleaning described in "Cleaning with Chamois Leather" section. The apparatus used is described in the following paragraph.

Apparatus: A round bottom flask of pyrex B or borosilicate glass of 1 L capacity is used. It has a long bent neck terminating in a long jet. The diameter of jet is around 2 mm. The flask is three-fourth filled with freshly doubled-distilled water. The water in the flask is heated though a round-shaped electric mantle of 350 W. With this heater, water would boil at the rate of about 0.5 L per hour.

Referring to Fig. 5.1, the standard is kept on a platinum–iridium plate, placed within a cylindrical shallow groove of the tripod. The upper part of the tripod can both be rotated about its vertical axis and can be raised or lowered by several centimetres.

Cleaning procedure: The steam jet is first pointed on the upper surface of the standard. The standard is rotated slowly so that all parts of the upper surface are

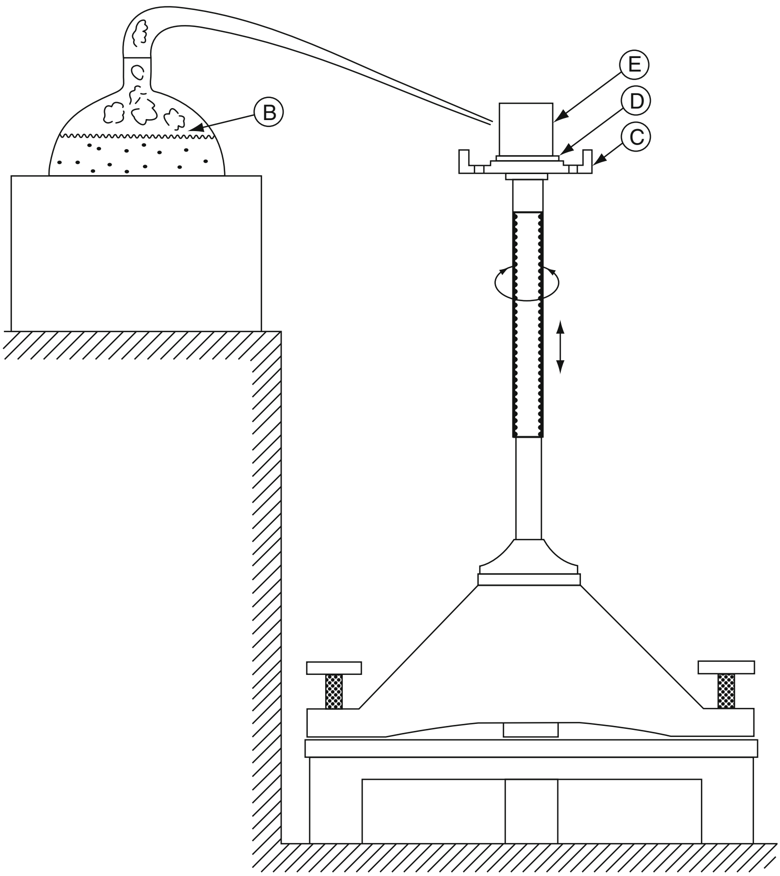

Fig. 5.1 Steam-washing apparatus

washed. After a few minutes of washing the upper surface, the platform with the standard is raised so that the steam jet may be directed on the curved surface. The standard is kept rotating and slowly moved up and down so that the steam jet covers all parts of the standard. The tapered glass tube terminating in the jet is kept about 5 mm away from the surface of the standard throughout the cleaning procedure.

The washing of the cylindrical surface continues for about 15–20 min. Water that was condensed on the surface of the standard and which has not trickled down is absorbed by an ash less filter paper. For this operation, a freshly cut edge of the paper is brought into contact with each drop and the water is allowed to flow into the paper by capillary action.

The standard is turned upside down so that it now rests on the base, which has just been washed. The washing procedure is repeated as described above. This includes washing of the curved surface.

Needless to say, the platinum–iridium disc is thoroughly cleaned with solvent and steam as described above before the procedure of cleaning the standard is undertaken.

After the procedure of cleaning and washing is completed, the standard is kept on its support and covered with a bell jar. No chemical desiccant is used.

Efficacy of the Cleaning Procedure

The BIPM conducted detailed experiments to find out the efficacy of the whole cleaning procedure. It has been found that about 90% of the cleaning is complete and that there is no significant change in mass of the platinum–iridium cylinder after the second cleaning. Hence it is recommended that a standard in platinum–iridium should be cleaned and washed twice before its mass is determined against a better known standard of mass.

5.2.2 Uncertainty in National Standards

The masses of 1 kg standards of the same alloy as that of the international prototype are compared in air against the international prototype by means of balances with a relative uncertainty approaching 1 part in 10^9.

In the case of stainless-steel 1 kg standards compared with the national prototype, the relative uncertainty is about 1 part in 10^8. Increased uncertainty is due to air buoyancy correction, mainly because of the limited knowledge of air density. The results of such comparisons made in vacuum, though unaffected by air buoyancy, are subject to additional corrections to account for changes in mass of the standard when cycled between vacuum and ambient air.

Mass standards representing multiples and submultiples of the kilogram are calibrated by the group weighing method.

5.3 The Second

A smaller number of national metrology laboratories realize the unit of time with the highest accuracy. They design and build primary frequency standards that produce electric oscillations at a frequency whose relationship to the transition frequency of the atom of caesium 133, which defines the second, is known. In 1997, the best of these primary standards produced the SI second with a relative combined standard uncertainty of 2 parts in 10^{15}. It is important to note that the definition of the second should be understood as the definition of the unit of proper time. It applies in a small spatial domain, which shares the motion of the caesium atom. In a laboratory sufficiently small to allow the effects of the non-uniformity of the gravitational field to be neglected when compared to the uncertainties of the realization of the second, the proper second is obtained after application of the special relativistic correction for the velocity of the atom in the laboratory. It is wrong to correct for the local gravitational field.

Primary frequency standards can also be used for calibration of the frequency of secondary time standards used in national time-service laboratories. These are generally commercial caesium clocks characterized by extreme long-term stability and are able to maintain a frequency with a stability of better than 1 part in 10^{14} over a few months. Such clocks are very good "timekeepers". The relative uncertainty of their frequencies is on the order of 10^{-12}. Time metrology laboratories also use hydrogen masers with good short-term stability. These instruments are used in all applications, which require a stable reference over intervals of less than one day, i.e. the stability of hydrogen masers is 1 part in 10^{15} in 10,000 s interval. In their basic form, hydrogen masers are subject to frequency drifts that become apparent when their mean frequencies are compared with that of a caesium clock over a few days. This drift is greatly reduced when the masers are operated in an active mode with a self-servo-controlled cavity. Caesium clocks and hydrogen masers must be operated under carefully controlled environmental conditions.

5.4 The Ampere

The realization of the ampere with a reasonable accuracy directly in terms of its definition is difficult and time-consuming. The cases are similar with the ohm and the volt (derived units of the SI). The best realization of the ampere is now obtained through combinations of realizations of the watt, the ohm and the volt. The watt realized electrically is compared by balance experiments with the watt realized mechanically. These experiments employ a coil in a magnetic flux and are devised in such a way that it is not necessary to know either the dimensions of the coil or the magnitude of the flux density. The ohm is realized using a Thompson–Lampard capacitor whose capacitance value can be changed by an amount that depends only on the magnitude of a linear displacement of a guard electrode. The volt is realized by means of a balance with which an electrostatic force is measured in terms of a mechanical force. The ampere may thus be deduced from the combinations of any of these two units. The relative uncertainty in the value of the ampere obtained in this way is estimated to be a few parts in 10^7. The ampere, ohm and volt may also be determined from measurements of various combinations of physical constants. Laboratory reference standards for the volt and the ohm based upon the Josephson and quantum Hall effects are, however, significantly more reproducible and stable than a few parts in 10^7. In order to take advantage of these highly stable methods of maintaining laboratory reference standards of the electrical units while at the same time taking care not to change their SI definitions, the 18th CGPM in 1987 adopted the resolution to represent the volt and the ohm by conventional values of the Josephson constant K_J and the von Klitzing constant R_K.

5.4.1 Josephson and Klitzing Constants

In 1988 the CIPM adopted Recommendations 1 (CI-1988) and 2 (CI-1988) which set exact values for the Josephson and von Klitzing constants, and called for laboratories to base their standards on these values from 1 January 1990.

However, at its meeting in 1988 the CCE specifically warned that the adoption of conventional values K_{J-90} and R_{K-90} do not constitute a redefinition of SI units but are only to be used to represent laboratory standards.

This recommendation is imperative; otherwise the status of μ_o would change from being exact and thereby abrogate the definition of the ampere, as well as producing electrical units, which would be incompatible with the definition of the kilogram and units derived from it.

Based on a review of the relevant R_K data as that existed at the time, the Consultative Committee for Electricity and Magnetism (CCEM) decided in September 2000 that the assigned relative standard uncertainty of a perfectly realized ohm representation based on the quantum Hall effect and R_{K-90} should be reduced to 1 part in 10^7. This decision was subsequently approved by the CIPM in October 2000.

5.4.2 Values of Josephson and Klitzing Constants

The recommended values for the two constants are:

483,597.9 GHz/V for Josephson constant denoted as $K_{J-}90$

25,812.807 for von Klitzing constant and denoted as R_{K-90}

5.5 The Candela

The definition of the candela is expressed strictly in physical terms. The objective of photometry, however, is to measure light in such a way that the result of the measurement correlates closely with the visual sensation experienced by a human observer of the same radiation. For this purpose, the International Commission on Illumination (CIE) introduced two special functions $V(\lambda)$ and $V'(\lambda)$, referred to as spectral luminous efficiency functions, which describe the relative spectral response of the average human eye for photopic (light-adapted) and scotopic (dark-adapted) vision, respectively. The more important of these two, the light-adapted function $V(\lambda)$, is expressed relative to its value for the monochromatic radiation to which the eye is most sensitive when adapted to high levels of luminance. That is, it is defined relative to radiation at 540×10^{12} Hz, which corresponds to a wavelength of 555.016 nm in standard air. For further details one may see [7].

The CIPM has approved the use of these functions with the effect that the corresponding photometric quantities are defined purely in physical terms as quantities proportional to the integral of a spectral power distribution, weighted according to a specified function of wavelength.

The candela has been one of its base units for a very long time; it remained a base unit even after being linked, in 1979, to the derived SI unit of power, the watt. The original photometric standards were light sources, and the earliest ones were candles; hence the name candela was given to the unit of luminous intensity. From 1948 to 1979 the radiation from a black body, Planck radiation, at the temperature of freezing platinum was used to define the candela. Today the definition is given in terms of monochromatic radiation rather than the broadband radiation implied by the black body definition. The value of 1/683 watt per steradian which appears in the present definition was chosen in 1979 to minimize any change in the mean representation of the photometric units maintained by the national standards laboratories.

The definition gives no indication as to how the candela should be realized, which has the great advantage that new techniques to realize the candela can be adopted without changing the definition of the base unit. Today, national metrology institutes realize the candela by radiometric methods. Standard lamps are still used, however, to maintain the photometric units. The standards provide as sources of either a known luminous intensity in a given direction or a known luminous flux.

5.6 The Kelvin

5.6.1 Triple Point of Water

The unit of the fundamental physical quantity known as thermodynamic temperature with the symbol T is the kelvin with the symbol K, and is defined as the fraction 1/273.16 of the thermodynamic temperature of the triple point of water [13th CGPM]. The International Committee for Weights and Measures (CIPM) [Recommendation 2, CI-2005] recently clarified the definition of the triple point of water by specifying the isotopic composition of the water to be that of Vienna Standard Mean Ocean Water (V-SMOW). For details one may see section "Unit of Thermodynamic Temperature (Kelvin)". Triple-point-of-water cells provide a convenient realization of this definition.

For temperatures other than the triple point of water, direct measurements of thermodynamic temperature require a primary thermometer based on a well-understood physical system whose temperature may be derived from measurements of other quantities. In practice, primary thermometry is difficult and time-consuming and not a practical means of disseminating the kelvin. As an alternative, the International Temperature Scale provides an internationally accepted method for realizing temperature in a practical way, known as temperature scales.

5.6.2 Temperature Scales

A temperature scale consists of assigned values of temperature to the phase transition points of some pure elements. Elements are taken in such a way that the temperature scale covers the maximum range. Besides these fixed points there are standardized thermometers like gas, platinum resistance and radiometric thermometers, and specific relations between the temperature and the quantity (pressure, resistance, etc.) that are measured.

5.6.3 ITS-90

The International Temperature Scale of 1990 (ITS-90) defines that both International Kelvin Temperature with symbol T_{90} and International Celsius temperature with symbol t_{90} are related as:

$$t(^{o}C) = T_{90}(\text{kelvin}) - 273.15$$

The ITS-90 covers the temperature range from 0.65 K to the highest temperature that can be determined practically by radiometric means.

The ITS-90 comprises a number of ranges and sub-ranges. Several of these ranges overlap, and where such overlapping exists, the differing definitions are of equal status. Normally the differences are negligible for practical purposes.

There were significant differences between the values of T_{90} and the corresponding values of T_{68} that the old scale used, till the adoption of T_{90}.

Defining Fixed Points on ITS-90

ITS-90 fixed points as adopted by CGPM are given in Table 5.2.

Here

e-H_2 is hydrogen at the equilibrium concentrations of the ortho- and para-molecular forms.

V is vapour pressure point.

T is triple point (temperature at which solid, liquid and vapour phases are in equilibrium).

Table 5.2 Defining fixed points of the ITS-90

S. No.	Temperature (K)	Substance	State	W_r (T_{90})
1	3 to 5	He	V	
2	13.803 3	$e - H_2$	T	0.001 190 07
3	≈17	$e - H_2$ (or He)	V or gas	
4	≈20.3	$e - H_2$ (or He)	V or Gas	
5	24.55 61	Ne	T	0.008 449 74
6	54.3584	O_2	T	0.091 718 04
7	83.805 8	Ar	T	0.215 895 75
8	243.315 6	Hg	T	0.844 142 11
9	273.16	H_2O	T	1.000 000 00
10	302.914 6	Ga	M	1.118 138 89
11	429.748 5	In	F	1.609 801 85
12	505.078	Sn	F	1.892 797 68
13	692.677	Zn	F	2.568 917 30
14	933.473	Al	F	3.376 008 60
15	1234.93	Ag	F	4.286, 420 53
16	1337.33	Au	F	
17	1357.77	Cu	F	

G is gas thermometer.
M is melting point and F is freezing point, each at a pressure of 101,325 Pa at which solid and liquid phases are in equilibrium.
All substances except ^{3}He are of natural isotopic composition.

Beginning in 1927, the CIPM, acting under the authority of the General Conference on Weights and Measures (CGPM) and, since 1937, on the advice of its Consultative Committee for Thermometry (CCT), has adopted a series of International Temperature Scales. Subsequent to the 1927 scale, new scales have been adopted in 1948, 1968 and 1990, with occasional minor revisions in intervening years.

ITS-90 and PLTS-2000

The International Temperature Scale of 1990 (ITS-90) was extended downward from 1 K to 0.9 mK in 2000. The CIPM adopted this as a supplemental scale from 0.9 mK to 1 K (PLTS-2000) [Recommendation 1, CI-2000]. The ITS-90 and the PLTS-2000 define temperatures T_{90} and T_{2000} that are good approximations to thermodynamic temperature.

Considerable research has been conducted on establishing a temperature scale extending to temperatures lower than 0.65 K; the PLTS-2000 is the resulting outcome. The PLTS-200 defines temperature from 1 K down to 0.9 mK. The PLTS-2000 is explicitly a provisional scale, as the data sets comprising the basis of the scale are somewhat inconsistent. In the temperature range 0.65 K to 1 K, temperatures may be defined on either the ITS-90 or the PLTS-2000. Either scale is acceptable; the choice of scale typically is dictated by convenience or the attainable uncertainty of realization. In those rare cases where the use of either scale is convenient, T_{2000} is a better approximation of thermodynamic temperature than ITS-90 in the region of overlap [1].

The low temperature scale from 0.m K to 1 K, PLTS-2000 is defined by the equation relating the melting pressure p of ^{3}He to temperature T_{2000}.

$$p = \sum_{p=-3}^{p=9} a_p (T_{2000})^p$$

Here p is expressed in MPa and T_{2000} in kelvin.

The values of a_i are:

$a_{-3} = -1.385\,5442 \cdot 10^{-12}$
$a_{-2} = -4.555\,702\,6 \cdot 10^{-9}$
$a_{-1} = -6.443\,086\,9 \cdot 10^{-6}$
$a_0 = 3.446\,743\,4 \cdot 10^{0}$
$a_1 = -4.417\,643\,8 \cdot 10^{0}$
$a_2 = 1.541\,743\,7 \cdot 10^{1}$
$a_3 = -3.578\,985\,3 \cdot 10^{1}$
$a_4 = 7.149\,912\,5 \cdot 10^{1}$
$a_5 = -1.041\,437\,9 \cdot 10^{2}$
$a_6 = 1.051\,853\,8 \cdot 10^{2}$
$a_7 = -6.944\,376\,7 \cdot 10^{1}$
$a_8 = 2.683\,3087 \cdot 10^{1}$
$a_9 = -4.587\,570\,9 \cdot 10^{0}$

Defining Fixed Points on PLTS-2000

Four fixed points in the range are:

Point	p/MPa	T_{2000}/mK
minimum	2.931 13	315.24
A	3.434 07	2.444
A-B	3.436 09	1.896
Néel	3.439 34	0.902

Here p is pressure in MPa and T_{2000} is in millikelvin.

Uncertainty is 0.3% at the temperature 25 mK, which increases to 2% at 0.9mK.

For further details one may refer to [8] or SI Appendix 2 on the BIPM website.

5.7 The Mole

All quantitative results of chemical analyses or of dosages can be expressed in units of amount of substance of the elementary entities, for which the base unit is the mole. The principle of physical measurement based on this unit is explained below.

The simplest case is that of a sample of a pure substance consisting of atoms. Let us consider X as the chemical symbol of these atoms. A mole of atoms X contains by definition as many atoms as there are ^{12}C atoms in 0.012 kg of carbon 12. As neither the mass $m(^{12}\mathrm{C})$ of an atom of carbon 12 nor the mass $m(X)$ of an atom X can be measured accurately, we use the ratio of these masses, $m(X)/m(^{12}\mathrm{C})$, which can be determined accurately, for example, by means of a Penning trap. The mass corresponding to 1 mol of X is then

$[m(X)/m(^{12}\mathrm{C})] \times 0.012\,\mathrm{kg}$, which is expressed by the statement that the molar mass $M(X)$ of X (quotient of mass by amount of substance) is

$$M(X) = [m(X)/m(^{12}\mathrm{C})] \times 0.012\,\mathrm{kg\,mol^{-1}}.$$

For example, the atom of fluorine ^{19}F and the atom of carbon ^{12}C have masses that are in the approximate ratio of 18.9984/12. The molar mass of the molecular gas F_2 is:

$$\mathrm{M(F_2)} = (2 \times 18.9984/12) \times 0.012 = 0.037\,9968\,\mathrm{kg\,mol^{-1}}$$

Hence the amount of substance in mol corresponding to a given mass is calculated on dividing by 0.037 996 8. For example, 0.0500 kg of F_2 is 0.0500/0.037 9988 = 1.316 mol.

