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C H A P T E R T W E N T Y - S E V E N

THE THEORY OF ACTIVE PORTFOLIO MANAGEMENT

Thus far we have alluded to active portfolio management in only three instances: the Markowitz methodology of generating the optimal risky portfolio (Chapter 8); security analysis that generates forecasts to use as inputs with the Markowitz procedure (Chapters 17 through 19); and fixed-income portfolio management (Chapter 16). These brief analyses are not adequate to guide investment managers in a comprehensive enterprise of active portfolio management. You may also be wondering about the seeming contradiction between our equilibrium analysis in Part III—in particular, the theory of efficient markets—and the real-world environment where profit-seeking investment managers use active management to exploit perceived market inefficiencies.

Despite the efficient market hypothesis, it is clear that markets cannot be perfectly efficient; hence there are reasons to believe that active management can have effective results, and we discuss these at the outset. Next we consider the objectives of active portfolio management. We analyze two forms of active management: market timing, which is based solely on macroeconomic factors; and security selection, which includes microeconomic forecasting. We show the use of multifactor models in active portfolio management, and we end with a discussion of the use of imperfect forecasts and the implementation of security analysis in industry.

27.1 THE LURE OF ACTIVE MANAGEMENT

How can a theory of active portfolio management be reconciled with the notion that markets are in equilibrium? You may want to look back at the analysis in Chapter 12, but we can interpret our conclusions as follows.

Market efficiency prevails when many investors are willing to depart from maximum diversification, or a passive strategy, by adding mispriced securities to their portfolios in the hope of realizing abnormal returns. The competition for such returns ensures that prices will be near their "fair" values. Most managers will not beat the passive strategy on a risk-adjusted basis. However, in the competition for rewards to investing, exceptional managers might beat the average forecasts built into market prices.

There is both economic logic and some empirical evidence to indicate that exceptional portfolio managers can beat the average forecast. Let us discuss economic logic first. We must assume that if no analyst can beat the passive strategy, investors will be smart enough to divert their funds from strategies entailing expensive analysis to less expensive passive strategies. In that case funds under active management will dry up, and prices will no longer reflect sophisticated forecasts. The consequent profit opportunities will lure back active managers who once again will become successful.[1] Of course, the critical assumption is that investors allocate management funds wisely. Direct evidence on that has yet to be produced.

As for empirical evidence, consider the following: (1) Some portfolio managers have produced streaks of abnormal returns that are hard to label as lucky outcomes; (2) the "noise" in realized rates is enough to prevent us from rejecting outright the hypothesis that some money managers have beaten the passive strategy by a statistically small, yet economically significant, margin; and (3) some anomalies in realized returns have been sufficiently persistent to suggest that portfolio managers who identified them in a timely fashion could have beaten the passive strategy over prolonged periods.

These conclusions persuade us that there is a role for a theory of active portfolio management. Active management has an inevitable lure even if investors agree that security markets are nearly efficient.

Suppose that capital markets are perfectly efficient, that an easily accessible market-index portfolio is available, and that this portfolio is for all practical purposes the efficient risky portfolio. Clearly, in this case security selection would be a futile endeavor. You would be better off with a passive strategy of allocating funds to a money market fund (the safe asset) and the market-index portfolio. Under these simplifying assumptions the optimal investment strategy seems to require no effort or know-how.

Such a conclusion, however, is too hasty. Recall that the proper allocation of investment funds to the risk-free and risky portfolios requires some analysis because y, the fraction to be invested in the risky market portfolio, M, is given by

$$y = \frac{E(r_M) - r_f}{.01A\sigma_M^2} \qquad (27.1)$$

where $E(r_M) - r_f$ is the risk premium on M, σ_M^2 its variance, and A is the investor's coefficient of risk aversion. Any rational allocation therefore requires an estimate of σ_M and $E(r_M)$. Even a passive investor needs to do some forecasting, in other words.

Forecasting $E(r_M)$ and σ_M is further complicated by the existence of security classes that are affected by different environmental factors. Long-term bond returns, for example, are

[1] This point is worked out fully in Sanford J. Grossman and Joseph E. Stiglitz, "On the Impossibility of Informationally Efficient Markets," *American Econonic Review* 70 (June 1980).

driven largely by changes in the term structure of interest rates, whereas equity returns depend on changes in the broader economic environment, including macroeconomic factors beyond interest rates. Once our investor determines relevant forecasts for separate sorts of investments, she might as well use an optimization program to determine the proper mix for the portfolio. It is easy to see how the investor may be lured away from a purely passive strategy, and we have not even considered temptations such as international stock and bond portfolios or sector portfolios.

In fact, even the definition of a "purely passive strategy" is problematic, because simple strategies involving only the market-index portfolio and risk-free assets now seem to call for market analysis. For our purposes we define purely passive strategies as those that use only index funds *and* weight those funds by fixed proportions that do not vary in response to perceived market conditions. For example, a portfolio strategy that always places 60% in a stock market–index fund, 30% in a bond-index fund, and 10% in a money market fund is a purely passive strategy.

More important, the lure into active management may be extremely strong because the potential profit from active strategies is enormous. At the same time, competition among the multitude of active managers creates the force driving market prices to near efficiency levels. Although enormous profits may be increasingly difficult to earn, decent profits to the better analysts should be the rule rather than the exception. For prices to remain efficient to some degree, some analysts must be able to eke out a reasonable profit. Absence of profits would decimate the active investment management industry, eventually allowing prices to stray from informationally efficient levels. The theory of managing active portfolios is the concern of this chapter.

27.2 OBJECTIVES OF ACTIVE PORTFOLIOS

What does an investor expect from a professional portfolio manager, and how does this expectation affect the operation of the manager? If the client were risk neutral, that is, indifferent to risk, the answer would be straightforward. The investor would expect the portfolio manager to construct a portfolio with the highest possible expected rate of return. The portfolio manager follows this dictum and is judged by the realized average rate of return.

When the client is risk averse, the answer is more difficult. Without a normative theory of portfolio management, the manager would have to consult each client before making any portfolio decision in order to ascertain that reward (average return) is commensurate with risk. Massive and constant input would be needed from the client-investors, and the economic value of professional management would be questionable.

Fortunately, the theory of mean-variance efficient portfolio management allows us to separate the "product decision," which is how to construct a mean-variance efficient risky portfolio, and the "consumption decision," or the investor's allocation of funds between the efficient risky portfolio and the safe asset. We have seen that construction of the optimal risky portfolio is purely a technical problem, resulting in a single optimal risky portfolio appropriate for all investors. Investors will differ only in how they apportion investment to that risky portfolio and the safe asset.

Another feature of the mean-variance theory that affects portfolio management decisions is the criterion for choosing the optimal risky portfolio. In Chapter 8 we established that the optimal risky portfolio for any investor is the one that maximizes the reward-to-variability ratio, or the expected excess rate of return (over the risk-free rate) divided by the standard deviation. A manager who uses this Markowitz methodology to construct the optimal risky portfolio will satisfy all clients regardless of risk aversion. Clients, for their part, can evaluate managers using statistical methods to draw inferences from realized rates of return about prospective, or ex-ante, reward-to-variability ratios.

William Sharpe's assessment of mutual fund performance[2] is the seminal work in the area of portfolio performance evaluation (see Chapter 24). The reward-to-variability ratio has come to be known as **Sharpe's measure:**

$$S = \frac{E(r_P) - r_f}{\sigma_P}$$

It is now a common criterion for tracking performance of professionally managed portfolios.