In the case of a pure substance made up of molecules B, which are combinations of atoms X, Y, ... according to the chemical formula $\mathrm{B} = \mathrm{X}_\alpha \mathrm{Y}_\beta \ldots$, the mass of one molecule is $m(\mathrm{B}) = \alpha m(\mathrm{X}) + \beta m(\mathrm{Y}) + \ldots$

This mass is not known precisely but the ratio $m(\mathrm{B})/m(^{12}\mathrm{C})$ can be determined accurately.

The molar mass of a molecular substance B is then:

$$\begin{aligned}\mathrm{M(B)} &= \mathrm{m(B)/m(^{12}C)}\\ &= \{\alpha \mathrm{m(X)/m(^{12}C)} + \beta \mathrm{m(Y)/m(^{12}C)} + \ldots\ldots\} \times 0.012\,\mathrm{kg\,mol^{-1}}.\end{aligned}$$

The same procedure is used in the more general case when the composition of substance B is specified as $\mathrm{X}_\alpha \mathrm{Y}_\beta$ even if $\alpha, \beta \ldots$ are not integers. If we denote the mass ratios $m(\mathrm{X})/m(^{12}\mathrm{C}), m(\mathrm{Y})/m(^{12}\mathrm{C}), \ldots$ by $r(\mathrm{X})$, $r(\mathrm{Y})$, ... respectively, the molar mass of substance B is given by the formula:

$$M(\mathrm{B}) = [\alpha\, r(\mathrm{X}) + \beta r(\mathrm{Y}) + \ldots] \times 0.012\,\mathrm{kg\,mol^{-1}}.$$

Other methods for the measurement of amount of substance are based on the laws of physics and physical chemistry. Three examples are:

1. With perfect gases, 1 mol of particles of any gas occupies the same volume at a temperature T and a pressure p (approximately 0.0224 m^3 at $T = 273.15$ K and $p = 101\,325$ Pa): this provides a method of measuring the ratio of amounts of substance for any two gases (the corrections to apply if the gases are not perfect are well known).
2. For quantitative electrolytic reactions the ratio of amounts of substance can be obtained by measuring quantities of electricity. For example, 1 mol of Ag and 0.5 mol of Cu are deposited on a cathode by the same quantity of electricity (approximately 96,483 C).
3. Application of the laws of extremely dilute solutions is yet another method of determining ratios of amounts of substance.

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Chapter 6
Derived Quantities and Their Units

6.1 Derived Quantities

Derived quantities are those that may be expressed in terms of base or derived quantities by means of the mathematical symbols of multiplication and division only (no addition or subtraction or any other sign). Basically they may be divided into two categories. The first group consists of derived quantities, which have proper units. Their units can be expressed in terms of units of base or derived units. The second class consists of quantities that do not have units expressed in terms of base or derived units. These quantities, as discussed in Chap. 2, are known as dimensionless quantities or quantities of dimension 1.

6.2 Units of Derived Quantities

The basic purpose of this book is to describe SI units. Derived units belonging to the International System of Units are coherent derived units in terms of base units, coherent derived units with special names, coherent derived units expressed in terms of derived units with special names and base units, dimensionless quantities or quantities with dimension 1 with special names. The dimensionless quantities are simple ratios of two similar quantities; hence they are pure numbers having 1 as their unit. Units outside the SI but accepted for use with SI units, non-SI units of those quantities whose values are determined in SI units because of their importance in physics, and some units that are being used for historical reasons will be discussed. Finally non-SI units associated with CGS and CGS–Gaussian system of units will also be given.

6.3 SI Derived Units

For convenience, coherent derived units are grouped in three sections: (1) derived units that are derived from the base units only; (2) derived units that have been assigned special names and symbols; and (3) derived units that are formed from derived units with special names and base units.

6.3.1 Units Expressed in Terms of Base Units

Table 6.1 lists some coherent derived units in terms of base units using their respective definitions except the intensity of magnetization.

The word coherent has been specifically used to distinguish between the units expressed in terms of coherent base units and units expressed in terms of their multiples or sub-multiples. For example, the metre is the base unit of length and not the centimetre or any multiple or sub-multiple of the metre. Hence the coherent unit of area is square metre (m^2) and not square centimetre (cm^2).

The unit of pole strength is derived from the relation

$$\text{Force} = \mu \cdot P_1 P_2 / r^2$$

Units of force and distance r are respectively newton and metre, and that of μ is NA^{-2}, giving the unit of pole strength as Am. The unit of magnetic moment, therefore, should be Am^2. The unit of intensity of magnetization, which is magnetic moment per unit volume, should be $Am^2/m^3 = Am^{-1}$.

Table 6.1 Examples of SI coherent derived units in terms of base units

Derived unit	Symbol	Name	Symbol
Area	A	square metre	m^2
Volume	V	cubic metre	m^3
Speed/velocity	v	metre per second	$m \cdot s^{-1}$
Acceleration	A	metre per second square	$m \cdot s^{-2}$
Wave number	σ, ν	reciprocal of metre	m^{-1}
Specific volume	υ	cubic metre per kilogram	$m^3 \cdot kg^{-1}$
Mass density	ρ	kilogram per metre cube	$kg \cdot m^{-3}$
Current density	j	ampere per square metre	$A \cdot m^{-2}$
Amount concentration	c	mole per cubic metre	$mol \cdot m^{-3}$
Concentration	ρ, γ	kilogram per metre cube	$kg \cdot m^{-3}$
Luminance	L_v	candela per square metre	$cd \cdot m^{-2}$
Intensity of magnetization (magnetic field)	H	ampere per metre	$A \cdot m^{-1}$

Note: Symbols of quantities are single alphabets or Greek letters and are written in italics. The following may be perused to understand the derivation of the unit of intensity of magnetization, H

The intensity of magnetization is a better nomenclature for H than the name magnetic field strength. The fact is that H is the ability to magnetize a material. After introducing the concept of permeability of free space μ_0, all formulae of magnetic and electromagnetic fields, will contain μ as a factor; hence, whenever the term magnetic field will occur it will denote the magnetic flux density whose unit is tesla.

$$B = \text{Magnetic flux density} = \mu H$$

Hence:

$$\{\mathrm{B}\} = \mathrm{NA}^{-2} \cdot \mathrm{A/m} = \mathrm{kg} \cdot \mathrm{s}^{-1}\mathrm{A}^{-1},$$

which is the same as tesla = Weber/m^2 (Table 6.4).

6.3.2 Derived Units with Special Names

Among these names and symbols, the last four entries in Table 6.2 are of particular note since they were accepted by the 15th CGPM (1975) [1] in Resolutions 8 and 9, the 16th CGPM (1979) [2] and the 21st CGPM (1999) [3], specifically with a view to safeguarding human health.

6.3.3 Derived Units Formed from the Derived Units with Special Names

Derived units, listed in Table 6.2, have been given special names and symbols for convenience and for honouring pioneer scientists. These names and symbols may themselves be used to express other derived units. Examples of such units are given in Table 6.3. The special names and symbols are a compact form for the expression of units that are used frequently. Relations used for arriving at the units are indicated in column 3 of Table 6.3.

6.3.4 Derived Quantities of Dimension 1

The quantities of dimension 1 or dimensionless quantities may be any of the following types:

1. The ratios of two quantities of the same kind. Examples of such quantities are angle, solid angle, refractive index, relative permeability, friction factor, Neper and decibel.

Table 6.2 Coherent derived units with special names

Quantity	Name	S	Relation used		
Plane angle	radian(1)	rad	Arc/radius of arc	1	m/m
Solid angle	steradian(1)	sr(2)	Surface area/ (radius)^2	1	m^2/m^2
Frequency	hertz(3)	Hz	Inverse of time period	–	s^{-1}
Force	newton	N	Mass.Acceleration	–	$m \cdot kg \cdot s^{-2}$
Pressure, stress	pascal	Pa	Force per unit area	$N \cdot m^{-2}$	$m^{-1} \cdot kg \cdot s^{-2}$
Energy of any kind	joule	J	Force. displacement	$N \cdot m$	$m^2 \cdot kg \cdot s^{-2}$
Power, radiant flux	watt	W	Energy per unit time	J/s	$m^2 \cdot kg \cdot s^{-3}$
Electric charge amount of charge	coulomb	C	ampere.time	–	$A \cdot s$
Electric potential or e.m.f	volt	V	Power = voltage.current	W/A	$m^2 \cdot kg \cdot s^{-3} \cdot A^{-1}$
Capacitance	farad	F	Charge =/Potential.capacitance	C/V	$m^{-2} \cdot kg^{-1} \cdot s^4 \cdot A^2$
Electric resistance	ohm	Ω	Ohms Law	V/A	$m^2 \cdot kg \cdot s^{-3} \cdot A^{-2}$
Electric conductance	siemens	S	Inverse of resistance	A/V	$m^{-2} \cdot kg^{-1} \cdot s^3 \cdot A^2$
Magnetic flux	weber	Wb	dΦ/dt = voltage	$V \cdot s$	$m^2 \cdot kg \cdot s^{-2} \cdot A^{-1}$
Magnetic flux density	tesla	T	magnetic flux per unit area	Wb/m^2	$kg \cdot s^{-2} \cdot A^{-1}$
Inductance	henry	H	Flux = Inductance.current	Wb/A	$m^2 \cdot kg \cdot s^{-2} \cdot A^{-2}$
Celsius temperature	degree Celsius(4)	°C(5)	By definition	(3,4)	K
Luminous flux	lumen	lm	By definition	cd.sr(1)	cd
Illuminance	lux	lx	lumen per unit area	lm/m^2	$m^{-2} \cdot cd$
Activity referred to radio- nuclide	becquerel (6)	Bq	Definition	Bq (5)	s^{-1}
Absorbed dose, Specific energy imparted, kerma,	gray	Gy	Definition	J/kg	$m^2 \cdot s^{-2}$
Dose equivalent, Ambient dose equivalent, Directional dose equivalent, Personal dose equivalent,	sievert(7)	Sv	Definition	J/kg	$m^2 \cdot s^{-2}$
Catalytic activity	katal	kat	Definition	–	$mol \cdot s^{-1}$

To honour the scientists concerned in a specific field a unit has been assigned his/her name.
S stands for symbol of quantity
Multiplication sign between the two symbols is a point (·) and is above the line midway of the height of the letter, but its use is optional.
The SI prefixes may be used with any of the special names and symbol, but when this is done the resulting unit will not be coherent

[1] The radian and steradian are special names for the number one that may be used to convey information about the quantity concerned. In practice the symbols rad and sr are used where appropriate, but symbol for the derived unit 1 is generally omitted in specifying the values of dimensionless quantities.
[2] In photometry the symbol sr is usually retained in expressions for other units.
[3] The hertz is used only for periodic phenomena and the becquerel is used for stochastic process in activity of radio-nuclide.
[4] The degree Celsius is the special name for the kelvin used to express Celsius temperature. The degree Celsius and the kelvin are equal in magnitude, so that numerical value of a temperature difference or temperature interval is the same when expressed in degree Celsius or in kelvins.
[5] The degree Celsius may be used in combination with SI prefixes, e. g. milli-degree Celsius, m $^\circ$C.
[6] Activity referred to a nuclide is sometimes incorrectly called radioactivity
[7] The sievert (symbol Sv) is an SI derived unit of equivalent dose or effective dose (of radiation), and so is dependent upon the biological effects of radiation as opposed to the physical aspects, characterised by the absorbed dose (measured in Gray).

Table 6.3 SI-derived units whose names and symbols include derived units with special names and symbols

Quantity	Name	Relation used	In terms of special names	In terms of base units
Dynamic viscosity	pascal second	Poiseulle's formula[a]	Pa · s	$m^{-1} \cdot kg \cdot s^{-1}$
Kinematic viscosity[b]	metre square per second	Dynamic viscosity/density	–	$m^2 \cdot s^{-1}$
Moment of force	newton metre	Definition	N · m	$m^2 \cdot kg \cdot s^{-2}$
Angular velocity	radian per second	Definition	rad/s	s^{-1}
Angular acceleration	radian per second square	Definition	rad/s^2	s^{-2}
Surface tension	newton per metre	Force per unit length	N/m	$kg \cdot s^{-2}$
Heat flux density, Irradiance	watt per square metre	Definition	W/m^2	$kg.s^{-3}$
Heat capacity, entropy	joule per kelvin	Energy required to change temperature through 1 K	J/K	$m^2 \cdot kg \cdot s^{-2} \cdot K^{-1}$
Specific heat capacity, specific entropy	joule per kilogram kelvin	Energy required to change unit temperature of 1 kg of substance	J/(kg · K)	$m^2 \cdot s^{-2} \cdot K^{-1}$
Specific energy	joule per kilogram	Definition		$m^2 \cdot s^{-2}$
Thermal conductivity	watt per metre kelvin	Conductivity formula[c]	W/m · K	$m \cdot kg \cdot s^{-3} \cdot K^{-1}$
Energy density	joule per cubic metre	Definition	J/m^3	$m^{-1} \cdot kg \cdot s^{-2}$
Electric field strength	volt/metre	dV/dr	V/m	$m \cdot kg \cdot s^{-3} \cdot A^{-1}$
Electric charge density	coulomb per cubic metre	Definition	C/m^3	$m^{-3} \cdot s \cdot A$
Electric flux density Electric displacement	coulomb per square metre	Definition	C/m^2	$m^{-2} \cdot s \cdot A$
Permittivity[d]	farad per metre	Capacitance of a spherical conductor	F/m	$m^{-3} \cdot kg^{-1} \cdot s^4 \cdot A^2$
Permeability[e]	henry per metre	NA^{-2}	H/m	$m \cdot kg \cdot s^{-2} \cdot A^{-2}$
Molar energy, Molar heat capacity	joule per mole	Definition	J/mol	$m^2\ kg\ s^{-1}\ mol^{-1}$
Exposure (X and rays)	coulomb per kg	Definition	C/kg	$kg^{-1} \cdot s \cdot A$
Absorbed dose rate	gray per second	Definition	Gy/s	$m^2 \cdot s^{-3}$
Radiant intensity	watt per steradian	Definition	W/sr	$m^2 \cdot kg \cdot s^{-3}$
Radiance	watt per square metre · steradian	Definition	W/m^2	$kg \cdot s^{-3}$
Catalytic (activity) Concentration	katal per cubic metre	Definition	kat/m^3	$m^{-3} \cdot s^{-1} \cdot mol$

Further it may, in general, be added that

A single SI unit may correspond to several different quantities, for example, work and moment of a force have the same unit. Table 6.3 provides several examples, although they are not exhaustive. Thus, joule per kelvin (J/K) is the SI unit for the quantity heat capacity as well as for the quantity entropy; similarly, the ampere (A) is the SI unit for the base quantity electric current as well as for the derived quantity magneto motive force. It is therefore important not to use the unit alone to specify the quantity. This rule applies not only to scientific and technical texts but also, for example, to measuring instruments (i.e. an instrument should indicate both the unit and the quantity measured).

A derived unit can often be expressed in different ways by combining the names of basic units with special names of derived units. This, however, is an algebraic freedom to be governed by common-sense physical considerations. Joule, for example, may be written as newton metre or kilogram metre per square second, but in a given situation some forms may be more helpful than others.

[a]Volume flow rate of a liquid of dynamic viscosity η in a capillary tube of radius a and length L under a pressure head P is given by Poiseulle's formula as

$$V/T = Pa^4/8\eta L$$

Expressing the quantities used in terms of base units we get:

$m^3/s = Pa \cdot m^4/(\text{unit of } \eta)m$, which gives

Unit $\eta = Pa \cdot m^3/m^3 \cdot s^{-1} = Pa \cdot s$

[b]Though the unit of kinematic viscosity is not associated with special name but its closeness by way of definition prompted the author to include it in this table

[c]Rate of flow of heat energy in the steady state is proportional to the difference in temperature and area of the conductor divided by its length. If k is conductivity, then we may write:

Heat energy/time = power = k · difference in temperature = area/length.

Expressing them in base units:

Unit (k) $= W \cdot m/K{\cdot}m^2 = W/m{\cdot}K$

[d]Capacitance of a spherical conductor $= 4\pi\varepsilon r$, where r is the radius of the spherical conductor and is ε is permittivity.

Expressing this quantity equation in terms of units, we get:

$\{\text{capacitance}\} = 1\{\varepsilon\} \cdot \{\text{radius}\}$, which gives

Unit (ε) = Farad/m

[e]Unit μ from the formula defining of electric current ampere $= NA^{-2}$, expressing N in base units.

Unit $\mu = m \cdot kg \cdot s^{-2} \cdot A^{-2}$

But unit henry $H = m^2 \cdot kg \cdot s^{-2} \cdot A^{-2}/m$; by dividing we get

Unit μ = H/m.

Since the expression used for defining current is the gateway between mechanical and electrical quantities, in the view of the author, the unit μ should be expressed as NA^{-2}.

2. These are defined as a complex product of simpler quantities in such a way that when each simpler quantity is expressed in terms of base units, the algebraic sum of the exponents of each base unit becomes zero, and hence the quantity is dimensionless or of dimension 1 depending upon the convention followed. For example, Reynolds number (R_e) is defined as

$$R_e = \rho \upsilon l/\eta$$
$$[Re] = ML^{-3} \cdot LT^{-1}L/M \cdot L^{-1}T^{-1} = M^0L^0T^0 = 1$$

3. Yet, there is another class of quantities which represent a count, such as a number of molecules, degeneracy (number of energy levels) and partition function in statistical thermodynamics etc.; these are also called quantities of dimension 1 or dimensionless quantities.

All of the quantities of dimension 1 have the coherent SI unit 1. Their values are simply expressed as numbers and, in general, the unit 1 is not explicitly shown. In a few cases, however, special names are given to such units, mainly to avoid confusion between some compound derived units involving such quantities. Examples of such units are the radian, steradian and refractive index, and examples of such quantities are given in Table 6.4.

1. For better understanding, the name steradian and the symbol sr are usually retained in expressions for units representing photometric units.
2. The neper or bel is used to express values of such logarithmic quantities as field level, power level, sound pressure level, and logarithmic decrement. Natural logarithms are used to obtain the numerical values of quantities expressed in nepers, and logarithms to the base 10 are expressed in Bel. The neper is coherent with the SI, but not yet adopted by the CGPM as an SI unit. For further information one may see ISO 31.

Table 6.4 Quantities of dimension 1

Derived quantity	Name of unit	Symbol	In terms of ratio of two quantities
Plane angle	radian	rad	m/m
Solid angle	steradian	sr	m^2/m^2
Ratio of permeability in the medium and vacuum	relative permeability	μ_r	NA^{-2}/NA^{-2}
Ratio of velocity of light in medium and vacuum	refractive index	μ,n	ms^{-1}/ms^{-1}
Logarithmic ratio of two	bel	B	Log_{10} (Pa/Pa)
quantities of same kind:	neper	Np	Log_e (Pa/Pa)
e.g. sound and pressure	decibel	dB	B/10
Percentage ratio of solutes in solutions	brix	°B	% of sugar by mass in cane sugar solution
	alcoholic degree	°A	% of pure alcohol by volume in water

6.4 Units Outside the SI

The CIPM (1969) recognized that users would wish to employ the SI with certain units that are not part of it but are important and widely used. So the CIPM listed such units in three classes, namely (1) units to be maintained as they are; (2) units that may be tolerated temporarily for some time; and (3) units to be avoided. Standards of Weights and Measures (National Standard) Rules, 1988, framed under the Standards of Weights and Measures Act, 1976 [4], followed these CIPM decisions in regard to non-SI units.