Briefly, mean-variance portfolio theory implies that the objective of professional portfolio managers is to maximize the (ex-ante) Sharpe measure, which entails maximizing the slope of the CAL (capital allocation line). A "good" manager is one whose CAL is steeper than the CAL representing the passive strategy of holding a market-index portfolio. Clients can observe rates of return and compute the realized Sharpe measure (the ex-post CAL) to evaluate the relative performance of their manager.

Ideally, clients would like to invest their funds with the most able manager, one who consistently obtains the highest Sharpe measure and presumably has real forecasting ability. This is true for all clients regardless of their degree of risk aversion. At the same time, each client must decide what fraction of investment funds to allocate to this manager, placing the remainder in a safe fund. If the manager's Sharpe measure is constant over time (and can be estimated by clients), the investor can compute the optimal fraction to be invested with the manager from equation 27.1, based on the portfolio long-term average return and variance. The remainder will be invested in a money market fund.

The manager's ex-ante Sharpe measure from updated forecasts will be constantly varying. Clients would have liked to increase their allocation to the risky portfolio when the forecasts are optimistic, and vice versa. However, it would be impractical to constantly communicate updated forecasts to clients and for them to constantly revise their allocation between the risky portfolios and risk-free asset.

Allowing managers to shift funds between their optimal risky portfolio and a safe asset according to their forecasts alleviates the problem. Indeed, many stock funds allow the managers reasonable flexibility to do just that.

27.3 MARKET TIMING

Consider the results of the following two different investment strategies:

1. An investor who put $1,000 in 30-day commercial paper on January 1, 1927, and rolled over all proceeds into 30-day paper (or into 30-day T-bills after they were introduced) would have ended on December 31, 1978, fifty-two years later, with $3,600.
2. An investor who put $1,000 in the NYSE index on January 1, 1927, and reinvested all dividends in that portfolio would have ended on December 31, 1978, with $67,500.

Suppose we defined perfect **market timing** as the ability to tell (with certainty) at the beginning of each month whether the NYSE portfolio will outperform the 30-day paper portfolio. Accordingly, at the beginning of each month, the market timer shifts all funds

[2] William F. Sharpe, "Mutual Fund Performance," *Journal of Business, Supplement on Security Prices* 39 (January 1966).

into either cash equivalents (30-day paper) or equities (the NYSE portfolio), whichever is predicted to do better. Beginning with $1,000 on the same date, how would the perfect timer have ended up 52 years later?

This is how Nobel Laureate Robert Merton began a seminar with finance professors 20 years ago. As he collected responses, the boldest guess was a few million dollars. The correct answer: $5.36 *billion.*[3]

CONCEPT CHECK ☞ QUESTION 1

What was the monthly and annual compounded rate of return for the three strategies over the period 1926 to 1978?

These numbers highlight the power of compounding. This effect is particularly important because more and more of the funds under management represent retirement savings. The horizons of such investments may not be as long as 52 years but are measured in decades, making compounding a significant factor.

Another result that may seem surprising at first is the huge difference between the end-of-period value of the all-safe asset strategy ($3,600) and that of the all-equity strategy ($67,500). Why would anyone invest in safe assets given this historical record? If you have internalized the lessons of previous chapters, you know the reason: risk. The average rates of return and the standard deviations on the all-bills and all-equity strategies for this period are:

	Arithmetic Mean	Standard Deviation
Bills	2.55	2.10
Equities	10.70	22.14

The significantly higher standard deviation of the rate of return on the equity portfolio is commensurate with its significantly higher average return.

Can we also view the rate-of-return premium on the perfect-timing fund as a risk premium? The answer must be "no," because the perfect timer never does worse than either bills or the market. The extra return is not compensation for the possibility of poor returns but is attributable to superior analysis. It is the value of superior information that is reflected in the tremendous end-of-period value of the portfolio.

The monthly rate-of-return statistics for the all-equity portfolio and the timing portfolio are:

Per Month	All Equities (%)	Perfect Timer No Charge (%)	Perfect Timer Fair Charge (%)
Average rate of return	0.85	2.58	0.55
Average excess return over return on safe asset	0.64	2.37	0.34
Standard deviation	5.89	3.82	3.55
Highest return	38.55	38.55	30.14
Lowest return	–29.12	0.06	–7.06
Coefficient of skewness	0.42	4.28	2.84

Ignore for the moment the fourth column ("Perfect Timer—Fair Charge"). The results of rows one and two are self-explanatory. The third row, standard deviation, requires some

[3] This demonstration has been extended to recent data with similar results.

discussion. The standard deviation of the rate of return earned by the perfect market timer was 3.82%, far greater than the volatility of T-bill returns over the same period. Does this imply that (perfect) timing is a riskier strategy than investing in bills? No. For this analysis standard deviation is a misleading measure of risk.

To see why, consider how you might choose between two hypothetical strategies: The first offers a sure rate of return of 5%; the second strategy offers an uncertain return that is given by 5% *plus* a random number that is zero with probability .5 and 5% with probability .5. The characteristics of each strategy are

	Strategy 1 (%)	Strategy 2 (%)
Expected return	5	7.5
Standard deviation	0	2.5
Highest return	5	10.0
Lowest return	5	5.0

Clearly, Strategy 2 dominates Strategy 1 because its rate of return is *at least* equal to that of Strategy 1 and sometimes greater. No matter how risk averse you are, you will always prefer Strategy 2, despite its significant standard deviation. Compared to Strategy 1, Strategy 2 provides only "good surprises," so the standard deviation in this case cannot be a measure of risk.

These two strategies are analogous to the case of the perfect timer compared with an all-equity or all-bills strategy. In every period the perfect timer obtains at least as good a return, in some cases a better one. Therefore the timer's standard deviation is a misleading measure of risk compared to an all-equity or all-bills strategy.

Returning to the empirical results, you can see that the highest rate of return is identical for the all-equity and the timing strategies, whereas the lowest rate of return is positive for the perfect timer and disastrous for all the all-equity portfolio. Another reflection of this is seen in the coefficient of skewness, which measures the asymmetry of the distribution of returns. Because the equity portfolio is almost (but not exactly) normally distributed, its coefficient of skewness is very low at .42. In contrast, the perfect timing strategy effectively eliminates the negative tail of the distribution of portfolio returns (the part below the risk-free rate). Its returns are "skewed to the right," and its coefficient of skewness is therefore quite large, 4.28.

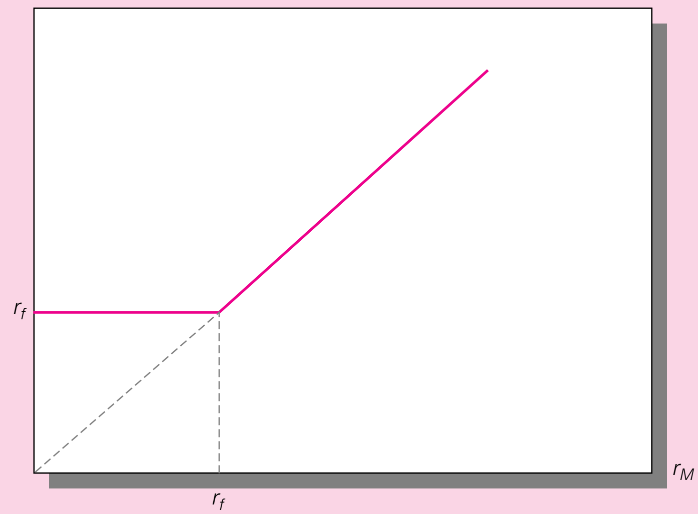
Now for the fourth column, "Perfect Timer—Fair Charge," which is perhaps the most interesting. Most assuredly, the perfect timer will charge clients for such a valuable service. (The perfect timer may have otherwordly predictive powers, but saintly benevolence is unlikely.)