However, on reviewing this categorization, the CIPM (1996) agreed to a new classification of non-SI units: units accepted for use with the SI whose values are obtained experimentally and other units currently accepted for use with the SI to satisfy the needs of special interests.

The CIPM (2004) has revised the classification of non-SI units again:

1. Non-SI units that are accepted for use with the International System by the CIPM, because they are widely used with the SI in matters of everyday life (Table 6.5). Their use is expected to continue indefinitely, and each has an exact definition in terms of an SI unit.
2. Non-SI units that are related to fundamental constants (Table 6.6), and their values, from time to time, have to be determined experimentally.
3. Some non-SI units have exactly defined values in terms of SI units, and are used in particular circumstances to satisfy the needs of commercial, legal or specialized scientific interests (Table 6.7). It is likely that these units will continue to be used for many years.
4. Non-SI units, which are important for the interpretation of older scientific texts (Table 6.8).
5. Non-SI units found in old literature (Table 6.9).

6.4.1 Units Accepted for Use with the SI

Table 6.5 lists non-SI units that are accepted for use with the SI. It includes units that are commonly used, in particular the traditional units of time and angle, together with a few other units that have assumed increasing technical importance. It also contains the hectare, the litre and the tonne, which are common in everyday use throughout the world, and which differ from the corresponding coherent SI units by an integer power of ten. The SI prefixes are used with several of these units, but not with the units of time.

6.4.2 Non-SI Units with Experimentally Obtained Values

Non-SI units indicated in Table 6.6 are also accepted for use with the SI, whose values in SI units have been obtained by experiment and are therefore not known

Table 6.5 Non-SI units accepted for use with the International System

Quantity	Name	Symbol	Value in SI units
Time	minute	min	1 min = 60 s
	hour[(a)]	h	1 h = 60 min = 3,600 s
	day	d	1 day = 24 h = 86,400 s
Plane angle	degree[(b,c)]	°	$1° = (\pi/180)$ rad
	minute	′	$1' = (1/60)° = (\pi/10\,800)$ rad
	second[(d)]	″	$1'' = (1/60)' = (\pi/648\,000)$ rad
Area	acre	a	$1 \text{dam}^2 = 100\ \text{m}^2$
	hectare[(e)]	ha	$1\ \text{hm}^2 = (100\text{m})^2 = 10^4\ \text{m}^2$
Volume	litre[(f)]	l, L	$1\ \text{l} = 1\ \text{dm}^3 = 10^{-3}\ \text{m}^3$
Mass	tonne[(g)]	t	$1\ \text{t} = 10^3$ kg

[a]The symbol of time was included in Resolution 7 of the 9th CGPM (1948; CR, 70).
[b]ISO 31 recommends that the degree be subdivided decimally rather than using the minute and the second. However, for navigation and surveying, the minute has the advantage that one minute of latitude on the surface of the earth corresponds to approximately one nautical mile.
[c]The gon (or grad) is an alternative unit of the plane angle to the degree, defined as $(\pi/200)$ rad. Thus, there are 100 gon in a right angle. The advantage of the gon in navigation is that one kilometre on the surface of the earth subtends an angle of one centigon at the centre of the earth. This is because of the fact that the distance from the pole to the equator is approximately 10,000 km; however, the gon is rarely used.
[d]For applications in astronomy, small angles are measured in arcseconds (i.e. seconds of plane angle) and denoted as ″. Its submultiples like milliarcseconds, microarcseconds and picoarcseconds are denoted as mas, μas and pas, respectively. The arcsecond is an alternative name for second of plane angle.
[e]The unit hectare and its symbol ha were adopted by the CIPM in 1879 (PV, 1879, 41). The hectare is used to express land area.
[f]The litre and its symbol l were adopted by the 3rd CGPM 1901 as a unit of volume of 1 kg mass of water at the temperature of its maximum density. The 12th CGPM (1964) by Resolution 6 abrogated the name litre as the unit of volume. However, the 16th CGPM (1979) adopted the use of litre with the symbols l or L as another name for decimetre cube. The alternative symbol L was adopted in order to avoid the risk of confusion between the letter l (el) and the numeral 1 (one). For details see [2].
[g]The tonne and its symbol t were adopted by the CIPM in 1879 (PV, 1879, 41). In English-speaking countries this unit is sometimes called the 'metric ton'

exactly. Their values are given with their combined standard uncertainties, which apply to the last two digits, shown in parentheses. These units are in common use in certain specialized fields.

6.4.3 Non-SI Units Used by Special Groups

People working in special fields who see an advantage to express their viewpoints in specific units, given in Table 6.7, are allowed to use these units. However, as SI units are the international meeting ground in terms of which all other units are defined, those who use units from Tables 6.7 or 6.8 should always give the definition of the units they use in terms of SI units.

Table 6.6 Non-SI units whose values in SI units must be obtained experimentally

Quantity	Name of unit	Symbol for unit	Value in SI units[(a)]
	Units accepted for use with the SI		
Energy	electronvolt[(b)]	eV	1 eV = 1.602 176 53 (14) × 10^{-19} J
Mass	dalton[(c)]	Da	1 Da = 1.660 538 86 (28) × 10^{-27} kg
	unified atomic mass unit	U	1 u = 1 Da
Length	astronomical unit[(d)]	ua	1 ua = 1.495 978 706 91 (6) × 10^{11}m
	Natural units (n.u.)		
Speed	n.u. of speed (speed of light in vacuum)	c_o	299 792 458 m/s (exact)
Action	n.u. of action (reduced Planck's constant)	$\hbar$	1.054 571 68 (18) × 10^{-34} J s
Mass	n.u. of mass (electron mass)	m_e	9.109 3826 (16) × 10^{-31} kg
Time	n.u. of time	$h/(m_e c_0^2)$	1.288 088 6677 (86) × 10^{-21} s
	Atomic units (a.u.)		
Charge	a.u. of charge, (elementary charge)	e	1.602 176 53 (14) × 10^{-19} C
Mass	a.u. of mass, (electron mass)	m_e	9.109 3826 (16) × 10^{-31} kg
Action	a.u. of action, (reduced Planck's constant)	h	1.054 571 68 (18) × 10^{-34} J s
Length	a.u. of length, bohr (Bohr radius)	a_0	0.529 177 2108 (18) × 10^{-10} m
Energy	a.u. of energy, hartree (Hartree energy)	E_h	4.359 744 17 (75) × 10^{-18} J
Time	a.u. of time	h/E_h	2.418 884 326 505 (16) × 010^{-17} s

[a]The values in SI units of all units in this table, except the astronomical unit, are taken from the 2002 CODATA set of recommended values of the fundamental physical constants [5]. The combined standard uncertainty in the last two digits is given in parenthesis

[b]The electronvolt is the kinetic energy acquired by an electron in passing through a potential difference of one volt in vacuum. The electronvolt is often combined with the SI prefixes

[c]The dalton (Da) and the unified atomic mass unit (u) are alternative names (and symbols) for the same unit, which is equal to 1/12 times the mass of a free carbon 12 atom, at rest and in its ground state. The dalton is often combined with SI prefixes, for example to express the masses of large molecules in kilodaltons, kDa, or megadaltons, MDa, or to express the values of small mass differences of atoms or molecules in nanodaltons, nDa, or even picodaltons, pDa

[d]The astronomical unit is approximately equal to the mean of the distances between Earth and Sun. It is the radius of an unperturbed circular Newtonian orbit about the Sun of a particle having infinitesimal mass, moving with a mean motion of 0.017 202 098 95 radians per day (known as the Gaussian constant). The value given for the astronomical unit is quoted from the IERS Conventions 2003 [6]. The value of the astronomical unit in metres comes from the JPL ephemerides DE403 [7]

6.4.4 Other Non-SI Units with Special Names

The quantities and their units given in Table 6.8 deal with the relationship between the CGS units and the SI, and the CGS units that were assigned special names are also given in the 6.8. In the field of mechanics, the CGS system of units was built

Table 6.7 Non-SI units

Quantity	Name	Symbol	Value in SI units
Distance	nautical mile[(a)]	M	1 M = 1,852 m
Speed	knot	kn	1 kn = (1,852/3,600) m/s
Pressure	bar[(b)]	bar[(3)]	1 bar = 0.1 MPa = 10^5 Pa
	millimetres of mercury[(c)]	mmHg	1 mmHg = 133.322 Pa
Length	angstrom[(d)]	Å	1 Å = 0.1 nm = 10^{-10} m
Area	barn[(e)]	b	1 b = 100 fm^2 = 10^{-28} m^2
Logarithmic ratio:	neper[(f)]	(Np)	Logarithmic ratio to the base e
sound/pressure	bel[(g)]	B	Logarithmic ratio to the base 10
	decibel	dB	1 dB = (1/10)B

[a]The nautical mile is a special unit employed for marine and aerial navigation to express distances. The first International Extraordinary Hydrographic Conference, Monaco, 1929, adopted the value given above under the name "International nautical mile". As yet there is no internationally agreed symbol. This unit was originally chosen because one nautical mile on the surface of the Earth subtends approximately one minute of angle at its centre. As yet there is no internationally agreed symbol, but the symbols M, NM, Nm, and nmi are all used. The unit was originally chosen, and continues to be used, because one nautical mile on the surface of the Earth subtends approximately one minute of angle at the centre of the Earth, which is convenient when latitude and longitude are measured in degrees and minutes of angle

[b]The bar and its symbol are included in Resolution 7 of the 9th CGPM (1948; CR, 70). Since 1982 one bar has been used as the standard pressure for tabulating all thermodynamic data. Prior to 1982 the standard pressure used to be the standard atmosphere, equal to 1.013 25 bar, or 101 325 Pa

[c]The millimetre of mercury is a legal unit for the measurement of blood pressure in some countries.

[d]The ångström is widely used by x-ray crystallographers and structural chemists because all chemical bonds lie in the range 1–3 ångströms. However it has no official sanction from the CIPM or the CGPM

[e]The barn is a special unit employed in nuclear physics to express effective cross-section

[f]The neper is used to express values of such logarithmic quantities as field level, power level, sound pressure level, and logarithmic decrement. Natural logarithms are used to obtain the numerical values of quantities expressed in nepers. The neper is coherent with the SI, but not yet adopted by the CGPM as a SI unit. For further information one may see ISO 31

[g]The bel is used to express values of such logarithmic quantities as field level, power level, sound pressure level and attenuation. Logarithms to base ten are used to obtain the numerical values of quantities expressed in bels. The sub-multiple decibel, dB, is commonly used. For further information one may see ISO 31

[h]p is enclosed in parentheses because, although the neper is coherent with the SI, it has not yet been adopted by the CGPM

[i]In using these units it is particularly important that the quantity be specified. The unit must not be used to imply the quantity

upon three quantities of length, mass and time with corresponding base units of centimetre, gram and second. In the field of electricity and magnetism, units were expressed in terms of these three base units. Because this can be done in different ways, it led to the establishment of several different systems, like the CGS electrostatic System, the CGS electromagnetic System and the CGS Gaussian System. In these three last-mentioned systems, the system of quantities and the corresponding system of equations differ from those used with SI units.

Table 6.8 Non-SI units for the interpretation of older scientific CGS-derived units with special names

Quantity	Name	Symbol	Value in SI units
Energy	energy[(a)]	erg	1 erg $= 10^{-7}$ J
Force	force[(a)]	dyn	1 dyn $= 10^{-5}$ N
Dynamic viscosity	poise[(a)]	P	1 P = 1 dyn $\cdot$ s/cm^2 = 0.1 Pa $\cdot$ s
Kinematic viscosity	stokes	St	1 St = 1 cm^2/s $= 10^{-4}$ m^2/s
Luminance	stilb[(a)]	sb	1 sb = 1 cdcm^{-2} $= 10^4$ cdm^{-2}
Illuminance	phot	ph	1ph = 1 cd sr cm^{-2} $= 10^4$ lx
Acceleration	gal[(b)]	Gal	1 Gal = 1 cm/s^2 $= 10^{-2}$ m/s^2
Magnetic flux	maxwell[(c)]	Mx	1 Mx ^ 1 0^{-8} W b
Magnetic flux density	gauss[(c)]	G	1 G ^ 10^{-4} T
Magnetic field	oersted[(c,d)]	Oe	1 Oe ^ $(1000/4\pi)$ A/m

[a]This unit and its symbol were approved by the 9th CGPM (1948)

[b]The gal is a special unit employed in geodesy and geophysics to express acceleration due to gravity

[c]This unit is part of the so-called “electromagnetic” three-dimensional CGS system and cannot strictly be compared with the corresponding unit of the International System, which has four dimensions when only mechanical and electric quantities are considered. For this reason, all such units are linked to its corresponding SI through the symbol (^), which stands for the words “corresponds to”

[d]These units are part of the so-called “electromagnetic” three-dimensional CGS system based on un-rationalised quantity equations, and must be compared with care to the corresponding unit of the International System which is based on rationalized equations involving four dimensions and four quantities for electromagnetic theory. The magnetic flux, Φ, and the magnetic flux density, B, are defined by similar equations in the CGS system and the SI, so that the corresponding units can be related as in the table. However, the un-rationalized magnetic field, H(un-rationalized) $= 4\pi \times H$ (rationalized). The equivalence symbol (^) is used to indicate that when H (un-rationalized) = 1 Oe, H (rationalized) $= (10^3/4\pi)$ Am^{-1}

6.4.5 Other Non-SI Units Found in the Old Literature

The units given in Table 6.9 are normally found in older literature. In the present context, it should be noted that if these units were continued to be used, the advantages of the SI would be lost. The relation of these units to the SI should be specified in every document in which they are used. In India, through the Weights and Measures Regulations, some of these units have been declared as prohibited units. In fact, there are too many non-SI units that are in use by some countries still using the fps system, or in some specific fields. However, it will be useful to know their conversion factors in terms of SI units. These one can find on the BIPM website at www.bipm.org/si/sibrochure/chapter4/conversion_factors.html.

Table 6.9 Examples of other non-SI units

Name	Symbol	Value in SI units
Curie[(a)]	Ci	1 Ci = 3.7×10^{10} Bq
Röntgen[(b)]	R	1 R = 2.58×10^{-4} C/k g
Rad[(c,d)]	Rad	1 rad = 1 cGy = 10^{-2} Gy
Rem[(d,e)]	rem	1 rem = 1 cSv = 10^{-2} S v
X unit[(f)]	–	1 X unit $\gg 1.002 \times 10^{-4}$ nm
Gamma[(d)]	γ	1 γ = 1 nT = 10^{-9} T
Jansky	Jy	1 Jy = 1 0^{-26} $\mathrm{W \cdot m^{-2} \cdot Hz^{-1}}$
Fermi[(d)]	–	1 fermi = 1fm = 10^{-15} m
Metric carat[(g)]	–	1 metric carat = 200 mg = $2{\cdot}10^{-4}$ kg
Torr	Torr	1 Torr = (101 325/760) Pa
Standard atmosphere	atm[(h)]	1 atm = 101 325 Pa
Calorie	cal[(i)]	
Micron[(d)]	μ[(j)]	1 μ = 10^{-3} mm = 10^{-6} m

[a]The curie is a special unit employed in nuclear physics to express the activity of radionuclides as retained by the 12th CGPM in 1964

[b]The röntgen is a special unit employed to express exposure to X or γ radiation

[c]The rad is a special unit employed to express absorbed dose of ionizing radiation. When there is risk of confusion with the symbol for radian, rd may be used as the symbol for rad

[e]The rem is a special unit used in radio- protection to express dose equivalent

[f]The X unit was employed to express the wavelengths of X rays. Its relationship with the SI unit is an approximate one

[d]These non-SI units are exactly equivalent to some sub-multiple of the corresponding SI unit

[g]The metric carat was adopted by the 4th CGPM in 1907 for commercial dealings in diamonds, pearls and precious stones. In India, carat is still being extensively used in the diamonds and precious stone industry as a unit of mass

[h]The designation "standard atmosphere" for a reference pressure of 101,325 Pa as approved by the 10th CGPM in 1954 was acceptable till 1982

[i]There are several "calories" in use:

(a) A calorie labelled "at 15 °C": 1 cal 15 = 4.1855 J value adopted by the CIPM in 1950 [8]

(b) A calorie labelled "IT" (International Table): 1 cal IT = 4.1868 J [9]

(c) A calorie labelled "thermo-chemical": 1 cal th = 4.184 J

[j]The micron and its symbol, adopted by the CIPM in 1879 (PV, 1879, 41) and repeated in Resolution 7 of the 9th CGPM (1948;) were abolished by the 13th CGPM (1967–1968)

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Chapter 7
Expressing SI Units

7.1 Introduction

We have discussed about the evolution of the International System of Units, base units, coherent derived units and non-SI units in the last few chapters. Continuing the subject, in this chapter we are going to discuss about SI prefixes, their use, and methods to express the quantities with proper numerals and units. The specific advantage of the International System of Units (SI) will also be discussed.

7.2 SI Prefixes

The 11th CGPM (1960) [1] adopted a series of prefixes and their symbols to form the names and symbols of the decimal multiples and submultiples of SI units ranging from 10^{12} to 10^{-12}. Prefixes for 10^{-15} and 10^{-18} were added by the 12th CGPM (1964) [2], for 10^{15} and 10^{18} by the 15th CGPM (1975) [3, 4] and for 10^{21}, 10^{24}, 10^{-21} and 10^{-24} by the 19th CGPM (1991) [5, 6]. All prefixes and their respective symbols, approved till date, are given in Table 7.1.

These SI prefixes refer strictly to powers of 10. They should not be used to indicate powers of 2 (e.g. one kilobit represents 1,000 bits and not 1,024 bits).

7.2.1 Rules for Using SI Prefixes

In accordance with the general principles adopted by the ISO (ISO 31), the CIPM recommended that the following rules be observed when using SI prefixes:

1. Regardless of the type used in the surrounding text, all prefix symbols are printed in roman (upright) type and are attached to unit symbols with no space between the prefix symbol and the unit symbol.
2. Similarly prefix names are also inseparable from the unit names to which they are attached. Thus, for example, millimetre, micropascal and meganewton are single words.

Table 7.1 SI prefixes

Factor	Name	Symbol	Factor	Name	Symbol
10^{24}	Yotta	Y	10^{-1}	Deci	d
10^{21}	Zetta	Z	10^{-2}	Centi	c
10^{18}	Exa	E	10^{-3}	Milli	m
10^{15}	Peta	P	10^{-6}	Micro	μ
10^{12}	Tera	T	10^{-9}	Nano	n
10^{9}	Giga	G	10^{-12}	Pico	p
10^{6}	Mega	M	10^{-15}	Femto	f
10^{3}	Kilo	k	10^{-18}	Atto	a
10^{2}	Hecto	ha	10^{-21}	Zepto	z
10^{1}	Deca	da	10^{-24}	Yocto	y

3. With the exception of da (deca), h (hecto) and k (kilo), all multiple prefix symbols are in capital (upper case) letters, and all submultiple prefix symbols are in lower case letters.
4. All prefix names are printed in lower case letters, except at the beginning of a sentence.
5. The grouping formed by the prefix symbol attached to the unit symbol constitutes a new inseparable symbol (of a multiple or submultiple of the unit concerned), which can be raised to a positive or negative power and combined with other unit symbols to form compound unit symbols.

For example:

$$
\begin{aligned}
1\,\mathrm{cm}^3 &= 10^{-2}\,\mathrm{m}^3 = 10^{-6}\,\mathrm{m}^3 \\
1\,\mu\mathrm{s}^{-1} &= 10^{-6}\,\mathrm{s}^{-1} \;= 10^{6}\,\mathrm{s}^{-1} \\
1\,\mathrm{V\,cm}^{-1} &= 1\,\mathrm{V}\,10^{-2}\,\mathrm{m}^{-1} = 10^{2}\,\mathrm{V\,m}^{-1} \\
1\,\mathrm{cm}^{-1} &= 10^{-2}\,\mathrm{m}^{-1} = 10^{2}\,\mathrm{m}^{-1}
\end{aligned}
$$

6. Compound prefixes, i.e. prefixes formed by the juxtaposition of two or more SI prefixes, are not permitted. This rule also applies to compound prefix names. Do not write nm as 1 mμm. Similarly microcentimetre with symbol μcm is also not allowed.
7. Prefix symbols can neither stand-alone nor be attached to the number 1, the symbol for the unit one. Similarly, prefix names cannot be attached to the name of the unit one, i.e. to the word ‘one’. We cannot say mega men for one million men.
8. Prefix names and symbols are used with a number of non-SI units, but they are never used with the units of time like minute (min), hour (h) and day (d). However, astronomers use milliarcsecond, which they denote as **mas**, and microarcsecond, denoted as **μas**, which they use as units for measuring very small angles.

7.2.2 Prefix About the Kilogram

Among the base units of the International System, the kilogram is the only one whose name and symbol, for historical reasons, include a prefix. Names and symbols for decimal multiples and submultiples of the unit of mass are, therefore, formed by attaching prefix names to the unit name ‘gram’, and prefix symbols to the unit symbol ‘g’ [7, 8].

7.3 Writing of SI Unit Symbols

7.3.1 Unit Symbols and Their Combinations

1. Every symbol of a unit is written in roman upright form. In general, unit symbols are written in lower case, but, if the name of the unit is derived from the proper name of a person, the first letter of the symbol is a capital. When the name of a unit is expressed in full, it is always written in lower case, except for the term ‘degree Celsius’ or when a sentence starts with the name of the unit.
2. The 16th CGPM (1979), Resolution 6, permitted the use of either capital L or lower case l for the litre. This was done in order to avoid possible confusion between the numeral 1 (one) and the lower case letter l (el).
3. A multiple or submultiple prefix, if used, is part of the unit and precedes the unit symbol without a separator. A prefix is never used in isolation, and compound prefixes are never used.
4. Unit symbols are mathematical entities and not abbreviations. Therefore, they are not followed by a period except at the end of a sentence.

For example, one may write a rod is 5.6 cm long, not a rod is 5.6 cm. long

5. Unit symbols are not to be changed in the plural.

For example, one should write 100 kg, not 100 kgs.

6. Unit symbols and unit names are not to be mixed within one expression, since names are not mathematical entities.

For example, for writing the unit of mass density use either kgm^{-3}, $\mathrm{kg/m}^3$ or kilogram per metre cube but not kilogram per m^3 or kg per metre cube.

7. In forming products and quotients of unit symbols the normal rules of algebraic multiplication or division apply. Multiplication must be indicated by a space or a middle dot (·), since otherwise some prefixes could be misinterpreted as a unit symbol. Indicate division by a horizontal line, a solidus (oblique stroke,/) or negative exponents. When several unit symbols are combined, care should be taken to avoid ambiguities, e.g. by using brackets or negative exponents. A solidus must not be used more than once in a given expression without brackets to remove ambiguities.

For example, one may write

m/s or ms^{-1} for metre per second
Write ms for millisecond and m s for metre times second the same may be written as m · s. Expressions like m kg/(s^3A), or m · kg s^{-3} A^{-1} are permitted but not m · kg/s^3/A, or m · kg/s^3 · A.

Work done in joules is expressed as m^2 kg s^{-2} and permeability as m kg $s^{-2}A^{-2}$; these are some examples for expressing quantities involving many units.

8. It is not permissible to use abbreviations for unit symbols or unit names, such as:

- sec for either s or second
- sq. mm for either mm^2 or square millimetre
- cc for either cm^3 or cubic centimetre
- mps for either m/s or metre per second

The use of the correct symbols for SI units, and for units in general, as listed in the book, is mandatory. In this way ambiguities and misunderstandings in the values of quantities are avoided.

7.3.2 Names of Units

1. Unit names are normally printed in roman (upright) type, and are treated like ordinary nouns. In English, the names of units start with a lower case letter (even when the symbol for the unit begins with a capital letter), except at the beginning of a sentence or in capitalized material such as a title. In keeping with this rule, the correct spelling of the name of the unit with the symbol °C is 'degree Celsius' (the unit degree begins with a lower case d and the modifier Celsius begins with an upper case C because it is a proper name).

Other examples are joule symbol (J); hertz symbol (Hz); second symbol (s), ampere symbol (A) and watt (W).

2. Although the values of quantities are normally expressed using symbols for numbers and symbols for units, if for some reason the unit name is more appropriate than the unit symbol, the unit name should be spelled out in full.

For example, we may write 5.9 N or 5.9 newtons; no abbreviation for the name of the unit can be used.

3. When the name of a unit is combined with the name of a multiple or submultiple prefix, no space or hyphen is used between the prefix name and the unit name. The combination of prefix name plus unit name is a single word; e.g. millimetre and not milli-metre. One should write kilopascal rather than kilo-pascal.
4. In both English and French, however, when the name of a derived unit is formed from the names of individual units by multiplication, either a space or a hyphen is used to separate the names of the individual units.

5. In both English and French, qualifiers such as ‘squared’ or ‘cubed’ are used in the names of units raised to powers, and they are placed after the unit name. However, in the case of area or volume, as an alternative the qualifiers ‘square’ or ‘cubic’ may be used, and these modifiers are placed before the unit name, but this applies only in English.

The rules enumerated in Sects 7.1 and 7.2 are enforced through Standard of Weights and Measures (National Standard) Rules [12].

7.3.3 Quantity Calculus

Value of Quantity

The value of a quantity is expressed as the product of a number and a unit. The number multiplying the unit is the numerical value of the quantity expressed in that unit.

Numerical Value of a Given Quantity

The numerical value of a given quantity depends on the choice of unit. Though the value of a given quantity is independent of the choice of unit, its numerical value will be different when expressed in different units. For example, the statements that the distance between the two given lines on the International Prototype of the Metre is 1 m, 100 cm or 1,000 mm are all correct. Numerical values are 1,100 and 1,000. Units are m, cm and mm. The same distance may be expressed as 1.093 613 3 yards.

Formatting the Value of a Quantity

1. The numerical value of the magnitude of the quantity always precedes the unit.
2. A space is always used to separate the unit from the number.
3. The space is regarded as a multiplication sign. Thus the value of the quantity is the product of the number and the unit.
4. The exceptions to rule 2 above are for the degree symbol ° and its submultiples, namely minute and second when representing a plane angle.
5. Degree symbol ° just precedes the Celsius symbol C when temperature is expressed in degree Celsius. However, a gap is always kept in between the symbol (degree Celsius) °C and the numerical value of the temperature, when expressed in degree Celsius. For example, write 27.5° C.
6. The symbols for degree, minute and second for a plane angle are, respectively, °, ′ and ″, for which no space is left between the numerical value and the unit symbol. The correct way of expressing a plane angle is 54° 33′ 44″.

7. Even when the value of a quantity is used as an adjective, a space is left between the numerical value and the unit symbol. Only when the name of the unit is spelled out would the ordinary rules of grammar apply, so that in English a hyphen would be used to separate the number from the unit. For example, write 15 kΩ resistor and 30-cm ruler.
8. It is preferable to use only one unit rather than several units and their submultiples. For example, to express a long distance quite accurately express its numerical value up to the desired decimal places. Express a distance as 10.532 m rather than 10 m 5 dm 3 cm and 2 mm.

For plane angles it is generally preferable to divide the degree decimally. Thus it is advisable to write 22.20° rather than $22°12'$.

Historically in fields of navigation, cartography, astronomy, and in the measurement of very small angles, minutes and seconds are used as subdivision of degrees expressing plane angles.

Symbols of Quantities

Symbols for quantities are generally single letters set in an italic font, although they may be qualified by further information in subscripts or superscripts or in brackets. Thus C is the recommended symbol for heat capacity, C_m for molar heat capacity, $C_{m,p}$ for molar heat capacity at constant pressure, and $C_{m,V}$ for molar heat capacity at constant volume.

For further details on symbols of quantities one may see standard references like [9–11]. However, one must remember that symbols for quantities are only recommendations, and one may or may not use them. But in contrast the symbols for units are mandatory and are to be used in that form alone. In particular circumstances authors may wish to use a symbol of their own choice for a quantity, for example, in order to avoid a conflict arising from the use of the same symbol for two different quantities. Invariably, the meaning of each symbol of the quantities used should be clearly stated.

Symbols for units are treated as mathematical entities. In expressing the value of a quantity as the product of a numerical value and a unit, both the numerical value and the unit may be treated by the ordinary rules of algebra. This procedure is described as the use of quantity calculus, or the algebra of quantities. For example, the equation $m = 1.235\,\mathrm{kg}$ may equally be written as $m/\mathrm{kg} = 1.235$. For the heading of a column in a table, it is often convenient to write the quotient of a quantity and its unit, so that the entries in the table are all simply numbers.

Quantity Symbols and Unit Symbols

Just as the quantity symbol does not imply any particular choice of unit, the symbol of a unit should not be used to provide specific information about the quantity. The unit used should never be the sole source of information on the quantity.

For example: If L is the symbol for the load on a balance, then L may be written as

$$L = 1.675\,\mathrm{kg}$$

and L_{max}, the capacity of the balance, should be written as

$$L_{\mathrm{max}} = 20\,\mathrm{kg}$$

But never write the capacity of the balance as

$$L = 20\,\mathrm{kg_{max}}$$

It means the symbol of a unit should never be qualified for further information about the nature of the quantity. Any extra information on the nature of the quantity should be attached to the quantity symbol and not to the unit symbol.

For example, if maximum voltage across two points is 1,000 V and U is symbol for voltage, then write

$$U_{\mathrm{max}} = 1,000\,\mathrm{V}$$

But never write

$$U = 1,000\,\mathrm{V_{max}}.$$

Taking w as a symbol of mass fraction, you may write the mass fraction of copper in the sample of silicon as

$$w\,\mathrm{Cu} = 1.3 \times 10^{-6}$$

but not $1.3 \times 10^{-6}\mathrm{w/w}$.

7.3.4 Stating Values of Quantities of Dimension 1

The coherent SI unit of quantities of dimension 1, also termed dimensionless quantities, is the numeral one with the symbol 1. Values of such quantities are expressed simply as numbers. The unit symbol 1 or unit name 'one' is not explicitly shown. In some special cases the unit symbol 1 is given a specific name. Such as

- For the quantity plane angle, the unit one is given the special name radian with the symbol rad
- For the quantity solid angle, the unit one is given the special name steradian with the symbol sr
- For the logarithmic ratio quantities, the special names neper, symbol Np, bel, symbol B and decibel, are used with the symbol dB

Because SI prefix symbols can neither be attached to the symbol 1 nor to the name 'one', powers of 10 are used to express the values of particularly large or small quantities of dimension 1. Do not write mega books for one million books.

Use of Symbol %

In mathematical expressions, the internationally recognized symbol % (percent) may be used with the SI to represent the number 0.01. Thus, it can be used to express the values of dimensionless quantities. When it is used, a space separates the number and the symbol %. In expressing the values of dimensionless quantities in this way, the symbol % should be used rather than the name "percent".

In written text, however, the symbol % generally takes the meaning of "parts per hundred". Phrases such as "percentage by mass", "percentage by volume" or "percentage by amount of substance" should not be used; the extra information on the quantity should instead be conveyed in the name and the symbol for the quantity. For example, $Q = 3.6\%$ is correct but to write $Q = 3.6\% V/V$ is not correct. To express the last expression one should define Q by writing it as Q_V and write $Q_V = 3.6\%$

In expressing the values of dimensionless fractions (e.g. mass fraction, volume fraction and relative uncertainties), the use of a ratio of two units of the same kind is sometimes useful. For example, for the molar concentration B with the symbol X_B, instead of writing

$$X_B = 2.5 \times 10^{-3}$$

It is better to write

$$X_B = 2.5\,\text{mmol mol}^{-1}$$

The term "ppm", meaning 10^{-6} relative value, or 1 in 10^6 or parts per million, is also used. This is analogous to the meaning of percent as parts per hundred. The terms "parts per billion" and "parts per trillion", and their respective abbreviations "ppb" and "ppt", are also used, but their meanings are language-dependent. For this reason the terms ppb and ppt should best be avoided, there is ambiguity in the values of the words billion and trillion.

- In English-speaking countries, billion is generally taken to be 10^9 and trillion to be 10^{12}.
- In French-speaking countries, billion is interpreted as 10^{12} and trillion as 10^{18}.
- The abbreviation ppt is also sometimes read as parts per thousand, adding further confusion.

When any of the terms %, ppm, etc. are used it is important to state the dimensionless quantity whose value is being specified.

7.4 Expression of Numbers

In India, the Standards of Weights and Measures (numeration) Rules, 1987 [12], deals with the numeration and expression of any number including large numbers and fractions. The rules are in accordance with international practices and are the same as prescribed in the International System of Units (SI).

SI rules governing the expression of numbers are as follows:

1. Every numeration shall be made in accordance with the decimal system.
2. Every number shall be represented on base 10.
3. In representing any number in digits, the international form of Indian numerals, namely 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9, are to be used. Quite often these digits are called the International form of Arabic numerals.
4. The symbol for the decimal marker (to separate the integral part of numbers from the decimal part) shall be either the point on the line or the comma on the line (22nd CGPM 2003). In India we adopted the point on the line to represent decimal marker.
5. In any number with digits exceeding three, the decimal point is taken as the starting point; and the digits, whether to the left or the right of it, are divided into groups of three; each group is separated by a space. No comma or full stop is to be inserted in the spaces between such groups (CGPM, 1948, reconfirmed by 22nd CGPM in 2003). However, when there are only four digits before or after the decimal marker, it is customary not to use a space to isolate a single digit.

Note: The practice of grouping digits in this way is a matter of choice; it is not always followed in certain specialized applications such as engineering drawings, financial statements and scripts to be read by a computer.

6. For numbers in a table, the format used should not vary within one column.
7. If the number is between $+1$ and -1, then the decimal marker is always preceded by a zero.

7.4.1 Formatting Numbers, and the Decimal Marker

The symbol used to separate the integral part of a number from its decimal part is called the decimal marker.

Following the decision of the CIPM made at its 86th meeting (1997), the BIPM now uses:

1. The dot (point on the line) as the decimal marker in all the English language versions of its publications.
2. A comma (on the line) as the decimal marker in all its French language publications.

However, some international bodies including some international standards organizations use the comma on the line as the decimal marker in all languages including English.

The use of the comma on the line or the point on the line as decimal marker varies from country to country. In some countries the symbol for the decimal marker varies from one native language to another. To cater to the need of every section of the society the CGPM in its 22nd meeting held in 2003, vide resolution 10, decided that the decimal marker 'shall be either the point on the line or the comma on the line'. Hence the decimal marker should be recognized in the context of the language and the country.

7.4.2 Expressing the Measurement Uncertainty

The uncertainty that is associated with the estimated value of a quantity should be evaluated and expressed in accordance with the Guide to the Expression of Uncertainty in Measurement [ISO, 1995]. The standard uncertainty associated with a quantity x is denoted by u (x). It is the estimated standard deviation with a coverage factor $k = 1$ A convenient way to represent uncertainty is given in the following example:

$$m_n = 1.67492728(29) \times 10^{-27}\,\text{kg}$$

where m_n is the symbol for the quantity (in this case the mass of a neutron), and the number in parentheses is the numerical value of the combined standard uncertainty of the estimated value of m_n referred to the last two digits of the quoted value.

In this case, u $(m_n) = 0.000\,000\,29 \times 10^{-27}$ kg.

In the literature, standard uncertainty is normally cited. However, some calibration laboratories give a combined expanded uncertainty with a coverage factor k of 2 or 3 depending upon whether the stated results are at 96 or 99.7% confidence level. Expanded uncertainty is k times the combined standard uncertainty. However, if any coverage factor k different from one is used, this factor must be stated.

7.5 Advantages of SI Units

7.5.1 Harmonization of Units

General Principle

Before the adoption of SI units, there were different units for the same quantity in different areas of measurements. For example, in the area of electrical measurements there were different sets of units for electrostatic, electromagnetic and Gaussian measurements. These sets used to be called electrostatic, electromagnetic and practical units. It was the same case with energy; there were separate units for mechanical, electrical, heat and optical energies.

To standardize the units of the given quantity in all areas of electrical measurements, the following steps are taken:

1. Introduced permittivity and permeability as the constants of proportionality in the Coulomb's law for force of attraction/repulsion between free charges and the corresponding formula for free magnetic poles, respectively.
2. As both formulae are valid in space, 4π, the constant of spherical symmetry, was introduced in the denominator.
3. The electric current is defined so that permeability of free space becomes $4\pi \times 10^{-7}$.
4. Using Maxwell's theory, the permittivity of free space is derived from the formula showing that the product of permittivity and permeability is an inverse of the square of the velocity of light.

Further details are given below.

Coulomb's formula is modified as

$$F = (1/\varepsilon_o 4\pi)\ Q_1 Q_2\, \mathrm{r}^{-2} \tag{7.1}$$

where ε_o is the permittivity of free space.

From the above, the unit of charge is defined as follows:

If two equal and opposite charges are placed in a vacuum at a distance of 1 m and each experiences a force equal to $1/\varepsilon_o 4\pi$ N, then each is a unit charge.

Similarly the force (F) experienced per unit length between the two parallel wires of infinite length and negligible cross section in a vacuum was modified to

$$F \text{ per unit length} = (\mu_o/4\pi)\ 2\mathrm{I}_1\mathrm{I}_2\mathrm{r}^{-1} \tag{7.2}$$

where μ_o is the permeability of free space.

By defining the unit of electric current, as described in Sect. 4.3.4, the value of μ_o, the permeability of free space, becomes $4\pi \times 10^{-7}$ exact. Following the Maxwell relation, we get

$$\varepsilon_o \mu o = \mathrm{c}^{-2} \tag{7.3}$$

Taking c^2 equal to $9 \times 10^{16}\ \mathrm{m}^2\ \mathrm{s}^{-2}$, (7.3) gives the numerical value of $(1/\varepsilon_o 4\pi)$ as 9×10^9.