Subtracting a fair fee (discussed later) from the monthly rate of return of the timer's portfolio gives us an average rate of return lower than that of the passive, all-equity strategy. However, because the fee is *constructed* to be fair, the two portfolios (the all-equity strategy and the market-timing-with-fee strategy) must be equally attractive after risk adjustment. In this case, again, the standard deviation of the market timing strategy (with fee) is of no help in adjusting for risk because the coefficient of skewness remains high, 2.84. In other words, mean-variance analysis is inadequate for valuing market timing. We need an alternative approach.

Valuing Market Timing as an Option

The key to analyzing the pattern of returns to the perfect market timer is to recognize that perfect foresight is equivalent to holding a call option on the equity portfolio. The perfect

Figure 27.1
Rate of return of a perfect market timer.



timer invests 100% in either the safe asset or the equity portfolio, whichever will yield the higher return. This is shown in Figure 27.1. The rate of return is bounded from below by r_f.

To see the value of information as an option, suppose that the market index currently is at S_0, and that a call option on the index has an exercise price of $X = S_0(1 + r_f)$. If the market outperforms bills over the coming period, S_T will exceed X, whereas it will be less than X otherwise. Now look at the payoff to a portfolio consisting of this option and S_0 dollars invested in bills:

	$S_T < X$	$S_T \geq X$
Bills	$S_0(1 + r_f)$	$S_0(1 + r_f)$
Option	0	$S_T - X$
Total	$S_0(1 + r_f)$	S_T

The portfolio pays the risk-free return when the market is bearish (i.e., the market return is less than the risk-free rate), and it pays the market return when the market is bullish and beats bills. Such a portfolio is a perfect market timer. Consequently, we can measure the value of perfect ability as the value of the call option, because a call enables the investor to earn the market return only when it exceeds r_f. This insight lets Merton[4] value timing ability using the theory of option of valuation, and calculate the fair charge for it.

The Value of Imperfect Forecasting

Unfortunately, managers are not perfect forecasters. It seems pretty obvious that if managers are right most of the time, they are doing very well. However, when we say "most of the time," we cannot mean merely the percentage of the time a manager is right. The

[4] Robert C. Merton, "On Market Timing and Investment Performance: An Equilibrium Theory of Value for Market Forecasts," *Journal of Business*, July 1981.

weather forecaster in Tucson, Arizona, who *always* predicts no rain, may be right 90% of the time. But a high success rate for a "stopped-clock" strategy clearly is not evidence of forecasting ability.

Similarly, the appropriate measure of market forecasting ability is not the overall proportion of correct forecasts. If the market is up two days out of three and a forecaster always predicts market advance, the two-thirds success rate is not a measure of forecasting ability. We need to examine the proportion of bull markets ($r_M > r_f$) correctly forecast *and* the proportion of bear markets ($r_M < r_f$) correctly forecast.

If we call P_1 the proportion of the correct forecasts of bull markets and P_2 the proportion for bear markets, then $P_1 + P_2 - 1$ is the correct measure of timing ability. For example, a forecaster who always guesses correctly will have $P_1 = P_2 = 1$, and will show ability of 1 (100%). An analyst who always bets on a bear market will mispredict all bull markets ($P_1 = 0$), will correctly "predict" all bear markets ($P_2 = 1$), and will end up with timing ability of $P_1 + P_2 - 1 = 0$. If C denotes the (call option) value of a perfect market timer, then $(P_1 + P_2 - 1)C$ measures the value of imperfect forecasting ability. In Chapter 24, "Portfolio Performance Evaluation," we saw how market timing ability can be detected and measured.

CONCEPT CHECK ☞ QUESTION 2

What is the market timing score of someone who flips a fair coin to predict the market?

27.4 SECURITY SELECTION: THE TREYNOR-BLACK MODEL

Overview of the Treynor-Black Model

Security analysis is the other form of active portfolio management besides timing the overall market. Suppose that you are an analyst studying individual securities. It is quite likely that you will turn up several securities that appear to be mispriced. They offer positive anticipated alphas to the investor. But how do you exploit your analysis? Concentrating a portfolio on these securities entails a cost, namely, the firm-specific risk that you could shed by more fully diversifying. As an active manager you must strike a balance between aggressive exploitation of perceived security mispricing and diversification motives that dictate that a few stocks should not dominate the portfolio.

Treynor and Black[5] developed an optimizing model for portfolio managers who use security analysis. It represents a portfolio management theory that assumes security markets are *nearly* efficient. The essence of the model is this:

1. Security analysts in an active investment management organization can analyze in depth only a limited number of stocks out of the entire universe of securities. The securities not analyzed are assumed to be fairly priced.
2. For the purpose of efficient diversification, the market index portfolio is the baseline portfolio, which the model treats as the passive portfolio.
3. The macro forecasting unit of the investment management firm provides forecasts of the expected rate of return and variance of the passive (market-index) portfolio.

[5] Jack Treynor and Fischer Black, "How to Use Security Analysis to Improve Portfolio Selection," *Journal of Business,* January 1973.

4. The objective of security analysis is to form an active portfolio of a necessarily limited number of securities. Perceived mispricing of the analyzed securities is what guides the composition of this active portfolio.
5. Analysts follow several steps to make up the active portfolio and evaluate its expected performance:
   a. Estimate the beta of each analyzed security and its residual risk. From the beta and macro forecast, $E(r_M) - r_f$, determine the *required* rate of return of the security.
   b. Given the degree of mispricing of each security, determine its expected return and expected *abnormal* return (alpha).
   c. The cost of less than full diversification comes from the nonsystematic risk of the mispriced stock, the variance of the stock's residual, $\sigma^2(e)$, which offsets the benefit (alpha) of specializing in an underpriced security.
   d. Use the estimates for the values of alpha, beta, and $\sigma^2(e)$ to determine the optimal weight of each security in the active portfolio.
   e. Compute the alpha, beta, and $\sigma^2(e)$ of the active portfolio from the weights of the securities in the portfolio.
6. The macroeconomic forecasts for the passive index portfolio and the composite forecasts for the active portfolio are used to determine the optimal risky portfolio, which will be a combination of the passive and active portfolios.

Treynor and Black's model did not take the industry by storm. This is unfortunate for several reasons:

1. Just as even imperfect market timing ability has enormous value, security analysis of the sort Treynor and Black proposed has similar potential value.[6] Even with far from perfect security analysis, proper active management can add value.
2. The Treynor-Black model is conceptually easy to implement. Moreover, it is useful even when some of its simplifying assumptions are relaxed.
3. The model lends itself to use in decentralized organizations. This property is essential to efficiency in complex organizations.

Portfolio Construction

Assuming that all securities are fairly priced, and using the index model as a guideline for the rate of return on fairly priced securities, the rate of return on the *i*th security is given by

$$r_i = r_f + \beta_i(r_M - r_f) + e_i \tag{27.2}$$

where e_i is the zero mean, firm-specific disturbance.

Absent security analysis, Treynor and Black (TB) took equation 27.2 to represent the rate of return on all securities and assumed that the market portfolio, *M,* is the efficient portfolio. For simplicity, they also assumed that the nonsystematic components of returns, e_i, are independent across securities. As for market timing, TB assumed that the forecast for the **passive portfolio** already has been made, so that the expected return on the market index, r_M, as well as its variance, σ_M^2, has been assessed.