If each charge is one coulomb, then denoting the unit charge with C, (7.1) can be interpreted as

$$9 \times 10^9\ \mathrm{N} = (1/\varepsilon_o 4\pi)\ \mathrm{C}^2/1\,\mathrm{m}^2$$

$$1\,\mathrm{N} = (1/\varepsilon_o 4\pi)\,\mathrm{C}^2/\{\mathrm{m}^2\,(9 \times 10^9)\} \tag{7.4}$$

Similarly, if $\mathrm{I}_1 = \mathrm{I}_2 = 1$ A and $r = 1$ m, then from (7.2) we get

$$10^{-7}\ \mathrm{N\ per\ metre} = (\mu_o/4\pi)\ \mathrm{A}^2\ 1\,\mathrm{m}^{-1}$$

which gives

$$1\,\mathrm{N} = (\mu_o/4\pi)\,\mathrm{A}^2/10^{-7} \tag{7.5}$$

Equating (7.4) and (7.5) we get

$$(1/\varepsilon_o 4\pi)\ \mathrm{C}^2\,\mathrm{m}^{-2}\left(9\cdot 10^9\right) = (\mu_o/4\pi)\,\mathrm{A}^2/10^{-7}$$

or

$$\mathrm{C}^2\,\mathrm{m}^{-2} = \mathrm{A}^2\varepsilon_o\mu_o 9\times 10^{16},$$

But

$$\varepsilon_o\mu_o = 1/\left(9\times 10^{16}\mathrm{m}^2\ \mathrm{s}^{-2}\right). \tag{7.6}$$

Substitution of the values $\varepsilon_o\mu_o$ with proper units in (7.6) gives

$$\begin{aligned}\mathrm{C}^2\,\mathrm{m}^{-2} &= \mathrm{A}^2\,\mathrm{s}^2\,\mathrm{m}^{-2};\ \text{thus}\\ \mathrm{C} &= \mathrm{A\ s}\end{aligned} \tag{7.7}$$

This means that the unit of charge in electrostatic is ampere· second. The unit of charge using the electromagnetic phenomenon is also A s. Hence the unit of charge is the same in both the phenomena.

7.5.2 Expressing the Values of μ_o and ε_o in Terms of SI Units

By defining the ampere, the unit of current, as in Sect. 4.3.4, (7.2) gives

$2\ \times 10^{-7}\,\mathrm{N} = (\mu_o/2\pi)\ \mathrm{A}^2$; hence
$\mu_o = 4\pi\times 10^{-7}\,\mathrm{N\,A}^{-2}$

$$\mu_o = 4\pi\times 10^{-7}\,\mathrm{m\,kg\,s}^{-2}\,\mathrm{A}^{-2}. \tag{7.8}$$

But from Table 6.2, the henry H can be expressed in terms of base units as

$$\mathrm{H} = \mathrm{m}^2\,\mathrm{kg\,s}^{-2}\,\mathrm{A}^{-2}$$

Or

$$\mathrm{H\,m}^{-1} = \mathrm{m\,kg\,s}^{-2}\,\mathrm{A}^{-2}$$

So from (7.8), we get

$$\mu_o = 4\pi\times 10^{-7}\,\mathrm{H\,m}^{-1}. \tag{7.9}$$

Therefore, the unit of permeability is expressed in henry per metre with the symbol $\mathrm{H\ m}^{-1}$

Using (7.3), ε_o is given by

$$\begin{aligned}\varepsilon_o &= 1/\left(\mu_o c^2\right)\\ &= \left[1/\left(4\pi\times 10^{-7}\times 9\times 10^{16}\right)\right]\ \mathrm{m}^{-1}\,\mathrm{kg}^{-1}\,\mathrm{s}^2\,\mathrm{A}^2\,\mathrm{m}^{-2}\,\mathrm{s}^2\\ &= 8.84\times 10^{-12}\,\mathrm{m}^{-3}\,\mathrm{kg}^{-1}\,\mathrm{s}^4\,\mathrm{A}^2\end{aligned}$$

Using Table 6.2, farad F expressed in terms of base units is $m^{-4}\,kg^{-1}\,s^4\,A^2$, which gives

$$\varepsilon_o = 8.84 \times 10^{-12}\,F\,m^{-1} \tag{7.10}$$

Therefore, the unit of permittivity is expressed in Faraday per metre.

G. L. T. Giorgi [13], in 1904, around 30 years after the Metre Treaty, also suggested that if permeability of free space μ_o is taken as $4\pi \times 10^{-7}$ in the definition of current, then units of the same electromagnetic and electrostatic quantities will be the same.

Comments of the author: However, in the opinion of the author μ_o should be expressed in terms of $N\,A^{-2}$ rather than in terms of henry per metre. The relation (7.2) is the gateway between mechanical and electrical domains. The mechanical quantity of a force is related with the electrical quantity through a constant of proportionality μ_o – the permeability of free space. Hence it should be expressed in terms of force and current only. Introducing the idea of the unit of inductance here appears hasty. The author's idea will definitely make the teaching of units of measurements much easier as the idea of inductance comes much later.

In fact, in a very popular book of Class XII, the author of the book started the chapter on electricity with Coulomb's law. The law connects the electric charges with force via ε_o – the permittivity of free space. The formula used was the same as in (7.1) above. Defining the unit of charge, he got the unit of ε_o as follows:

The force F in newtons between two unit charges placed one metre apart in a vacuum is equal to

$$F = (1/4\pi\varepsilon_o)\ C^2\,m^{-2}$$

Writing $F = f\,N$ where f is a number, we get

$$\varepsilon_o = (4\pi/f)\ C^2\,m^{-2}N^{-1}$$

Hence the unit of $\varepsilon_o = C^2m^{-2}\,N^{-1}$
He used this unit throughout the chapter. However, he did mention that the SI unit of ε_o is Farad per metre or $F\,m^{-1}$.

It is therefore desirable that there be some uniform, intelligible procedure to arrive at the units of different electrical quantities, starting from the base quantity, i.e. electrical current.

7.5.3 Expressing Electrostatic and Electromagnetic Quantities in SI Units

Charge and Current

esu of Charge

Coulomb's law in SI units is

$$F\ N = Q_1Q_2/4\pi\varepsilon_o\,r^2 = 9 \times 10^9\ Q_1Q_2\,r^{-2}$$

Assuming $Q_1 = Q_2 = 1$ coulomb and $r = 1$ m, we get

$$1\ (\text{coulomb})^2/1\,\text{m}^2 = 9 \times 10^9\ \text{N}$$

The law defines the unit of charge as:

If two equal charges placed in a vacuum one metre apart experience a force of $9 \cdot 10^9$ newtons, then each is equal to one coulomb of charge.

Coulomb's law in CGS units is written as

Force in dynes $= Q_1 Q_2\, \text{r}^{-2}$

which gives

$$1\,\text{dyn} = (\text{esu})^2\ 1\,\text{cm}^{-2} = 10^4\ (\text{esu})^2\ 1\,\text{m}^{-2}.$$

Multiplying each side by 10^5, we get

$$10^5\ \text{dyn} = 1\,\text{N} = 10^9\ (\text{esu})^2\ 1\,\text{m}^{-2}.$$

Multiplying each side by 9×10^9, we get

$$9 \times 10^9\ \text{N} = 9 \times 10^{18}\ (\text{esu})^2\ 1\,\text{m}^{-2}.$$

But

$$9 \times 10^9\ \text{N} = 1\,(\text{c})^2\ 1\,\text{m}^{-2}$$

So

$$\begin{aligned}(\text{coulomb})^2 &= 9 \times 10^{18}\ (\text{esu})^2\\ 1\,\text{C} &= 3 \times 10^9\ esu\ of\ charge\\ 1\ \text{esu of charge} &= \left(10^{-9}/3\right)\ \text{C}\end{aligned} \tag{7.11}$$

emu of Current and Charge

Electromagnetic unit of charge is derived from the emu of current, whose unit is abampere. By definition if 1 abampere of steady current is passing through two parallel infinite wires of negligibly small cross section placed one cm apart, then each wire experiences a force of one dyne per cm length of wire (relation used is F per unit length $= \text{I}_1\text{I}_2/r$), which is

$$\begin{aligned}1\,\text{dyn cm}^{-1} &= 1\ \text{abampere}^2\ 1\,\text{cm}^{-1}\\ 1\,\text{dyn} &= 1\ \text{abampere}^2\end{aligned}$$

In SI units, when 1 A current is flowing through the aforesaid wires and placed 1 m apart, then the force acting is $2 \times 10^{-7}\,\text{N m}^{-1}$, which gives

$$10^{-7}\,\text{N m}^{-1} = \text{A}^2\ \text{m}^{-1}$$

Or

$$10^{-2}\,\text{dyn} = \text{A}^2$$
$$1\,\text{dyn} = 100\,\text{A}^2$$

Thus

$$1\,\text{dyn} = \text{abampere}^2 = 100\,\text{A}^2$$
$$1\,\text{abampere} = 10\,\text{A}. \tag{7.12}$$

It may be mentioned that the initial definition of ampere comes from the fact that ampere was taken as one-tenth of abampere – the electromagnetic unit current.

Further

$$1\,\text{C} = 1\,\text{s} \times 1\,\text{A} = 1\,\text{s} \times 0.1\,\text{abampere}$$

Or

$$1\,\text{emu of charge} = 10\,\text{C}. \tag{7.13}$$

Potential

esu Potential

By definition, the potential at a point is the work done on a unit positive charge in bringing it from infinity to that point. Hence, the unit of potential in SI units may be written as 1 J/1 C and is given a special name, volt. Similarly, the unit of potential in the electrostatic field will be 1.erg/1. esu of charge:

$$\begin{aligned} 1\,\text{V} &= 1\,\text{J}/1\,\text{C} \\ &= 10^7\,\text{ergs}/3 \times 10^9\,\text{esu of charge} \\ &= (1/300)\ (\text{erg/esu of charge}) \\ &= (1/300)\ \text{esu of potential} \end{aligned}$$

Thus

$$1\,\text{esu of potential} = 300\,\text{V}. \tag{7.14}$$

emu Potential

$$1\,\text{C} = 3 \cdot 10^9\,\text{esu of charge} = 0.1\,\text{emu of charge}$$
$$1\,\text{emu of charge} = 3 \cdot 10^{10}\,\text{esu of charge}$$

But

$$300\,\text{V} = 1\,\text{esu of potential} = 1\,\text{erg/esu of charge} = 3 \times 10^{10}\,\text{erg/emu of charge}$$
$$300\,\text{V} = 3 \times 10^{10}\,\text{emu of potential}$$

$$\text{emu of potential} = 10^{18}\,\text{V}. \tag{7.15}$$

Electrical Resistance

esu of Resistance

In the field of electrostatics, there is no motion, and the concept of resistance is against motion, so as such there should be no concept of resistance in the field of electrostatics. However, there is potential, so charge has a capacity to move, moreover, in the field of electromagnetic there is current with unit abampere, so we may proceed as follows:

Potential/current = resistance

$1\,\Omega = 1\text{ V }1\text{ A}^{-1} = (1/300)$ esu potential/0.1 abampere
$1\,\Omega = 1/30$ esu of potential per abampere

Abampere is chosen, but as in the electrostatic field, the current has no meaning.

If we wish to express resistance in esu, we will replace abampere by 3×10^{10} esu of current, which gives

$1\,\Omega = (1/9) \times 10^{-11}$ esu of resistance

Hence

$$1 \text{ esu of resistance} = 9 \times 10^{11}\,\Omega \tag{7.16}$$

emu of Resistance

$$1\,\Omega = 10^{8} \text{ emu } 0.1\,aA^{-1}$$
$$1\,\Omega = 10^{9} \text{ emu of resistance}$$
$$1 \text{ emu of resistance} = 10^{-9}\,\Omega \tag{7.17}$$

Electrical Capacitance

esu of Capacitance

farad = $1\text{ C}/1\text{ V} = 3 \cdot 10^{9}$ esu of charge/(1/300) of esu potential
$1\text{ f} = 9 \cdot 10^{11}$ unit of capacity in esu

$$1 \text{ esu of capacity} = \left(10^{-11}/9\right) \text{f} \tag{7.18}$$

emu of Capacitance

$1\text{ f} = 1\text{ C}1\text{ V}^{-1} = 10^{-1}$ emu of charge/10^{8} emu of potential
10^{-9} emu of capacitance

Hence

$$1 \text{ emu of capacitance} = 10^{9}\text{ f} \tag{7.19}$$

emu of Magnetic Flux

The unit of flux in SI is weber with the symbol Wb. We know that

$$1\ \mathrm{Wb} = 1\ \mathrm{V}\ 1\ \mathrm{s}$$

Therefore

$$\text{weber} = 10^8\ \text{emu of potential} \cdot \#1\ \mathrm{s}$$

But the unit of emu of flux is called maxwell with the symbol Mx. Hence

$$\begin{aligned} 1\ \mathrm{Wb} &= 10^8\ \mathrm{Mx} \\ 1\ \mathrm{Mx} &= 10^{-8}\ \mathrm{Wb} \end{aligned} \tag{7.20}$$

Magnetic field Strength (Flux Density)

Magnetic flux density = magnetic flux per square metre

$$\begin{aligned} 1\ \mathrm{T} &= 1\ \mathrm{Wb\,m^{-2}} \\ &= 10^8\ \mathrm{Mx}\ 10^4\ \mathrm{cm^{-2}} \\ &= 10^4\ \mathrm{Mx\ cm^{-2}} \end{aligned}$$

But the unit of emu of the magnetic field strength is called gauss with the symbol G. Hence

$$1\ \mathrm{T} = 10^4\ \mathrm{G}$$

Or

$$1\ \mathrm{G} = 10^{-4}\ \mathrm{T} \tag{7.21}$$

Electric field

$$\begin{aligned} 1\ \mathrm{V}\ 1\ \mathrm{m^{-1}} &= (1/300)\ \text{esu of potential}/100\ \mathrm{cm} \\ 1\ \mathrm{V\,m^{-1}} &= (1/3) \times 10^{-4}\ \text{esu of electric field} \\ 1\ \text{esu of electric field} &= 3 \times 10^4\ \mathrm{V\,m^{-1}} \end{aligned} \tag{7.22}$$

Inductance

The unit of inductance is henry = $\mathrm{d}\Phi/\mathrm{d}i$, where Φ is flux and i is current

$$\begin{aligned} 1\ \mathrm{H} &= 10^8\ \text{emu of flux}/0.1\ \text{emu of current} \\ 1\ \mathrm{H} &= 10^9\ \text{emu of inductance} \\ 1\ \text{emu of inductance} &= 10^{-9}\ \mathrm{H} \end{aligned} \tag{7.23}$$

7.5.4 SI Units of Quantities in Magnetic Field

SI Unit of Magnetic Pole Strength

The force between two poles of strength P_1 and P_2 placed r metres apart experience a force F in newtons given by

$$F = \mu_0 \, P_1 P_2 / 4\pi r^2$$

But

$$m_0 = 4\pi \times 10^{-7} \, \mathrm{N\,A^{-2}} \tag{7.24}$$

Defining unit pole as follows:

If two free unit poles P placed in a vacuum at a distance of 1 metre, then each will experience a force of 10^{-7} newtons. Then (7.22) gives us

$$10^{-7}\,\mathrm{N} = 10^{-7}\,\mathrm{N\,A^{-2}}\,P^2\,1\,\mathrm{m^{-2}}$$

Hence

$$P^2 = A^2\,m^2$$

or

$$\text{SI unit of Pole strength P is A} \times \text{m} \tag{7.25}$$

Magnetic Flux Density/Magnetizing Force

The magnetic flux density and magnetizing force are two quantities that are quite often misunderstood. In SI units in all the relations where magnetic field is used, it is the magnetic flux density that represents the intensity of a magnetic field. One way of defining the quantity of the magnetic flux density is as follows:

A charge q moving with a velocity v in a field of magnetic flux density B experiences a force F normal to the plane containing v and B given by the relation:

$$F = q\,(v \cdot B \cdot \sin\theta) \tag{7.26}$$

where θ is the angle between the velocity of the moving charge q and the direction of the magnetic flux density B.

Assigning values of 1 to q and v each and taking θ as 90° in (7.24) we get

$$F = B \tag{7.27}$$

Hence, the magnetic flux density or intensity of magnetic induction is the force acting on a unit positive charge moving with unit velocity at right angles to the magnetic field B, and its unit has a special name, tesla.

Writing the quantity equation of the magnetic flux density from (7.24), we get

$$B = F/v \cdot q$$

which gives the unit equation of B as

$$\begin{aligned}\{\mathrm{B}\} &= \{F\}/\{v\}\{q\}\\ &= \mathrm{N/m\,s^{-1}\,C}\\ &= \mathrm{N/m\,C\,s^{-1}}\\ &= \mathrm{N\,Am^{-1}}\end{aligned} \tag{7.28}$$

On the other hand, the magnetizing force or intensity of magnetization H is the degree to which a magnetic field can magnetize a material. For example, a toroidal solenoid having n turns per unit length wound over a circular ring of magnetic material and carrying a steady current I produces a magnetic field B. Given by the relation:

$$B = \mu\,\mathrm{n}\,I \tag{7.29}$$

The product nI is called the magnetizing force or magnetic intensity H given by

$$H = \mathrm{n}\,I$$

Hence, the magnitude of the magnetizing force is defined as the product of current (in amperes) and number of turns of a solenoid wound round the unit length of a toroidal solenoid. So its unit is ampere per metre with the symbol A/m.

If inside the toroidal solenoid, there is a free space, then the magnetic induction is

$$B = \mu_0 H \tag{7.30}$$

where μ_0 is the magnetic permeability of free space. So the SI unit of H is given by

$$\text{Unit of } H = \left(\mathrm{Wb\,m^{-2}}\right)/\mathrm{N\,A^{-2}}$$

Expressing this in base units, we get

$$\begin{aligned}\text{Unit of } H &= \mathrm{m^2\,kg\,s^{-2}\,A^{-1}\,m^{-2}/m\,kg\,s^{-2}\,A^{-2}}\\ \text{Unit of } H &= \mathrm{A\,m^{-1}}\end{aligned} \tag{7.31}$$

Intensity of Magnetization

The intensity of magnetization represents the ability to magnetize, same as magnetizing force. Numerically it is equal to the magnetic moment per unit volume of the material, i.e. the intensity of magnetization H is given by

$$H = \text{Magnetic moment/Volume of the material}$$

Hence

The unit of intensity of magnetization = ampere·metre^2/metre^3 = ampere/metre

$$\text{Unit of } H = \text{A m}^{-1}$$

For example, for a rectangular solid bar of uniform cross section of area a and length $2l$, if P is the pole strength developed in it, then

$$H = P \cdot 2l/a \cdot 2l = P/a$$

So

$$\text{Unit of H-the intensity of magnetisation} = \text{A} \times \text{m m}^{-2} = \text{A m}^{-1} \tag{7.32}$$

In the field of magnetism, there is another term – the strength of a magnetic field. Let us denote this also by I. The strength of a magnetic field due to a pole of strength P at a point distance r from it is the force experienced by a unit positive pole at that point given by the relation

$$I = \mu_0 \text{P r}^{-2} \tag{7.33}$$

Expressing each quantity of the above equation in SI units, I can be written as

$$\{I\} = \text{NA}^{-2}\,\text{A m m}^{-2}$$

$$= \text{N A m}^{-1}. \tag{7.34}$$

This expression is the same as that of the magnetic induction or flux density in (7.28).

Hence, magnetic induction, magnetic flux density and strength of the magnetic field are one and the same quantity. Wherever the term magnetic field occurs, the tesla should be taken as its unit. Referring to the last item of Table 6.1 of Chap. 6, the term intensity of magnetization with the unit ampere per metre should be preferable over the term magnetic field.

The CGS unit of H is oerested and that of B is gauss. So from (7.24)

$$\text{Gauss/oerested} = \mu_0 = 4\pi \times 10^{-7}\,\text{N A}^{-2}$$

So

$$\begin{aligned} \text{oersted } &= 10^{-4}\,\text{T}/\left(4\pi \times 10^{-7}\,\text{NA}^{-2}\right) \\ &= \left(10^3/4\pi\right)\,\text{Wb m}^{-2}\,\text{N}^{-1}\,\text{A}^2 \end{aligned}$$

Expressing this in SI base units, we get

$$1\,\text{Oe} = \left(10^3/4\pi\right)\,\text{V s m}^{-2}\,\text{m}^{-1}\,\text{kg}^{-1}\,\text{A}^{-2}$$

$$\left(10^3/4\pi\right)\,\text{m}^2\,\text{kg s}^{-3}\,\text{A}^{-1}\,\text{s m}^{-3}\,\text{kg}^{-1}\,\text{s}^2\,\text{A}^2$$

$$1\,\text{Oe} = \left(10^3/4\text{p}\right)\,\text{A/m} \tag{7.35}$$

7.5.5 Homogenizing of Units of Energy in Heat

Prior to the adoption of SI units, the units used for mechanical energy, heat energy and electrical energy were different, as all were independently defined. The erg – the unit of mechanical energy – was a dot product of two vectors, namely force and displacement. The calorie was the heat exchanged by 1 gram of water through a change of one degree centigrade. While electrical energy was the product of current, potential difference and time, current was measured in ampere, potential difference in volts and time in seconds.

To solve this problem, mechanical energy was expressed in terms of base units of mass, length and time. Watt unit of power is obtained from mechanical energy, and electromotive force or potential difference is obtained by dividing watt by electrical current, thus making energy units the same in mechanical and electrical fields. To bring homogeneity in the area of heat, the unit of energy is taken as joule and corresponding changes have been made in the specific heat capacity of water, which roughly comes to 4,200 j per kilogram kelvin, instead of 1 calorie per gram degree Celsius.

7.5.6 Coherent System

Any system of measurements is said to be coherent if all derived units from base units have no numerical multiplier other than one. In this system, as seen in Chap. 6, all derived units are expressed in terms of base units which have only one as the multiplier. Every derived unit is expressed as a product or division of a relevant base or derived unit with a special name but always has unity as its coefficient. So it is a coherent system.

Coherent Derived Unit

The coherent derived unit, for a given system of quantities and for a chosen set of base units, is a product of powers of base units with no proportionality factor other than one.

It may be noted that

1. A power of base unit is the base unit raised to an exponent.
2. Coherence can be determined only with respect to a particular system of quantities and a given set of base units.

Examples are:

If the metre, the second and the mole are base units, the metre per second is the coherent derived unit of velocity when velocity is defined by the quantity equation $v = \mathbf{d}\boldsymbol{r}/\mathrm{d}t$, and the mole per cubic metre is the coherent derived unit of amount of substance concentration when the amount of substance concentration is defined by the quantity equation $c = n/V$. The kilometre per hour and the knot, given as examples of derived units in Chap. 6, are not coherent derived units in SI.

3. A derived unit can be coherent with respect to one system of quantities, but not to another.

For example: The centimetre per second is the coherent derived unit of speed in the CGS system of units but is not a coherent derived unit in the SI.

4. The coherent derived unit for every derived quantity of dimension 1, in a given system of units, is the number one, symbol 1. The name and symbol of the measurement unit one are generally not indicated.

7.5.7 Well-Defined Units

All the base units are very precisely defined and, except the unit of mass, all other units are realizable in a good metrological laboratory. So, except the kilogram, no artefact is maintained as the representative of the unit.

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Chapter 8
Future Definitions of SI Units

In the present chapter, a glimpse has been given about futuristic view on definitions of base units of the international system of units SI. Methods of defining base units in terms of

- physical constants of one element namely hydrogen
- one standard namely frequency
- indestructible and invariable physical constants
- some invariant physical phenomenon like Einstein matter energy relation

have been discussed. The latest thinking about redefinitions of kilogram, ampere, kelvin and mole have also been discussed

8.1 In Terms of Physical Constants

In the previous chapters we have seen quite a number of sets of physical constants to define a system of units.

8.1.1 Basis of SI Units

The International System of Units (SI) is based on the following values of fundamental constants [1]:

Velocity of light	299,792,458 m s^{-1} for defining metre
Transition periods of caesium atom 133	9,192,631,770 for defining second
Triple point of water	273.16 for defining temperature
Permeability of free space	$4\pi \times 10^{-7}$ for defining unit of electric current

But this is not the only set of constants that we can use for defining a system of units. In fact, using the same constants one can arrive at several other sets of base units. For example, instead of using μ_0– the permeability of free space for defining electric current – we can use it as well as the velocity of light c to define impedance of the free space. This will make ohm the base unit instead of electric current. We know $c \cdot \mu_0$ has the unit of resistance defining:

$$1 \text{ ohm} = c \cdot \mu_0 / 376.731{,}119{,}44 \tag{8.1}$$

As $c \cdot \mu_0 = 376.73111944$, because of the presence of π in μ_0, the value of $c \cdot \mu_0$ may be taken up to a large number of decimal places; hence this value is not unique. Petley [2] suggested this definition of ohm in 1990.

8.2 From Single Source

From above we see that we are using different elements or compounds and sources to arrive at certain fundamental constants. Let us consider the possibility of defining the base units including the kilogram in terms of fundamental constants obtained from a single source.

8.2.1 In Terms of Hydrogen Atom

Professor Kose et al. [3], in 2003, suggested that among many possibilities one is to consider hydrogen for defining base units: for example, kilogram in terms of nucleus of the atom, second in terms of period of hydrogen atom in ground state (Fig. 8.1). The definition of metre could be unchanged as it is already defined in terms velocity of light c_o, a fundamental constant. Other units could be defined as shown below.
Time: Second – the unit of time – is defined as the duration of 466,8xx,xxx,xxx,xxx periods of the radiation corresponding to the transitions between the levels $2S_{1/2}$ and $3P_{1/2}$ of the hydrogen atom. As the radiation transitions in the hydrogen atom are measured eight times more accurately than the hyperfine structure in the caesium atom it will prove to be a better option.
Mass: Kilogram – the unit of mass – is 5.973386xxx $\times 10^{26}$ hydrogen atoms.
Electric current: Ampere – unit of electric current – corresponds to $6.241509484 \times 10^{18}$ of elementary charges (charge on electron) per second.
Thermodynamic temperature: Kelvin – unit of temperature – is the fraction 1/13.80xx of the triple point of hydrogen.
Amount of substance: Mole – the unit of substance – is the amount of substance in a system that contains 6.022141xxx $\times 10^{25}$ elementary entities [4].

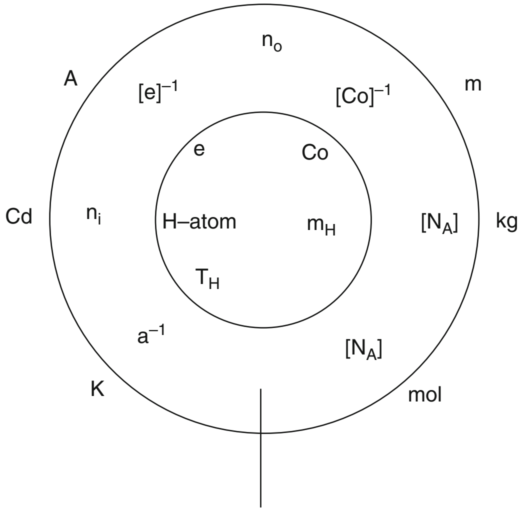

Fig. 8.1 SI units in terms of hydrogen atom

Definitions of metre and luminous intensity remain the same as metre is already defined in terms of velocity of light, a fundamental constant; and luminous intensity in SI is only a derived unit.

Please note that xxx after the decimals indicate that specific numerals will be substituted according to the latest internationally accepted values of physical constants involved.

Please remember that the second, the kilogram and the kelvin are independent base units, whereas the other four units are metrologically dependent, just as metre depends upon the velocity of light. Candela is a derived unit from the mechanical units like watt, frequency, etc.

Here T_H represents the triple point of hydrogen and a^{-1} is numerically equal to T_H. The symbols C_0, e, m_H and N_A are respectively velocity of light, charge on the electron, rest mass of hydrogen atom and the Avogadro constant. The fundamental constants within square brackets represent the numerical values in SI units.

The values, in SI units, of the constants used are shown in the ring with hydrogen and its constants are inside the inner circle. The corresponding SI symbols are indicated on the outer periphery of the two circles.

8.2.2 In Terms of Only One Standard (Frequency)

Another approach may be taking frequency as the base standard and expressing all other base units in terms of frequency and other fundamental constants like Planck's constant, Boltzman's constant, velocity of light and electronic charge. The basic reason for this proposal is the fact that frequency can be measured with uncertainty several orders of magnitude better than any other base unit.

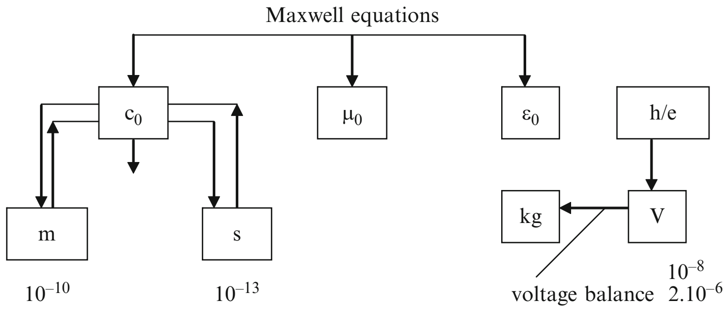

Fig. 8.2 Maxwell equations

8.2.3 In Terms of Fundamental Constants Using Maxwell Equations

Using Maxwell equations and permeability of free space μ_0 and velocity of light c_0 in vacuum we may find permittivity of free space, and it gives electrical quantities. The use of the velocity of light defines metre, the unit of length. Josephson's constant $(h/2e)$ gives voltage, so kilogram may be obtained in terms of volt through voltage balance. The system may not be as good as to get the kilogram from volt, and the realizable uncertainty is 2×10^{-6} only. While two mass standards of the same material can be compared with an uncertainty of 5×10^{-9}; the relationships with various units are shown in Fig. 8.2 diagrammatically.

8.2.4 A Consistent Set of Fundamental Constants by BIPM

BIPM [5] has given another way to link base units in SI with the fundamental physical and atomic constants. The relationship of physical constants (shown in boxes) with base units is represented in Fig. 8.3. The figure shows that the base units of the SI are linked to measurable quantities through the unchanging and universal constants of physics.

In Fig. 8.3, the surrounding boxes, lines and uncertainties represent measurable quantities. The numbers marked next to the base units are estimates of the standard uncertainties of their best practical realizations; the fractions shown next to the fundamental constants represent the uncertainty of our knowledge of these constants [6]. The lighter boxes reflect the unknown long-term stability of the kilogram artefact and its consequent effects on the practical realization of the definitions of the ampere, mole and candela.

The definition of the ampere, for example, involves the kilogram, but an alternative link is the Josephson constant (K_{J-90}) and von Klitzing's quantum–Hall resistance (R_{J-90}), both of which were given fixed conventional values in 1990.

For further reading, references in [7] and [8] may be seen.

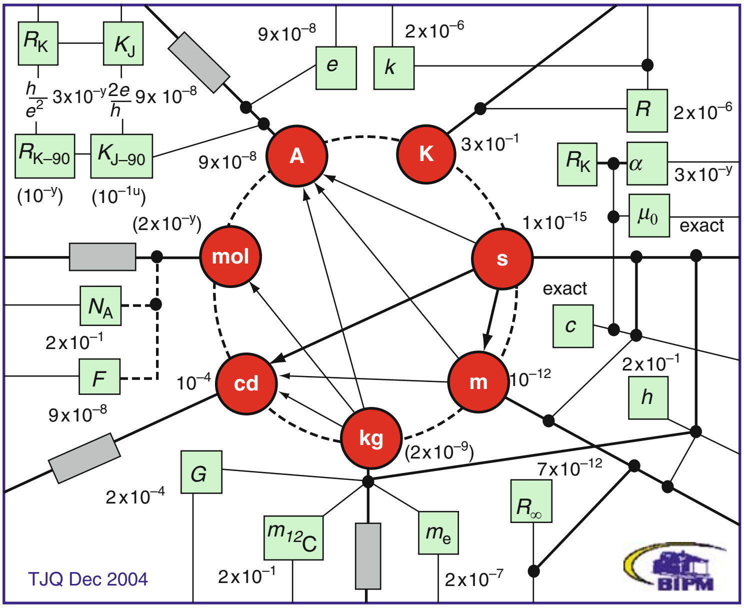

Fig. 8.3 A consistent set of fundamental constants by BIPM

8.3 CIPM Recommendation 1 (CI-2005)

On the direction of the 21st CGPM (1999), the International Committee for Weights and Measures (CIPM) held its meeting on 7 October 2005. The CIPM, in this meeting, in consultation with CCU, CCM, CCEM, CCQM and CCT, suggested preparing new definitions and practical methods (*mises en pratique*) for each unit, namely kilogram, ampere, kelvin and mole.

If the results of experimental measurements over the next few years are indeed acceptable by all and agreed by the various Consultative Committees and other relevant bodies, then the CIPM should:

1. Prepare proposals well in time, for possible adoption by the 24th CGPM in 2011
2. Urge member states for funding National Metrology Institutes to pursue continued relevant research in order to facilitate the changes suggested here and improve knowledge of the relevant fundamental constants, with a view to further improvement in the SI
3. Explore the possibility of redefining, at the same time, the mole in terms of a fixed value of the Avogadro constant
4. Invite all Consultative Committees, gparticularly the CCM, CCEM, CCQM and CCT, to consider the implications of changing the definitions of the above-mentioned base units of the SI, and to submit a report to the CIPM not later than June 2007

8.4 A Proposal to Redefine Kilogram, Ampere, Kelvin and Mole

Following the aforesaid recommendations of the CIPM 2005, a paper about the proposed definitions of kilogram, ampere, kelvin and mole appeared in the 2006 edition of *Metrologia*. The paper contains at least two ways of defining each unit.

8.4.1 Kilogram

The kilogram may be defined in one of the following ways:

1. The kilogram is the mass of a body whose equivalent energy is equal to that of a number of photons whose frequencies sum up to exactly $[(299,792,458)^2/66,260,693] \times 10^{41}$ Hz or $1.356\,313\,8 \times 10^{50}$ Hz.
2. The kilogram is the mass of a body whose de Broglie–Compton frequency is equal to exactly $[(299,792,458)^2/(6.6260693 \times 10^{-34})]$ Hz or 1.3563138×10^{50} Hz.
3. The kilogram, the unit of mass, is such that Planck's constant is exactly $6.6260693 \times 10^{-34}$ J/s.

8.4.2 Ampere

1. The ampere is the electric current in the direction of the flow of exactly $1/\left(1.60217653 \times 10^{-19}\right) = 6.24150948 \times 10^{18}$ elementary charges in one second.
2. The ampere, the unit of electric current, is such that the elementary charge is exactly $1.60217653 \times 10^{-19}$ C.

8.4.3 Kelvin

1. The kelvin is the change of thermodynamic temperature that results in a change of thermal energy kT by exactly $1.3806505 \times 10^{-23}$ J.
2. The kelvin, the unit of thermodynamic temperature, is such that the Boltzmann constant is exactly $1.3806505 \times 10^{-23}$ J/K.

8.4.4 Mole

1. The mole is the amount of substance of a system that contains exactly 6.0221415×10^{23} specified elementary entities, which may be atoms, molecules, ions, electrons, other particles or specified groups of such particles.
2. The mole, the unit of amount of substance of a specified elementary entity, which may be an atom, molecule, ion, electron, any other particle or a specified group of such particles, is such that the Avogadro constant is exactly $6.0221415 \times 10^{23}\ \mathrm{mol}^{-1}$.

8.5 The Values of *h*, *e*, *k* and N_A

The kilogram, ampere, kelvin and mole link to the exact values of the Planck constant h, elementary charge e, Boltzmann constant k and Avogadro constant N_A, respectively. However, only the values of Planck's constant h and molar gas constant R will be taken from the adjusted values of CODATA 2000 or will be taken from the latest CODATA values; the other three values, namely of the elementary charge e, Boltzman's constant k and Avagadro's constant N_A, will be derived from the following formulae:

$$
\begin{aligned}
e &= (2\alpha h/\mu_0 c_0)^{1/2} \\
k &= R/N_A \\
N_A &= c_0 A_r(e)\, M_u \alpha^2/2R_\infty h
\end{aligned}
$$

where $\mu_0 = 4\pi \times 10^{-7} N_A^{-2}$ is the magnetic constant (permeability of vacuum), $M_u = 10^{-3}$ kg/mol is the molar mass constant, and α, $A_r(e)$ and R_∞ are the fine-structure constant, the relative atomic mass of the electron and the Rydberg constant, respectively. These are also adjusted constants.

The respective relative uncertainties u of the 2002 recommended values of R, the Rydberg constant, the fine structure constant and the relative atomic mass are:

$$
\begin{aligned}
u_r(R) &= 1.7 \times 10^{-6}, \\
u_r(R_\infty) &= 6.6 \times 10^{-12}, \\
u_r(\alpha) &= 3.3 \times 10^{-9} \text{ and} \\
u_r[A_r(e)] &= 4.4 \times 10^{-10}.
\end{aligned}
$$

The values of h, e, k and N_A, derived from the above formulae of e, k, and N_A, are as follows:

$$
\begin{aligned}
h &= 6.6260693\,(11) \times 10^{-34}\ \mathrm{J\,s} \quad \left[1.7 \times 10^{-7}\right] \\
e &= 1.60217653\,(14) \times 10^{-19}\ \mathrm{C} \quad \left[8.5 \times 10^{-8}\right] \\
k &= 1.3806505\,(24) \times 10^{-23}\ \mathrm{J/K} \quad \left[1.8 \times 10^{-6}\right] \\
N_A &= 6.022141510 \times 10^{23}\ \mathrm{mol}^{-1} \quad \left[1.7 \times 10^{-7}\right]
\end{aligned}
$$

where, as usual, the number in parentheses is the numerical value of the standard uncertainty referred to the last two digits of the quoted value. Relative standard uncertainties are indicated within square brackets.

8.5.1 Observations

Thus, the uncertainty of h plays the dominant role by far in determining the uncertainty of e and N_A, while the uncertainty of R plays a similar role in determining the value of k.

Here one may observe that the measurement capability of each base unit is better than that with which these constants are known. The advantage of taking these constants as fixed, with zero uncertainty, is that the uncertainty of the host of fundamental constants involving these constants will be considerably reduced.