[6] Alex Kane, Alan Marcus, and Robert Trippi, "The Valuation of Security Analysis," *Journal of Portfolio Management,* Spring 1999.

Now a portfolio manager unleashes a team of security analysts to investigate a subset of the universe of available securities. The objective is to form an active portfolio of positions in the analyzed securities to be mixed with the index portfolio. For each security, *k,* that is researched, we write the rate of return as

$$r_k = r_f + \beta_k (r_M - r_f) + e_k + \alpha_k \quad (27.3)$$

where α_k represents the extra expected return (called the *abnormal return*) attributable to any perceived mispricing of the security. Thus, for each security analyzed the research team estimates the parameters α_k, β_k, and $\sigma^2(e_k)$. If all the α_k turn out to be zero, there would be no reason to depart from the passive strategy and the index portfolio *M* would remain the manager's choice. However, this is a remote possibility. In general, there will be a significant number of nonzero alpha values, some positive and some negative.

One way to get an overview of the TB methodology is to examine what we should do with the active portfolio once we determine it. Suppose that the **active portfolio** (*A*) has been constructed somehow and has the parameters α_A, β_A, and $\sigma^2(e_A)$. Its total variance is the sum of its systematic variance, $\beta_A^2\sigma_M^2$, plus the nonsystematic variance, $\sigma^2(e_A)$. Its covariance with the market index portfolio, *M,* is

$$\text{Cov}(r_A, r_M) = \beta_A\sigma_M^2$$

Figure 27.2 shows the optimization process with the active and passive portfolios. The dashed efficient frontier represents the universe of all securities assuming that they are all fairly priced, that is, that all alphas are zero. By definition, the market index, *M,* is on this efficient frontier and is tangent to the (dashed) capital market line (CML). In practice, the analysts do not need to know this frontier. They need only to observe the market-index portfolio and construct a portfolio resulting in a capital allocation line that lies above the CML. Given their perceived superior analysis, they will view the market-index portfolio as inefficient: The active portfolio, *A,* constructed from mispriced securities, must lie, by design, above the CML.

To locate the active portfolio *A* in Figure 27.2, we need its expected return and standard deviation. The standard deviation is

$$\sigma_A = [\beta_A^2\sigma_M^2 + \sigma^2(e_A)]^{1/2}$$

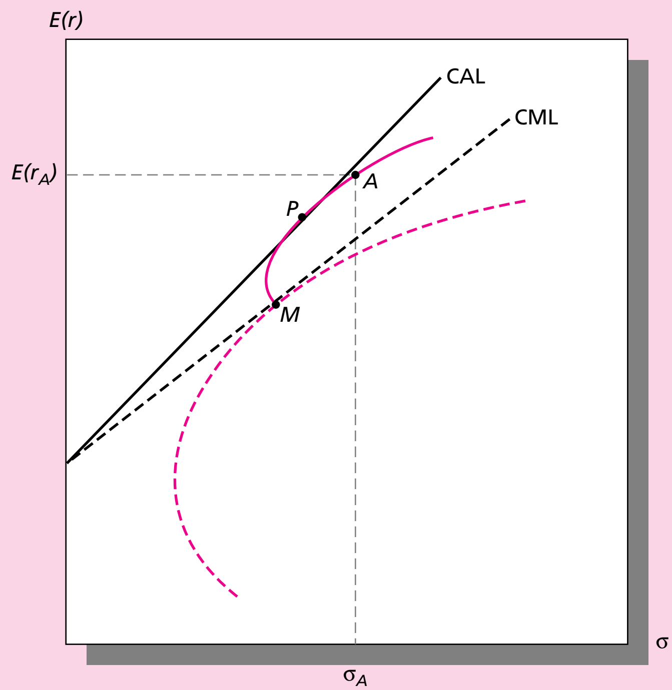
Because of the positive alpha value that is forecast for *A,* it may plot above the (dashed) CML with expected return

$$E(r_A) = \alpha_A + r_f + \beta_A[E(r_M) - r_f]$$

The optimal combination of the active portfolio, *A,* with the passive portfolio, *M,* is a simple application of the construction of optimal risky portfolios from two component assets that we first encountered in Chapter 8. Because the active portfolio is not perfectly correlated with the market-index portfolio, we need to account for their mutual correlation in the determination of the optimal allocation between the two portfolios. This is evident from the solid efficient frontier that passes through *M* and *A* in Figure 27.2. It supports the optimal capital allocation line (CAL) and identifies the optimal risky portfolio, *P,* which combines portfolios *A* and *M* and is the tangency point of the CAL to the efficient frontier. The active portfolio *A* in this example is not the ultimate efficient portfolio, because we need to mix *A* with the passive market portfolio to achieve optimal diversification.

Let us now outline the algebraic approach to this optimization problem. If we invest a proportion, *w,* in the active portfolio and 1 – *w* in the market index, the portfolio return will be

Figure 27.2
The optimization process with active and passive portfolios.



$$r_p(w) = wr_A + (1 - w)r_M$$

To find the weight, w, which provides the best (i.e., the steepest) CAL, we use equation 8.7 from Chapter 8, which describes the optimal risky portfolio composed of two risky assets (in this case, A and M) when there is a risk-free asset:

$$w_A = \frac{[E(r_A) - r_f]\sigma_M^2 - [E(r_M) - r_f]\text{Cov}(r_A, r_M)}{[E(r_A) - r_f]\sigma_M^2 + [E(r_M) - r_f]\sigma_A^2 - [E(r_A) - r_f + E(r_M) - r_f]\text{Cov}(r_A, r_M)} \qquad (8.7)$$

Now recall that

$$E(r_A) - r_f = \alpha_A + \beta_A R_M \qquad \text{where } R_M = E(r_M) - r_f$$
$$\text{Cov}(R_A, R_M) = \beta_A \sigma_M^2 \qquad \text{where } R_A = E(r_A) - r_f$$
$$\sigma_A^2 = \beta_A^2 \sigma_M^2 + \sigma^2(e_A)$$

$$[E(r_A) - r_f] + [E(r_M) - r_f] = (\alpha_A + \beta R_M) + R_M = \alpha_A + R_M(1 + \beta_A)$$

Substituting these expressions into equation 8.7, dividing both numerator and denominator by σ_M^2, and collecting terms yields the expression for the optimal weight in portfolio A, w^*,

$$w^* = \frac{\alpha_A}{\alpha_A(1 - \beta_A) + R_M \dfrac{\sigma^2(e_A)}{\sigma_M^2}} \qquad (27.4)$$

Let's begin with the simple case where $\beta_A = 1$ and substitute into equation 27.4. Then the optimal weight, w_0, is

$$w_0 = \frac{\dfrac{\alpha_A}{R_M}}{\dfrac{\sigma^2(e_A)}{\sigma_M^2}} = \frac{\alpha_A/\sigma^2(e_A)}{R_M/\sigma_M^2} \tag{27.5}$$

This is a very intuitive result. If the systematic risk of the active portfolio is average, that is, $\beta_A = 1$, then the optimal weight is the "relative advantage" of portfolio A as measured by the ratio: alpha/[market excess return], divided by the "disadvantage" of A, that is, the ratio: [nonsystematic risk of A]/[market risk]. Some algebra applied to equation 27.4 reveals the relationship between w_1 and w^*:

$$w^* = \frac{w_0}{1 + (1 - \beta_A)w_0} \tag{27.6}$$

w^* increases when β_A increases because the greater the systematic risk, β_A, of the active portfolio, A, the smaller is the benefit from diversifying it with the index, M, and the more beneficial it is to take advantage of the mispriced securities. However, we expect the beta of the active portfolio to be in the neighborhood of 1.0 and the optimal weight, w^*, to be close to w_0.