8.6 Practical Standards to Realize Kilogram

The kilogram, the unit of mass in SI, may be realized by any of the following methods:

1. The present International Prototype of the Kilogram may be used, after cleaning as per BIPM procedure, with an uncertainty of

$$\left[\left(1\times10^{-9}\right)^2+\left(<5\times10^{-9}\right)^2\right]^{1/2}\approx5\times10^{-9}.$$

2. The electron volt as given in the SI Brochure by BIPM [9].
3. The unified atomic mass unit u.
4. The Planck constant h or h/c^2, which may be inserted in the forthcoming SI Brochure. It may also be advisable to include realization of h through Josephson and von Klitzing effects, with designation h_{s-90}, given by the relation

$$h_{s-90}/c^2=4/\left[k_{j-90}^2\times R_{k-90}\times c^2\right]. \tag{8.2}$$

This will avoid disruption to the disseminated representations of electrical units and will also provide a stable mass reference for the moving coil watt realizations and a method to compare with each other.

8.6.1 Other Methods of Redefining Kilogram

- Through Avogadro's constant
- Levitation of pure diamagnetic bodies in the superconducting state

- Ion collection of heavy metals like bismuth or gold
- Voltage balance
- Watt balance

8.6.2 Conclusion in Regard to the Kilogram

The problem is that none of the constants that can be related with mass are known at a better uncertainty than 1.7×10^{-7}, whereas the uncertainty with which two kilograms of the same material and polish can be inter-compared is of 5×10^{-9}. The uncertainty in the comparison of two mass standards even when the material of the two standards is different is a few parts in 100 million. It will therefore be advisable to wait for some time, first, to ascertain the change in mass of prototype kilograms and, second, to improve upon the uncertainty of related fundamental constants. The balance available at the time of defining the mass (1875) had a readability on the order of $100\,\mu$ g. It is only in 1975 when a new balance of readability, $1\,\mu$g, was made available to the BIPM that a third comparison of the National Prototypes was carried out and a conclusion drawn that kilograms are increasing at the rate of $0.5\,\mu g/year$. The change is small in comparison to the uncertainty of comparison; hence it will be worthwhile to wait till the fourth comparison is carried out. In the mean time the National Metrology Institutes may improve upon the uncertainty of the concerned physical constants.

8.6.3 Measurement Standards

Embodiment of Units of Measurements

Though in the initial stages, every unit was in the form of an artefact, like the cubit in Egypt, later on, it was found that the material standard of length might shrink in winter and expand in summer, so distances measured in winter will show higher values than those measured in summer. People thought of this problem and came to the conclusion that instead of taking the length of the rod as unit, it would be better to consider some dimension of a natural body. The obvious natural body for them must have been our earth. So a certain fraction of a specific meridian of the earth was taken as the unit of length. The next most common thing was water. So people used the properties of water to define the units. Defining the kilogram as the mass of one decimetre cube of water at its temperature of maximum density is one such example. The rotation of the earth provided the unit of time in terms of one full day and night. They estimated time by the number of nights that passed. Nights are distinguishable from days by the appearance of the moon at night.

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Chapter 9
Scientists Associated with Units of Measurements

It is interesting to know about the life of the scientists who worked throughout their life in the field of measurement. To honour some such eminent scientists, some of the base and derived units have been assigned their respective names.

Among the base units, the unit of electric current (ampere), thermodynamic temperature (kelvin) and temperature difference (Celsius) have been respectively named after Andre Marie Ampere, Lord Kelvin and Anders Celsius.

9.1 Scientists Associated with Base Units

9.1.1 Lord Kelvin

June 26, 1824 to December 1907

The real name of Lord Kelvin was William Thomson. He was born on 26 June 1824 at Belfast, Ireland. He was an outstanding leader in the physical sciences of the nineteenth century. He did important work in the mathematical analysis of electricity and thermodynamics, and did much to unify the emerging discipline of physics in its modern form. He also enjoyed a second career as a telegraph engineer and inventor, a career that propelled him into the public eye and ensured his fame and honour.

He was the first Baron Kelvin, and his barony was named after the river Kelvin which runs through the university of Glasgow. He was awarded many meritorious awards like GCVO, OM, PC and PRS. He died on 17 December 1907 in Largs, Ayshire, Scotland.

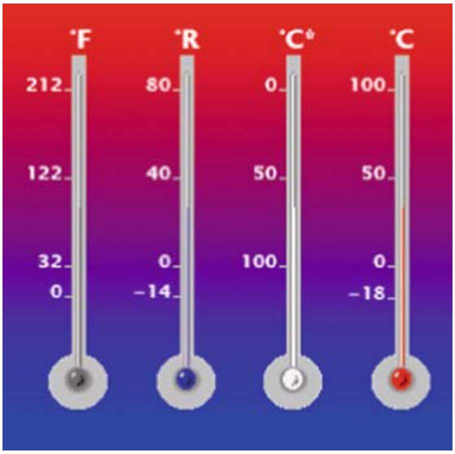
The unit of thermodynamic temperature, kelvin, has been named after him to honour this great mathematician, physicist and engineer.

9.1.2 Anders Celsius

1701–1744

The child born on 27 January 1701 in Uppsala, Sweden, grew up to become famous as Anders Celsius. After his completing his education in his home town north of Stockholm, he became a professor of astronomy in 1730. At that time, there was no larger observatory anywhere in Sweden. Therefore, Celsius made a round trip to some of the famous European astronomy sites from 1732 to 1734.

The director of the Paris observatory, which was founded in 1672, was Jaques Cassini (1677–1756), son of Jean-Dominique Cassini (1625–1712). At this time, there was a dispute between English and French astronomers about the actual shape of the earth. To find the true answer to this question, expeditions had to be sent to the "ends" of the world to measure the local positions exactly. The expedition to the north was commissioned to Pierre Louis de Maupertuis (1696–1759), which Celsius joined as assistant. The journey to Lapland, the northernmost part of Sweden, lasted from 1736 to 1737. After all the measurements were made, Newton's theory about the flattening of the earth at the poles was confirmed in 1744.



Originally Celsius defined its temperature scale as shown 3rd from the left

After the expedition, Celsius returned to Uppsala and worked on the erection of an observatory, which was finished in 1740. Celsius was one of the first to examine the changes of the earth's magnetic field at the time of a northern light. He was also one of the first to measure the brightness of stars with measuring tools. After the first Swedish observatory was completed, he was appointed as its director. Celsius became famous for his recommendation in 1742 to divide the temperature scale of a mercury thermometer at air pressure of 760 mm of mercury into 100°C, where 100 was taken as the freezing point and 0 the boiling point of water. Because of the detailed fixation of the measuring environment and methods, this definition was considered more exact than those given by Gabriel Daniel Fahrenheit (1686–1736) and the aristocrat and biologist René-Antoine Ferchault de Réaumur (1683–1757). Later the reversion of the Celsius scale with 0 as the freezing point and 100 as the boiling point of water was introduced, and with this modification it became widely accepted.

Celsius was an active supporter of the introduction of the Gregorian calendar, which was accepted in Sweden in 1753, just nine years after his death. The unit of temperature interval, "degree Celsius", has been named after him to honour this great scientist.

9.1.3 Andre Marie Ampere

The ampere – the unit of electric current – is named in the honour of Andre Marie Ampere.

He was born on 20 January 1775 near Lyon, France. He enjoyed a happy youth only until the age of 17, when his father died at the guillotine.

1775–1836

He worked systematically on topics in mathematics, chemistry, optics and metaphysics, always aiming at an orderly taxonomy of knowledge and the reduction of observations to rational principles.

In 1820, Oersted announced his discovery of the magnetic effect of an electric current. Ampere performed his experiments more thoroughly than Oersted could manage. For all his work Maxwell called him "the Newton of electricity". It was only through the framework developed by Faraday, Weber and Maxwell himself that the merits of Ampere's obsessive devotion could be appreciated. He died on 10 June 1836 in Marseille and was buried in the Cimetière de Montmartre, Paris, France. The practical international unit ampere defined with the help of electrolysis of silver nitrate is 0.99985 A.

9.2 Scientists Associated with Derived Units

It will be also interesting to know a little more about the work of the scientists who have been honoured by assigning their names to some SI-derived units.

9.2.1 Sir Isaac Newton

The unit of force has been named after this great scientist, Sir Isaac Newton. The unit of force is the newton with the symbol N.

(1642–1727)

Isaac Newton was born in Lincolnshire, near Grantham, on 25 December 1642. Newton was the contemporary and friend of Wallis Huygens. He was educated at Trinity College, Cambridge, and lived there from 1661 to 1696, during which time he produced the bulk of his work in mathematics. Though most of his mathematical work was done between 1665 and 1686, the bulk of it was not printed till some years later.

The development of mathematics in Great Britain was, for a century, entirely in the hands of the Newtonian school.

In October 1669, Barrow resigned from the Lucasian chair in favour of Newton. Newton laid down the foundation of differential and integral calculus. Around 1696, he discovered the binomial theorem. His work in gravitation and optics made him one of the greatest scientists the world has ever known.

He died at Kensington, London, on 20 March 1727.

9.2.2 Heinrich Rudolf Hertz

Heinrich Rudolf Hertz was born on 22 February 1857, in Hamburg, Germany. In 1888, he was the first to experimentally prove the existence of electromagnetic waves. His apparatus consisted of a wire connected to an induction coil to produce waves and a small loop of wire with a spark gap to detect the waves. When currents were induced in the detection loop, the induced current produced a spark across the gap. He further showed with the help of mirrors, prisms and metal gratings that electromagnetic waves do have analogous properties to light. In the process of his investigation, he discovered the photoelectric effect also but could not recognize it.

1875–1894

In 1892, he published an important paper "Investigations on the Propagation of Electric Force". Through this work, he reformulated Maxwell's complicated field equations into a symmetric and compact form. He criticized Maxwell's definition of electric charge and his concept of displacement of current, which he replaced with a mathematical formalism. Hertz's simplification of Maxwell's theory to a mathematical formalism led to its widespread acceptance. He died on 1 January 1894.

9.2.3 Blaise Pascal

Blaise Pascal was born on 19 June 1623 at Clemont, France. He died at the young age of 39 on 19 August 1662. His father Etienne Pascal had unorthodox views regarding education and decided to teach his son himself and removed all books on mathematics from the house. This raised a curiosity in Blaise and he studied geometry on his own and discovered that the sum of the angles of a triangle is two right angles. His first work, *Essay on Conic Sections*, was published in February 1640. He invented the first digital calculator to help his father with his work of collecting taxes after working on it for three years between 1642 and 1645. The device, called the Pascaline, resembled a mechanical calculator of the 1940s.

From about this time Pascal began a series of experiments on atmospheric pressure. By 1647 he had proved to his satisfaction that a vacuum existed, but Descartes, a great scientist of that time, discarded his idea and wrote a sarcastic letter to

1623–1662

Huygens, another big name of that time, saying that "Pascal has too much vacuum in his head".

In August 1648 Pascal observed that the pressure of the atmosphere decreases with height and deduced that a vacuum existed above the atmosphere. In October 1647 he wrote "New Experiments Concerning Vacuums". From May 1653, he worked on mathematics and physics and wrote the "Treatise on the Equilibrium of Liquids" (1653) in which he explains Pascal's law of pressure.

Blaise Pascal was a very influential French mathematician and philosopher who contributed to many areas of mathematics. He worked on conic sections and projective geometry and, in correspondence with Fermat, he laid the foundations for the theory of probability.

The pascal – the unit of pressure – has been named after this great man to honour him.

9.2.4 James Prescott Joule

James Prescott Joule was born on 24 December 1818 in Salford near Manchester, UK. He studied the nature of heat, and discovered its relationship to mechanical energy. This led to the theory of conservation of energy (the First Law of Thermodynamics). He worked with Lord Kelvin to develop the absolute scale of temperature, and made observations on magnetostriction. Magnetostriction is a property of ferromagnetic materials that causes them to change their shape when subjected to a magnetic field. Joule was the first to identify it in 1842 while observing a sample of nickel. He found the relationship between the flow of current through a resistance and the heat dissipated, now called Joule's law.

1818–1889

The SI unit of energy – the joule – is named after him, and is pronounced to rhyme with “tool.”

9.2.5 James Watt

James Watt was born on 19 January 1736 in Greenock, Scotland. He was the son of a ship’s chandler. His father set him up in his workshop with his own bench and tools, and there the young James made models and became familiar with instruments used in ships.

In 1755, he decided to manufacture scientific instruments and so he took up an apprenticeship in London and got a job to make instruments at Glasgow University. He did not confine himself to scientific instruments only, but made violas, guitars, fiddles, flutes and organs also.

In 1763, when asked to repair a steam Newcomen engine, Watt realized that the efficiency of the machine could be greatly improved by having a separate, but linked, condenser. This invention of Watt saved between two-thirds and three-quarters of the coal consumed by the older type of engine. In 1781 he patented five methods of converting the reciprocating motion of a steam engine’s piston into continuous rotary motion.

In 1782 he patented the double-acting steam engine, in which steam is admitted alternately at either end of the cylinder. In 1784 he patented the parallel motion, an

1736–1819

arrangement of connected rods which he described as "one of the most ingenious, simple pieces of mechanism I have contrived". Then, in 1788, again at Matthew Boulton's urging, he designed the centrifugal governor for controlling the speed of an engine. He also invented a gauge for measuring steam pressure and a revolution counter.

He helped Joseph Priestley with his investigations into gases. He experimented on the strength of materials, a subject of acute interest to his manufacturer friends. He further developed accurate means of measuring dimensions, furnace temperatures and the like, again vital to the advancement of manufacturing processes. In 1780 he had patented what was probably the earliest form of a press-copier, which he marketed through his own company, James Watt & Co. The process involved writing with ink mixed with Arabic gum. When a sheet of damp tissue paper was pressed against the manuscript, some of the ink stuck to it, creating a mirror image of the original on the tissue paper. By turning the copy over it could then be read through the tissue paper. He died on 25 August 1819 in Heathfield, England.

The watt (symbol: W) is the SI unit of power that is named in honour of this great physicist.

9.2.6 Charles Augustin Coulomb

The coulomb (symbol: C) – the SI unit of electric charge – is named after Charles Augustin Coulomb. Coulomb was a French physicist. He was born in Angoulême,

1736–1806

France, on 14 June 1736 in a well-to-do family. He chose to be a military engineer, and spent three years in this profession, till significant injury to his health forced him to leave military service. Upon his return, he was employed at La Rochelle, the Isle of Aix and Cherbourg. He discovered an inverse relationship on the force between charges and the square of its distance, later named after him as Coulomb's law.

In 1781, he was stationed permanently at Paris. In 1784, he published his work on theory and practice on torsional forces on a metallic strip. In 1785, he published his three reports on Electricity and Magnetism in the Royal Academy of Sciences on:

1. The construction and use of electric (torsion) balance and experimentally verified the laws of force of attraction or repulsion between two charged bodies
2. The laws according to which both the magnetic and the electric fluids act, either by repulsion or by attraction
3. The quantity of electricity that an isolated body loses in a certain time period by contact with less humid air

Coulomb explained the laws of attraction and repulsion between electric charges and magnetic poles, although he did not find any relationship between the two phenomena. He thought that the attraction and repulsion were due to different kinds of fluids.

On the outbreak of the Revolution in 1789, he resigned his post and retired to a small estate which he owned at Blois. He was recalled to Paris in order to take part in the new determination of weights and measures, which had been decreed by the Revolutionary government. He was one of the first members of the National

Institute, and he was appointed inspector of public instruction in 1802. But his health became very feeble, and four years later he died on 23 August 1806 in Paris.

9.2.7 Alessandro Volta

1745–1827

The volt (symbol V) – the SI-derived unit of electric potential difference – is named in honour of the Italian physicist Alessandro Volta, who invented the voltaic pile, the first chemical battery. He was born on 18 February 1745 and died on 3 March 1827 near Como, Lombardy, Italy.

In 1800, Volta built the voltaic pile and discovered the first practical method of generating electricity. The voltaic pile comprised of alternating discs of zinc and copper, with pieces of cardboard soaked in brine between the metals. It produced electrical current. The metallic conducting arc was used to carry the electricity over a greater distance. Volta's voltaic pile was the first battery that produced a reliable, steady current of electricity.

One contemporary of Volta was Luigi Galvani; in fact, it was Volta's disagreement with Galvani's theory of galvanic responses (animal tissue contained a form of electricity) that led Volta to build the voltaic pile to prove that electricity did not come from the animal tissue but was generated by the contact of different metals, brass and iron, in a moist environment. Ironically, both scientists were right. He also worked on the chemistry of gases and discovered methane.

9.2.8 Michael Faraday

1791–1867

The SI unit Farad is named after the English chemist and physicist Michael Faraday. He was born on 22 September 1791, in Newington, Surrey, near London. He is known for his pioneering experiments in electricity and magnetism. Many consider him the greatest experimentalist who ever lived. Several concepts that he derived directly from experiments, such as lines of magnetic force, have become common ideas in modern physics.

He received little more than a primary education, and at the age of 14 he was apprentice to a bookbinder. There he became interested in the physical and chemical works of time. After hearing a lecture by the famous chemist Humphry Davy, he sent Davy the notes he had made from his lectures. As a result, Faraday was appointed, at the age of 21, as assistant to Davy in the laboratory of the Royal Institution in London.

During the initial years of his scientific work, Faraday occupied himself mainly with chemical problems. He discovered two new chlorides of carbon and succeeded in liquefying chlorine and other gases. He isolated benzene in 1825, the year he was appointed director of the laboratory.

Davy, who had the greatest influence on Faraday's thinking, had shown in 1807 that the metals sodium and potassium could be precipitated from their compounds by an electric current. He called this process electrolysis. Faraday's vigorous pursuit of these experiments led, in 1834, to what came to be known as Faraday's laws of electrolysis.

He demonstrated the principle of electromagnetic induction in 1831. Faraday expressed the electric current induced in the wire in terms of the number of lines of force that are cut by the wire. The principle of induction was a landmark in applied science, for it made possible the invention of the dynamo and the generator, which produce electricity by mechanical means.

Faraday's discovery (1845) that an intense magnetic field can rotate the plane of polarized light is known today as the Faraday Effect. The phenomenon has been used to elucidate molecular structure and has yielded information about galactic magnetic fields.

Faraday described his numerous experiments in electricity and electromagnetism in three volumes entitled *Experimental Researches in Electricity* (1839, 1844 and 1855); his chemical work was chronicled in *Experimental Researches in Chemistry and Physics* (1858). Faraday ceased research work in 1855 because of declining mental powers, but he continued as a lecturer until 1861. A series of six children's lectures published in 1860 as *The Chemical History of a Candle* has become a classic of science literature. He died on 25 August 1867.

9.2.9 Wilhelm Eduard Weber

1804–1891

Wilhelm Eduard Weber was born on 24 October 1804 in Wittenberg, Germany, and died on 23 June 1891. The weber – the unit of magnetic flux – is named in honour of this German scientist, Weber. One weber is equal to 108 maxwells.

He joined the University of Halle in 1822 and wrote his doctoral dissertation in 1826. After that he taught at Halle. In 1831, Weber was appointed to the chair of physics at Göttingen, and there followed his six years of close friendship and collaboration with Gauss. Weber developed sensitive magnetometers and other magnetic instruments during this time.

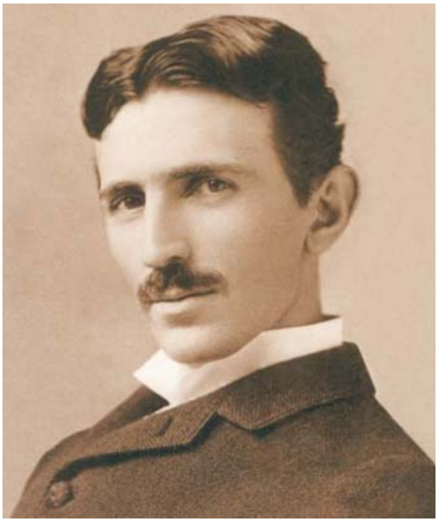
When Victoria became Queen of Britain in 1837 her uncle became ruler of Hanover and revoked the liberal constitution. Weber was one of the seven professors at Göttingen to sign a protest and all were dismissed. He remained at Göttingen without a position until 1843, when he became a professor of physics at Leipzig.

In 1848, he returned to his old position in Göttingen and, in 1855, he became temporary director of the astronomical observatory there. His work on the ratio between the electrodynamic and electrostatic units of charge in 1855 proved extremely important and was crucial to Maxwell in his electromagnetic theory of light. Weber found that the ratio was $3.1074 \times 10^8\ \mathrm{ms}^{-1}$ but failed to take any notice of the fact that this was close to the speed of light.

Weber's later years at Göttingen were devoted to work in electrodynamics and the electrical structure of matter. He was described by Thomas Hirst in the following way:

> "He speaks and stutters on unceasingly, one has nothing to do but listen. Sometimes he laughs for no earthly reason, and one feels sorry at being not able to join him."

9.2.10 Nickola Tesla



1856–1943

Nickola Tesla was born in Croatia (then part of Austria–Hungary) on 9 July 1856. He was the electrical engineer who invented the AC (alternating current) induction motor, which made the universal transmission and distribution of electricity possible. Tesla began his studies in physics and mathematics at Graz Polytechnic, and then took up philosophy at the University of Prague. He worked as an electrical engineer in Budapest, Hungary, and subsequently in France and Germany. In 1888, his discovery that a magnetic field could be made to rotate if two coils at right angles are supplied with AC current 90 C out of phase made possible the invention of the AC induction motor. The major advantage of this motor is its brushless operation, which many at the time believed was impossible.

Tesla moved to the United States in 1884, where he worked for Thomas Edison who quickly became a rival, Edison being an advocate of the inferior DC power transmission system. During this time, Tesla was commissioned with the design of the AC generators installed at Niagara Falls. George Westinghouse purchased the patents to his induction motor, and made it the basis of the Westinghouse power system, which still underlies the modern electrical power industry today. He also did notable research on high-voltage electricity, at one point creating an earthquake which shook the ground for several miles around his New York laboratory. He also devised a system which anticipated worldwide wireless communication: fax machines, radar, radio-guided missiles and aircraft. He died on 7 January 1943.

9.2.11 Joseph Henry

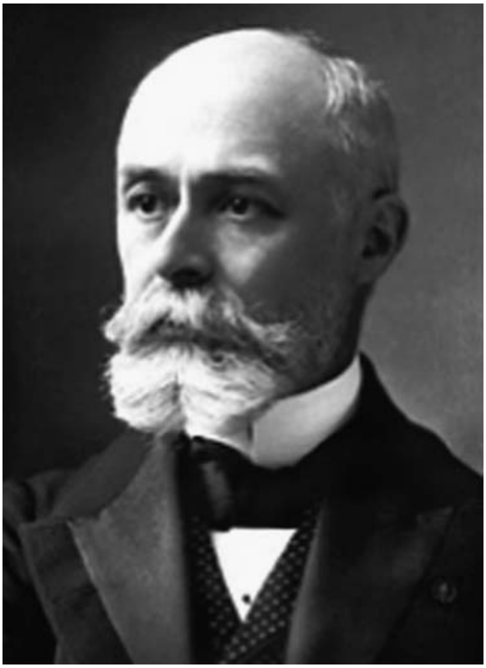
1797–1878

Joseph Henry, physicist and scientific administrator, was born on 17 December 1797. Although he was largely self-educated, Henry studied at the New York

Academy at Albany from 1819 to 1822. He began teaching at the Academy in Albany in 1826 where he remained until 1832 when he accepted a position at the College of New Jersey (now Princeton University).

He is widely considered the foremost American scientist of the nineteenth century. Although Henry at an early age appeared to be headed for a career in theatre, a chance encounter with a book of lectures on scientific topics turned his interest to science. Henry's early investigations concerned electromagnetic phenomena, and his discovery of electromagnetic self-induction in 1831 established his reputation in America. Henry appears to have discovered the principle of electromagnetic induction independently of British scientist Michael Faraday, but because Faraday published his results before Henry, he is credited with the discovery. In 1846, Henry was nominated first Secretary of the newly established Smithsonian Institution, a position he held until his death. In 1868, he was elected President of the Academy; this position, too, he held until his death. He died on 13 May 1878. The henry – the unit of inductance – has been named in honour of this American scientist, Joseph Henry.

9.2.12 Antoine Henri Becquerel



1852–1908

Antoine Henri Becquerel was born on 15 December 1852 into a family of scientists in Paris, France. He was the son of Alexandre Becquerel, who studied light and phosphorescence and invented the phosphoroscope, and the grandson of Antoine César Becquerel, one of the founders of electrochemistry. His son also became a great scientist later on.

In 1892, he became the third in his family to occupy the physics chair at the National Natural History Museum, France. In 1894, he became chief engineer in the Department of Bridges and Highways.

In 1896, Becquerel accidentally discovered radioactivity while investigating phosphorescence in uranium salts. This discovery led him to investigate the spontaneous emission of nuclear radiation. In 1903, he shared the Nobel Prize in Physics with Pierre and Marie Curie "in recognition of the extraordinary services he has rendered by his discovery of spontaneous radioactivity".

In 1908, he was elected permanent secretary of the Académie des Sciences. He died in the same year at the age of 55 on 25 August 1908 in Le Croisic.

The SI unit for radioactivity, the becquerel (Bq), is named after him. The craters on the Moon and on Mars are also named as Becquerels in his honour.

9.2.13 Louis Harold Gray

1905–1965

Louis Harold Gray was born on 10 November 1905 and died on 9 July 1965. He obtained his PhD at the Cavendish Laboratory under Rutherford at a time when the laboratory was a world centre for fundamental research in nuclear physics. Although W. H. Bragg had stated the principle in 1912, Gray worked out the consequences in far greater detail. In 1936, he developed the Bragg–Gray principle, which provides the basis for the cavity-ionization method for measuring gamma-ray energy.

Gray was interested in the biological effects of neutrons. Realizing that more powerful sources were required, Gray, together with John Read and technician J. G. Wyatt, constructed a neutron generator at the Mount Vernon Hospital, where Gray had been hospital physicist since 1933. With this tool, Gray and his colleagues made important contributions to understanding the relative biological effectiveness (RBE) of neutrons, discovering that it depended on dose, dose rate and level of biological damage.

In a 1940 paper, Gray and Read employed their energy unit, "that amount of neutron radiation which produces an increment of energy in unit volume of tissue equal to the increment of energy produced in unit volume of water by one röntgen of radiation".

After World War II, Gray joined the new radiotherapeutic research unit at Hammersmith where a cyclotron for radiobiology research and radioisotope production was built. As Deputy Director, he oversaw important research on radiobiology and DNA.

Leaving the Hammersmith group, Gray constructed a laboratory at the Mount Vernon Hospital, the nucleus of the present Gray Laboratory, which attracted many famous researchers. The unit became known as a centre for radiation chemistry, and studies were carried out on the irradiation of bacteria, and of tumours. Gray himself worked with Eleanor Deschner on the oxygen effect, and with Dewey he developed the Hersch cell for the measurement of oxygen.

Gray was Vice Chairman of the International Commission on Radiation Units and Measurements (ICRU) from 1956 to 1962, and assisted in the formation of the IARR. He received many awards for his work, notably the Betrner Medal in 1964. He was elected Fellow of the Royal Society (FRS) in 1961.

9.2.14 Rolf M. Sievert

Rolf Maximilian Sievert was born in Stockholm on 6 May 1896. His dissertation for PhD was on measuring methods on röntgen, radium and ultraviolet. The same year he became associate professor in medical physics at Stockholm University.

Between 1924 and 1937 he was head of the physics laboratory at Radiumhemmet. In 1937, he was appointment head of the department of radiation physics at Karolinska Institute, and in 1941, as professor in radiation physics at the same institute.

After his retirement in 1965 and till his death on 3 October 1966, he continued to take a very active part in the national and international cooperation in his particular

1896–1966

field of interest, radiation dose measurement and radiation protection. He had played a pioneering role in his area of activities.

Already in 1919, Rolf Sievert made contact with radiologists – physicians using ionizing radiation in their work – and offered them his cooperation in the attempt to solve the physical problems linked with the usage of radiation for diagnosis and therapy. His unremunerated cooperation continued until 1924, when the Cancer Society in Sweden decided to remunerate Sievert as head of the physics laboratorium of Radiumhemmet, which he had organized and financed on his own. Under the leadership of Sievert the laboratory was developed into a world-renowned centre for radiation physics. In 1938, he moved to the Karolinska Hospital and was made Head of the Department of Radiation Physics.

During the early part of the 1920s, no standardization of patient doses for different hospitals was performed. For this reason, Sievert started an organization in 1925 which was made responsible for the continuous control of dosage levels at all clinics in the country performing radiation treatment. The control programme was extended as time went by, and eventually included control of all work with radiation, medical as well as industrial. On the initiative of Sievert, the government passed Sweden's first radiation protection law in 1941. The law gave the Department of Radiation Physics the task to supervise all such activities.

During the years 1920–1940, Sievert made his most important contributions to the field of clinical physics. He developed the basics of how to calculate the absorbed dose to tumours; he developed new devices for patient irradiation and pointed out the importance of the contribution of secondary radiation. Furthermore, he invented a number of ingenious instruments for dose measurements, such as the world-renowned Sievert chamber, among others.

During the 1930s, Sievert worked primarily with the biological effects of ionizing radiation, and particularly the effects of the low doses received by radiologists in their daily work, and for comparison, the effects caused by unavoidable natural radiation background that we all are exposed to. Several years before the question of radioactive fallout was raised Sievert studied the matter by gathering available data on the spreading of aches in the atmosphere after volcano eruptions.

During the last 20 years of his life, Sievert spent most of his time working with radiation protection issues, and made plans for what was to become the Swedish Radiation Protection Institute (SSI).

To honour Rolf Sievert, the CGPM conference of 1979 accepted the sievert (symbol Sv) as the unit for dose equivalent for ionizing radiation. This unit is a part of the SI.

$$1\mathrm{Sv} = 1\mathrm{J/kg}$$

He was the pioneer in the field of radiation protection, and one of the main initiators of both ICRP and ICRU in 1929. He was Chairman of ICRP (1956–1962) and of UNSCEAR (1958–1960), and Professor at the Department of Medical Radiation Physics in Stockholm (1941–1965). He died in Stockholm on 3 December 1966.

Sievert Chamber

In its most usual form, the Sievert chamber is a sphere or cylinder of a magnesium alloy (electron metal) placed in the centre of a hollow sphere of the same material. The inner electrode is fixed in its position by amber isolators. Through an opening in the outer sphere they could be charged to a known potential. The opening is closed with a lid equipped with a staff acting as a holder of the chamber. If the chamber is exposed to ionizing radiation, the air in the cavity between the inner and the outer spheres becomes conducting through the ion pairs that are formed, and the charge of the chamber is reduced by the leakage current that is formed. The charged reduction can then easily be measured on a separate instrument at some other location. The reduction of the charge is a measure of the radiation dose received by the chamber. Such a chamber can be transported long distances without affecting the reading. The diameter of the chamber can be as small as a few millimetres.

9.2.15 Georg Simon Ohm

Georg Simon Ohm was born on 16 March 1787 in Erlangen, Germany. He was a German physicist who was best known for his research in electrical currents. He was educated at the University of Erlangen. Unfortunately, when Ohm published

1787–1854

his finding in 1827, *that current flowing in a given resistor was proportional to the voltage applied across it*, his colleagues dismissed his ideas. Ohm was forced to resign from his high school teaching position and he lived in poverty and shame until he accepted a position at Nuremberg in 1833.

From 1833 to 1849 he was Director of the Polytechnic Institute of Nuremberg, and from 1852 until his death he was professor of experimental physics at the University of Munich. His formulation of the relationship between current, electromotive force and resistance, known as Ohm's law, is the basic law of current flow.

In 1841, the Royal Society in London recognized the significance of his discovery and awarded him the Copley medal. The following year, they admitted him as a member. In 1849, just five years before his death, Ohm's lifelong dream was realized when he was given a professorship of Experimental Physics at the University of Munich. On 7 July 1854 he passed away in Munich, at the age of 65.

Electricity was not the only topic on which Ohm undertook research, and not the only topic in which he ended up in controversy. In 1843, he stated the fundamental principle of physiological acoustics, concerned with the way in which one hears combination tones. However, the assumptions which he made in his mathematical derivation were not totally justified, and this resulted in a bitter dispute with the physicist August Seebeck, who succeeded in discrediting Ohm's hypothesis and Ohm had to acknowledge his error.

The ohm with the symbol Ω – the unit of electrical resistance – has been named in his honour.

9.2.16 Werner von Siemens

1816–1892

Ernst Werner von Siemens was a German inventor and industrialist and was known as Werner von Siemens.

Werner Siemens was born in Lenthe, near Hanover, Germany, on 13 December 1816. He was the fourth child out of the fourteen children of a tenant farmer. He left school without finishing his education, but joined the army to undertake training in engineering.

Siemens invented a telegraph that used a needle to point to the right letter, instead of using the Morse code. Based on this invention, he founded the company Telegraphen-Bauanstalt von Siemens & Halske on 1 October 1847, with the company taking occupation of its workshop on 12 October.

Soon after founding the company, it was internationalized. One of his brothers represented him in England (Sir William Siemens) and another in St. Petersburg, Russia (Carl von Siemens), each earning separate recognitions in their own right. Following his industrial career, Siemens was ennobled in 1888, becoming Werner von Siemens. He retired from his company in 1890 and died on 6 December 1892.

The company, reorganized as Siemens & Halske AG, Siemens–Schuckertwerke and – since 1966 – Siemens AG, was later led by his brothers; his three sons, Arnold, Wilhelm and Carl Friedrich; and his nephews, Hermann, Ernst and Peter von Siemens. Siemens AG is still one of the largest electro-technological firms in the world.

In addition to the pointer telegraph, Siemens made several contributions to the development of electrical engineering and is therefore known as the founding father of the discipline in Germany. On 14 December 1877, he received the German patent

No. 2355 for an electromechanical "dynamic" or moving-coil transducer, which was adapted by A. L. Thuras and E. C. Wente for the Bell System in the late 1920s for use as a loudspeaker. Wente's adaptation was issued the US patent 1,707,545 in 1929. Siemens is also the father of the trolleybus, which he initially tried and tested with his "Elektromote" on 29 April 1882.

The name Siemens has been adopted for the SI unit of electrical conductance.

9.3 Some Units Not Named After Any Scientist

It may be noted that the derived units katal, lumen and lux are not named after any scientists. Lumen is the Latin word for light or window and lux is the Latin word for light.

Appendix A
National Physical laboratory

Measurement Uncertainties of Base Standards at NPLI

Each modernized country, including India, has a National Metrology Institute (NMI), which maintains the standards of measurements. The National Physical Laboratory (NPL) in India is situated very close to The Indian Agricultural Research Institute, Pusa, New Delhi. The NPL is built over an area of 30 ha. Through a subordinate legislation [1] passed under the Act of the parliament [2], the responsibility of realization, establishment, custody, maintenance, determination, reproduction and updating has been entrusted to the NPL. The NPL maintains the following units to fulfil its obligation.

A.1 Metre

The standard unit of length – the metre – is realized by employing an iodine- stabilized helium–neon laser as a source of light. Its frequency is measured experimentally. The stability in He–Ne laser is 2.5 parts in 10^{11}. From the value of frequency and the internationally accepted value of the speed of light $\left(299,792,458\,\mathrm{m\,s^{-1}}\right)$, the wavelength is determined using the relation:

Wavelength = Velocity of light/frequency

By a sophisticated optical interferometer, any length can be measured in terms of the wavelength of laser light. The nominal value of the wavelength employed at NPL is 633 nanometres.

The present level of uncertainty attained at NPL for its length standard is $\pm 3 \times 10^{-9}$ [3]. However, for most practical measurements, an uncertainty of $\pm 1 \times 10^{-6}$ is adequate.

A.2 Kilogram

The Indian National Standard of mass – the kilogram – is copy No. 57 of the International Prototype Kilogram supplied by the International Bureau of Weights and Measures (BIPM: French name – Bureau International des Poids et des Mesures), Paris. This is a platinum iridium cylinder, bearing the number 57, whose mass is measured against the International Prototype at BIPM. Besides this, the NPL also maintains a group of transfer standard kilograms made of non-magnetic stainless steel and nickel–chromium alloy.

The standard uncertainty in the mass value assigned to National Standard No. 57 is 4.6×10^{-9}. The expanded uncertainty at 95% level of confidence with coverage factor $k = 2$ is 1×10^{-8} [3]. Intermediate mass standards are calibrated with an uncertainty of 1 part in 10 million.

Normally standard weights, ranging from 2 t to 1 mg, received from Industry and Research Institutes are calibrated with an uncertainty of one part in one million.

A.3 Second

For national standards of time, a Caesium atomic clock is maintained. The standard uncertainty of the Caesium atomic clock is 7.6×10^{-9}s [11]. The uncertainty in frequency measurement is 8×10^{-14} [1].

The standard maintained at NPL is linked to different users. This process, known as dissemination, is carried out in a number of ways. For applications requiring low levels of uncertainty, there is a satellite-based dissemination service, which utilizes the Indian national satellite, INSAT. Time is also disseminated through television, radio, and special telephone services. The Caesium atomic clock maintained at NPL is linked to other atomic clocks maintained by NMIs all over the world through a set of global positioning satellites (GPS).

A.4 Ampere

Ampere, the unit of electric current, is realized at NPL by measuring Volt and Ohm separately. The uncertainty in measurement of ampere is $\pm 2 \times 10^{-6}$ [3].

A.5 Kelvin

The standard of temperature is based on the International Temperature Scale of 1990 (ITS - 90). This is based on the assigned temperatures to several fixed points. One of the most fundamental temperatures is the triple point of water. At this temperature, ice, water and its vapours are in equilibrium with each other. This temperature has been assigned the value of 273.16 kelvins, and is realized, maintained and measured in the laboratory. The standard uncertainty is 0.00017 K [3]. At present, temperature standards maintained at NPL cover a range of 54–2,473 K.

A.6 Candela

The base unit candela is realized through absolute radiometers and weighting with human eye response. The unit of luminous intensity, candela, is also realized through a set of standard lamps. For practical work, a group of tungsten incandescent lamps are used. The level of uncertainty is $\pm 1.3 \times 10^{-2}$ [1].

A.7 Mole

Experimental work has been initiated to realize the mole, the SI unit for the amount of substance.

A.8 Radiation

The NPL does not maintain standards of measurements for ionising radiations. This is the responsibility of the Homi J. Bhabha Atomic Research Centre, Mumbai.

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