What is the reward-to-variability ratio of the optimal risky portfolio once we find the best mix, w^*, of the active and passive index portfolios? It turns out that if we compute the square of Sharpe's measure of the risky portfolio, we can separate the contributions of the index and active portfolios as follows:

$$S_P^2 = S_M^2 + \frac{\alpha_A^2}{\sigma^2(e_A)} = \left[\frac{R_M}{\sigma_M}\right]^2 + \left[\frac{\alpha_A}{\sigma(e_A)}\right]^2 \tag{27.7}$$

This decomposition of the Sharpe measure of the optimal risky portfolio, which by the way is valid *only* for the optimal portfolio, tells us how to construct the active portfolio. Equation 27.7 shows that the highest Sharpe measure for the risky portfolio will be attained when we construct an active portfolio that maximizes the value of $\alpha_A/\sigma(e_A)$. The ratio of alpha to residual standard deviation of the active portfolio will be maximized when we choose a weight for the kth analyzed security as follows:

$$w_k = \frac{\alpha_k/\sigma^2(e_k)}{\sum_{i=1}^{n} \alpha_i/\sigma^2(e_i)} \tag{27.8}$$

This makes sense: The weight of a security in the active portfolio depends on the ratio of the degree of mispricing, α_k, to the nonsystematic risk, $\sigma^2(e_k)$, of the security. The denominator, the sum of the ratio across securities, is a scale factor to guarantee that portfolio weights sum to one.

Note from equation 27.7 that the square of Sharpe's measure of the optimal risky portfolio is increased over the square of the Sharpe measure of the passive (market-index) portfolio by the amount

$$\left[\frac{\alpha_A}{\sigma(e_A)}\right]^2$$

The ratio of the degree of mispricing, α_A, to the nonsystematic standard deviation, $\sigma(e_A)$, is therefore a natural performance measure of the active component of the risky portfolio. Sometimes this is called the **appraisal ratio.**

We can calculate the contribution of a single security in the active portfolio to the portfolio's overall performance. When the active portfolio contains n analyzed securities, the total improvement in the squared Sharpe measure equals the sum of the squared appraisal ratios of the analyzed securities,

$$\left[\frac{\alpha_A}{\sigma(e_A)}\right]^2 = \sum_{i=1}^{n}\left[\frac{\alpha_i}{\sigma(e_i)}\right]^2 \tag{27.9}$$

The appraisal ratio for each security, $\alpha_i/\sigma(e_i)$, is a measure of the contribution of that security to the performance of the active portfolio.

The best way to illustrate the Treynor-Black process is through an example that can be easily worked out in a spreadsheet. Suppose that the macroforecasting unit of Drex Portfolio Inc. (DPF) issues a forecast for a 15% market return. The forecast's standard error is 20%. The risk-free rate is 7%. The macro data can be summarized as follows:

$$R_M = E(r_M) - r_f = 8\%; \qquad \sigma_M = 20\%$$

At the same time the security analysis division submits to the portfolio manager the following forecast of annual returns for the three securities that it covers:

Stock	α	β	$\sigma(e)$	$\alpha/\sigma(e)$
1	7%	1.6	45%	.1556
2	−5	1.0	32	−.1563
3	3	0.5	26	.1154

Note that the alpha estimates appear reasonably moderate. The estimates of the residual standard deviations are correlated with the betas, just as they are in reality. The magnitudes also reflect typical values for NYSE stocks. Equations 27.9, 27.7, and the analyst input table allow a quick calculation of the DPF portfolio's Sharpe measure.

$$S_P = [(8/20)^2 + .1556^2 + .1563^2 + .1154^2]^{1/2} = \sqrt{.2220} = .4711$$

Compare the result with the Sharpe ratio for the market-index portfolio, which is only 8/20 = .40. We now proceed to compute the composition and performance of the active portfolio.

First, let us construct the optimal active portfolio implied by the security analyst input list. To do so we compute the appraisal ratios as follows (remember to use decimal representations of returns in the formulas):

Stock	$\alpha/\sigma^2(e)$		$\frac{\alpha_k}{\sigma^2(e_k)} \Big/ \sum_{i=1}^{3}\frac{\alpha_i}{\sigma^2(e_i)}$	
1	$.07/.45^2 =$	.3457	$.3457/.3012 =$	1.1477
2	$-.05/.32^2 =$	−.4883	$-.4883/.3012 =$	−1.6212
3	$.03/.26^2 =$	.4438	$.4438/.3012 =$	1.4735
Total		.3012		1.0000

The last column presents the optimal positions of each of the three securities in the active portfolio. Obviously, Stock 2, with a negative alpha, has a negative weight. The magnitudes of the individual positions in the active portfolio (e.g., 114.77% in Stock 1) seem quite extreme. However, this should not concern us because the active portfolio will later be mixed with the well-diversified market-index portfolio, resulting in much more moderate positions, as we shall see shortly.

The forecasts for the stocks, together with the proposed composition of the active portfolio, lead to the following parameter estimates (in decimal form) for the active portfolio:

$$\alpha_A = 1.1477 \times .07 + (-1.6212) \times (-.05) + 1.4735 \times .03$$
$$= .2056 = 20.56\%$$
$$\beta_A = 1.1477 \times 1.6 + (-1.6212) \times 1.0 + 1.4735 \times .5 = .9519$$
$$\sigma(e_A) = [1.1477^2 \times .45^2 + (-1.6212)^2 \times .32^2 + 1.4735^2 \times .26^2]^{1/2}$$
$$= .8262 = 82.62\%$$
$$\sigma^2(e_A) = .8262^2 = .6826$$

Note that the negative weight (short position) on the negative alpha stock results in a positive contribution to the alpha of the active portfolio. Note also that because of the assumption that the stock residuals are uncorrelated, the active portfolio's residual variance is simply the weighted sum of the individual stock residual variances, with the squared portfolio proportions as weights.

The parameters of the active portfolio are now used to determine its proportion in the overall risky portfolio:

$$w_0 = \frac{\alpha_A/\sigma^2(e_A)}{R_M/\sigma_M^2} = \frac{.2056/.6826}{.08/.04} = .1506$$

$$w^* = \frac{w_1}{1 + (1 - \beta_A)w_1} = \frac{.1506}{1 + (1 - .9519) \times .1506} = .1495$$

Although the active portfolio's alpha is impressive (20.56%), its proportion in the overall risky portfolio, before adjustment for beta, is only 15.06%, because of its large nonsystematic standard deviation (82.62%). Such is the importance of diversification. As it happens, the beta of the active portfolio is almost 1.0, and hence the adjustment for beta (from w_0 to w^*) is small, from 15.06% to 14.95%. The direction of the change makes sense. If the beta of the active portfolio is low (less than 1.0), there is more potential gain from diversification, hence a smaller position in the active portfolio is called for. If the beta of the active portfolio were significantly greater than 1.0, a larger correction in the opposite direction would be called for.

The proportions of the individual stocks in the active portfolio, together with the proportion of the active portfolio in the overall risky portfolio, determine the proportions of each individual stock in the overall risky portfolio.

Stock	Final Position		
1	.1495 × 1.1477	=	.1716
2	.1495 × (–1.6212)	=	−.2424
3	.1495 × 1.4735	=	.2202
Active portfolio			.1495
Market portfolio			.8505
			1.0000

The parameters of the active portfolio and market-index portfolio are now used to forecast the performance of the optimal, overall risky portfolio. When optimized, a property of the risky portfolio is that its squared Sharpe measure exceeds that of the passive portfolio by the square of the active portfolio's appraisal ratio:

$$S_P^2 = \left[\frac{R_M}{\sigma_M}\right]^2 + \left[\frac{\alpha_A}{\sigma(e_A)}\right]^2$$
$$= .16 + .0619 = .2219$$

and hence the Sharpe measure of the DPF portfolio is $\sqrt{.2219} = .4711$, compared with .40 for the passive portfolio.

Another measure of the gain from increasing the Sharpe measure is the M^2 statistic, as described in Chapter 24. M^2 is calculated by comparing the expected return of a portfolio on the capital allocation line supported by portfolio *P,* CAL(*P*), with a standard deviation equal to that of the market index, to the expected return on the market index. In other words, we mix portfolio *P* with the risk-free asset to obtain a new portfolio P^* that has the same standard deviation as the market portfolio. Since both portfolios have equal risk, we can compare their expected returns. The M^2 statistic is the difference in expected returns. Portfolio P^* can be obtained by investing a fraction σ_M/σ_P in *P* and a fraction $(1 - \sigma_M/\sigma_P)$ in the risk-free asset.

The risk premium on CAL(P^*) with total risk σ_M is given by (see Chapter 24)

$$R_{P*} = E(r_{P*}) - r_f = S_P\sigma_M = .4711 \times .20 = .0942, \text{ or } 9.42\% \qquad (27.10)$$

and

$$M^2 = [R_{P*} - R_M] = 9.42 - 8 = 1.42\% \qquad (27.11)$$

At first blush, an incremental expected return of 1.42% seems paltry compared with the alpha values submitted by the analyst. This seemingly modest improvement is the result of diversification motives: To mitigate the large risk of individual stocks (verify that the standard deviation of stock 1 is 55%) and maximize the portfolio Sharpe measure (which compares excess return to total volatility), we must diversify the active portfolio by mixing it with *M.* Note also that this improvement has been achieved with only three stocks, and with forecasts and portfolio rebalancing only once a year. Increasing the number of stocks and the frequency of forecasts can improve the results dramatically.

For example, suppose the analyst covers three more stocks that turn out to have alphas and risk levels identical to the first three. Use equation 27.9 to show that the squared appraisal ratio of the active portfolio will double. By using equation 27.7, it is easy to show that the new Sharpe measure will rise to .5327. Equation 27.11 then implies that M^2 rises to 2.65%, almost double the previous value. Increasing the frequency of forecasts and portfolio rebalancing will deploy the power of compounding to improve annual performance even more.

CONCEPT CHECK ☞ QUESTION 3

a. When short positions are prohibited, the manager simply discards stocks with negative alphas. Using the preceding example, what would be the composition of the active portfolio if short sales were disallowed? Find the cost of the short-sale restriction in terms of the decline in performance (M^2) of the new overall risky portfolio.

b. What is the contribution of security selection to portfolio performance if the macro forecast is adjusted upward, for example, to $R_M = 12\%$, and short sales are again allowed?

27.5 MULTIFACTOR MODELS AND ACTIVE PORTFOLIO MANAGEMENT

Portfolio managers use various multifactor models of security returns. So far our analytical framework for active portfolio management seems to rest on the validity of the index model, that is, on a single-factor security model. Using a multifactor model will not affect the construction of the active portfolio because the entire TB analysis focuses on the residuals of the index model. If we were to replace the one-factor model with a multifactor model, we would continue to form the active portfolio by calculating each security's alpha relative to its fair return (given its betas on *all* factors), and again we would combine the active portfolio with the portfolio that would be formed in the absence of security analysis. The multifactor framework, however, does raise several new issues.

You saw in Chapter 10 how the index model simplifies the construction of the input list necessary for portfolio optimization programs. If

$$r_i - r_f = \alpha_i + \beta_i(r_M - r_f) + e_i$$

adequately describes the security market, then the variance of any asset is the sum of systematic and nonsystematic risk: $\sigma^2(r_i) = \beta_i^2\sigma_M^2 + \sigma^2(e_i)$, and the covariance between any two assets is $\beta_i\beta_j\sigma_M^2$.

How do we generalize this rule to use in a multifactor model? To simplify, let us consider a two-factor world, and let us call the two factor portfolios *M* and *H*. Then we generalize the index model to

$$\begin{aligned} r_i - r_f &= \beta_{iM}(r_M - r_f) + \beta_{iH}(r_H - r_f) + \alpha_i + e_i \\ &= R_\beta + \alpha_i + e_i \end{aligned} \tag{27.12}$$

β_{iM} and β_{iH} are the betas of the security relative to portfolios *M* and *H*. Given the rates of return on the factor portfolios, r_M and r_H, the fair excess rate of return over r_f on a security is denoted R_β and its expected abnormal return is α_i.

How can we use equation 27.12 to form optimal portfolios? As before, investors wish to maximize the Sharpe measures of their portfolios. The factor structure of equation 27.12 can be used to generate the inputs for the Markowitz portfolio selection algorithm. The variance and covariance estimates are now more complex, however:

$$\sigma^2(r_i) = \beta_{iM}^2\sigma_M^2 + \beta_{iH}^2\sigma_H^2 + 2\beta_{iM}\beta_{iH}\text{Cov}(r_M, r_H) + \sigma^2(e_i)$$
$$\text{Cov}(r_i, r_j) = \beta_{iM}\beta_{jM}\sigma_M^2 + \beta_{iH}\beta_{jH}\sigma_H^2 + (\beta_{iM}\beta_{jH} + \beta_{jM}\beta_{iH})\text{Cov}(r_M, r_H)$$

Nevertheless, the informational economy of the factor model still is valuable, because we can estimate a covariance matrix for an *n*-security portfolio from

n estimates of β_{iM}
n estimates of β_{iH}
n estimates of $\sigma^2(e_i)$
1 estimate of σ_M^2
1 estimate of σ_H^2

rather than $n(n + 1)/2$ separate variance and covariance estimates. Thus the factor structure continues to simplify portfolio construction data requirements.

The factor structure also suggests an efficient method to allocate research effort. Analysts can specialize in forecasting means and variances of different factor portfolios. Having established factor betas, they can form a covariance matrix to be used together with expected security returns generated by the CAPM or APT to construct an optimal passive risky portfolio. If active analysis of individual stocks also is attempted, the procedure of constructing the optimal active portfolio and its optimal combination with the passive portfolio is identical to that followed in the single-factor case.

In the case of the multifactor market even passive investors (meaning those who accept market prices as "fair") need to do a considerable amount of work. They need forecasts of the expected return and volatility of each factor return, *and* they need to determine the appropriate weights on each factor portfolio to maximize their expected utility. Such a process is straightforward in principle, but it quickly becomes computationally demanding.

27.6 IMPERFECT FORECASTS OF ALPHA VALUES AND THE USE OF THE TREYNOR-BLACK MODEL IN INDUSTRY

Suppose an analyst is assigned to a security and provides you with a forecast of $\alpha = 20\%$. It looks like a great opportunity! Using this forecast in the Treynor-Black algorithm, we'll end up tilting our portfolio heavily toward this security. Should we go out on a limb? Before doing so, any reasonable manager would ask: "How good is the analyst?" Unless the answer is a resounding "good," a reasonable manager would discount the forecast. We can quantify this notion.

Suppose we have a record of an analyst's past forecast of alpha, α^f. Relying on the index model and obtaining reliable estimates of the stock beta, we can estimate the true alphas (after the fact) from the average realized excess returns on the security, $\bar{R}$, and the index, $\bar{R}_M$, that is,

$$\alpha = \bar{R} - \beta\bar{R}_M$$

To measure the forecasting accuracy of the analyst, we can estimate a regression of the forecasts on the realized alpha:

$$\alpha^f = a_0 + a_1\alpha + \varepsilon$$

The coefficients a_0 and a_1 reflect potential bias in the forecasts, which we will ignore for simplicity; that is, we will suppose $a_0 = 0$ and $a_1 = 1$. Because the forecast errors are uncorrelated with the true alpha, the variance of the forecast is

$$\sigma^2_{\alpha^f} = \sigma^2_\alpha + \sigma^2_\varepsilon$$

The quality of the forecasts can be measured by the squared correlation coefficient between the forecasts and realization, equivalently, the ratio of explained variance to total variance

$$\rho^2 = \frac{\sigma^2_\alpha}{\sigma^2_\alpha + \sigma^2_\varepsilon}$$

This equation shows us how to "discount" analysts' forecasts to reflect their precision. Knowing the quality of past forecasts, ρ^2, we "shrink" any new forecast, α^f, to $\rho^2\alpha^f$, to minimize forecast error. This procedure is quite intuitive: If the analyst is perfect, that is, $\rho^2 = 1$, we take the forecast at face value. If analysts' forecasts have proven to be useless, with $\rho^2 = 0$, we ignore the forecast. The quality of the forecast gives us the precise shrinkage factor to use.

Suppose the analysts' forecasts of the alpha of the three stocks in our previous example are all of equal quality, $\rho^2 = .2$. Shrinking the forecasts of alpha by a factor of .2 and repeating the optimization process, we end up with a much smaller weight on the active portfolio (.03 instead of .15), a much smaller Sharpe measure (.4031 instead of .4711), and a much smaller M^2 (.06% instead of 1.42%).

The reduction in portfolio expected performance does not reflect an inferior procedure. Rather, accepting alpha forecasts without acknowledging and adjusting for their imprecision would be naïve. We must adjust our expectations to the quality of the forecasts.

In reality, we can expect the situation to be much worse. A forecast quality of .2, that is, a correlation coefficient between alpha forecasts and realizations of $\sqrt{.2} = .45$, is most likely unrealistic in nearly efficient markets. Moreover, we don't even know this quality, and its estimation introduces yet another potential error into the optimization process. Finally, the other parameters we use in the TB model—market expected return and variance, security betas and residual variances—are also estimated with errors. Thus, under realistic circumstances, we would be fortunate to obtain even the meager results we have just uncovered.

So, should we ditch the TB model? Before we do, let's make one more calculation. The "meager" Sharpe measure of .4031 squares to .1625, larger than the market's squared Sharpe measure of .16 by .0025. Suppose we cover 300 securities instead of three, that is, 100 sets identical to the one we analyzed. From equations 27.7 and 27.9 we know that the increment to the squared Sharpe measure will rise to $100 \times .0025 = .25$. The squared Sharpe measure of the risky portfolio will rise to $.16 + .25 = .41$, a Sharpe measure of .64, and an M^2 of 4.8%! Moreover, some of the estimation errors of the other parameters that plague us when we use three securities will offset one another and be diversified away with many more securities covered.[7]

What we see here is a demonstration of the value of security analysis we mentioned at the outset. In the final analysis, the value of the active management depends on forecast quality. The vast demand for active management suggests that this quality is not negligible. The optimal way to exploit analysts' forecasts is with the TB model. We therefore predict the technique will come to be more widely used in the future.

SUMMARY

1. A truly passive portfolio strategy entails holding the market-index portfolio and a money market fund. Determining the optimal allocation to the market portfolio requires an estimate of its expected return and variance, which in turn suggests delegating some analysis to professionals.
2. Active portfolio managers attempt to construct a risky portfolio that maximizes the reward-to-variability (Sharpe) ratio.
3. The value of perfect market timing ability is considerable. The rate of return to a perfect market timer will be uncertain. However, its risk characteristics are not measurable by standard measures of portfolio risk, because perfect timing dominates a passive strategy, providing "good" surprises only.
4. Perfect timing ability is equivalent to the possession of a call option on the market portfolio, whose value can be determined using option valuation techniques such as the Black-Scholes formula.
5. With imperfect timing, the value of a timer who attempts to forecast whether stocks will outperform bills is determined by the conditional probabilities of the true outcome given the forecasts: $P_1 + P_2 - 1$. Thus if the value of perfect timing is given by the option value, C, then imperfect timing has the value $(P_1 + P_2 - 1)C$.
6. The Treynor-Black security selection model envisions that a macroeconomic forecast for market performance is available and that security analysts estimate abnormal

[7] Empirical work along these lines can be found in: Alex Kane, Tae-Hwan Kim, and Halbert White, "The Power of Portfolio Optimization," UCSD Working Paper, July 2000.

expected rates of return, α, for various securities. Alpha is the expected rate of return on a security beyond that explained by its beta and the security market line.

7. In the Treynor-Black model the weight of each analyzed security is proportional to the ratio of its alpha to its nonsystematic risk, $\sigma^2(e)$.
8. Once the active portfolio is constructed, its alpha value, nonsystematic risk, and beta can be determined from the properties of the component securities. The optimal risky portfolio, *P,* is then constructed by holding a position in the active portfolio according to the ratio of α_A to $\sigma^2(e_A)$, divided by the analogous ratio for the market-index portfolio. Finally, this position is adjusted by the beta of the active portfolio.
9. When the overall risky portfolio is constructed using the optimal proportions of the active portfolio and passive portfolio, its performance, as measured by the square of Sharpe's measure, is improved (over that of the passive, market-index, portfolio) by the amount $[\alpha_A/\sigma(e_A)]^2$.
10. The contribution of each security to the overall improvement in the performance of the active portfolio is determined by its degree of mispricing and nonsystematic risk. The contribution of each security to portfolio performance equals $[\alpha_i/\sigma(e_i)]^2$, so that for the optimal risky portfolio,

$$S_P^2 = \left[\frac{E(r_M) - r_f}{\sigma_M}\right]^2 + \sum_{i=1}^{n}\left[\frac{\alpha_i}{\sigma(e_i)}\right]^2$$

11. Applying the Treynor-Black model to a multifactor framework is straightforward. The forecast of the market-index mean and standard deviation must be replaced with forecasts for an optimized passive portfolio based on a multifactor model. The proportions of the factor portfolios are calculated using the familiar efficient frontier algorithm. The active portfolio is constructed on the basis of residuals from the multifactor model.
12. Implementing the model with imperfect forecasts requires estimation of bias and precision of raw forecasts. The adjusted forecast is obtained by applying the estimated coefficients to the raw forecasts.

KEY TERMS

Sharpe's measure
market timing
passive portfolio
active portfolio
appraisal ratio

PROBLEMS

1. The five-year history of annual rates of return in excess of the T-bill rate for two competing stock funds is

The Bull Fund	The Unicorn Fund
−21.7%	−1.3%
28.7	15.5
17.0	14.4
2.9	−11.9
28.9	25.4

a. How would these funds compare in the eye of a risk-neutral potential client?
b. How would these funds compare by Sharpe's measure?

c. If a risk-averse investor (with a coefficient of risk aversion $A = 3$) had to choose one of these funds to mix with T-bills, which fund should he choose, and how much should be invested in that fund on the basis of available data?

2. Historical data suggest that the standard deviation of an all-equity strategy is about 5.5% per month. Suppose that the risk-free rate is now 1% per month and that market volatility is at its historical level. What would be a fair monthly fee to a perfect market timer, based on the Black-Scholes formula?

3. In scrutinizing the record of two market timers, a fund manager comes up with the following table:

Number of months that $r_M > r_f$	135	
Correctly predicted by timer *A*		78
Correctly predicted by timer *B*		86
Number of months that $r_M < r_f$	92	
Correctly predicted by timer *A*		57
Correctly predicted by timer *B*		50

a. What are the conditional probabilities, P_1 and P_2, and the total ability parameters for timers *A* and *B*?

b. Using the data of problem 2, what is a fair monthly fee for the two timers?

4. A portfolio manager summarizes the input from the macro and micro forecasters in the following table:

Micro Forecasts

Asset	Expected Return (%)	Beta	Residual Standard Deviation (%)
Stock A	20	1.3	58
Stock B	18	1.8	71
Stock C	17	0.7	60
Stock D	12	1.0	55

Macro Forecasts

Asset	Expected Return (%)	Standard Deviation (%)
T-bills	8	0
Passive equity portfolio	16	23

a. Calculate expected excess returns, alpha values, and residual variances for these stocks.

b. Construct the optimal risky portfolio.

c. What is Sharpe's measure for the optimal portfolio and how much of it is contributed by the active portfolio? What is the M^2?

d. What should be the exact makeup of the complete portfolio for an investor with a coefficient of risk aversion of 2.8?

5. Recalculate problem 4 for a portfolio manager who is not allowed to short-sell securities.

a. What is the cost of the restriction in terms of Sharpe's measure and M^2?

b. What is the utility loss to the investor ($A = 2.8$) given his new complete portfolio?

6. A portfolio management house approximates the return-generating process by a two-factor model and uses two factor portfolios to construct its passive portfolio. The input table that is constructed by the house analysts looks as follows:

Micro Forecasts

Asset	Expected Return (%)	Beta on *M*	Beta on *H*	Residual Standard Deviation (%)
Stock A	20	1.2	1.8	58
Stock B	18	1.4	1.1	71
Stock C	17	0.5	1.5	60
Stock D	12	1.0	0.2	55

Macro Forecasts

Asset	Expected Return (%)	Standard Deviation (%)
T-bills	8	0
Factor *M* portfolio	16	23
Factor *H* portfolio	10	18

The correlation coefficient between the two factor portfolios is .6.

a. What is the optimal passive portfolio?

b. By how much is the optimal passive portfolio superior to the single-factor passive portfolio, *M,* in terms of Sharpe's measure?

c. Analyze the utility improvement to the $A = 2.8$ investor relative to holding portfolio *M* as the sole risky asset that arises from the expanded macro model of the portfolio manager.

7. Construct the optimal active and overall risky portfolio with the data of problem 6 with no restrictions on short sales.

a. What is the Sharpe measure of the optimal risky portfolio and what is the contribution of the active portfolio?

b. Analyze the utility value of the optimal risky portfolio for the $A = 2.8$ investor. Compare to that of problem 6.

8. Recalculate problem 7 with a short-sale restriction. Compare the results to those from problem 7.

9. Suppose that based on the analyst's past record, you estimate that the relationship between forecast and actual alpha is:

$$\text{Actual abnormal return} = .3 \times \text{Forecast of alpha}$$

Use the alphas from problem 4. How much is expected performance affected by recognizing the imprecision of alpha forecasts?

SOLUTIONS TO CONCEPT CHECKS

1. We show the answer for the annual compounded rate of return for each strategy and leave the monthly rate for you to compute.

Beginning-of-period fund:

$$F_0 = \$1{,}000$$

SOLUTIONS TO CONCEPT CHECKS

End-of-period fund for each strategy:

$$F_1 = \begin{cases} \$3{,}600 & \text{Strategy = Bill only} \\ \$67{,}500 & \text{Strategy = Market only} \\ \$5{,}360{,}000{,}000 & \text{Strategy = Perfect timing} \end{cases}$$

Number of periods: $N = 52$ years

Annual compounded rate:

$$[1 + r_A]^N = \frac{F_1}{F_0}$$

$$r_A = \left(\frac{F_1}{F_0}\right)^{1/N} - 1$$

$$r_A = \begin{cases} 2.49\% & \text{Strategy = Bills only} \\ 8.44\% & \text{Strategy = Market only} \\ 34.71\% & \text{Strategy = Perfect timing} \end{cases}$$

2. The timer will guess bear or bull markets completely randomly. One-half of all bull markets will be preceded by a correct forecast, and similarly for bear markets. Hence $P_1 + P_2 - 1 = ½ + ½ - 1 = 0$.

3. *a.* When short positions are prohibited, the analysis is identical except that negative-alpha stocks are dropped. In that case the sum of the ratios of alpha to residual variance for the remaining two stocks is .7895. This leads to the new composition of the active portfolio. If x denotes new portfolio weights, then

$$x_1 = .3457/.7895 = .4379$$
$$x_2 = .4438/.7895 = .5621$$

The alpha, beta, and residual standard deviation of the active portfolio are now

$$\alpha_A = .4379 \times .07 + .5621 \times .03 = .0475$$
$$\beta_A = .4379 \times 1.6 + .5621 \times .5 = .9817$$
$$\sigma(e_A) = [.4379^2 \times .45^2 + .5621^2 \times .26^2]^{1/2} = .2453$$

The cost of the short sale restriction is already apparent. The alpha has shrunk from 20.56% to 4.75%, while the reduction in the residual standard deviation is more moderate, from 82.62% to 24.53%. In fact, a negative-alpha stock is potentially more attractive than a positive-alpha one: Since most stocks are positively correlated, the negative position that is required for the negative-alpha stock creates a better diversified active portfolio.

The optimal allocation of the new active portfolio is

$$w_0 = \frac{.0475/.6019}{.08/.04} = .3946$$

$$w^* = \frac{.3946}{1 + (1 - .9817) \times .3946} = .3918$$

Here, too, the beta correction is essentially irrelevant because the portfolio beta is so close to 1.0.

SOLUTIONS TO CONCEPT CHECKS

Finally, the performance of the overall risky portfolio is estimated at

$$S_P^2 = .16 + \left[\frac{.0475}{.2453}\right]^2 = .1975; \qquad S_P = .44$$

It is clear that in this case we have lost about half of the original improvement in the Sharpe measure. Note, however, that this is an artifact of the limited coverage of the security analysis division. When more stocks are covered, then a good number of positive-alpha stocks will keep the residual risk of the active portfolio low. This is the key to extracting large gains from the active strategy.

We calculate the "Modigliani-square," or M^2 measure, as follows:

$$E(r_{P*}) = r_f + S_P\,\sigma_M = .07 + .44 \times .20 = .158 \text{ or } 15.8\%$$

$$M^2 = E(r_{P*}) - E(r_M) = 15.8\% - 15\% = 0.8\%$$

which is a bit less than half the M^2 value of the unconstrained portfolio.

b. When the forecast for the market-index portfolio is more optimistic, the position in the active portfolio will be smaller and the contribution of the active portfolio to the Sharpe measure of the risky portfolio also will be smaller. In the original example the allocation to the active portfolio would be

$$w_0 = \frac{.2056/.6826}{.12/.04} = .1004$$

$$w^* = \frac{.1004}{1 + (1 - .9519) \times .1004} = .0999$$

Although the Sharpe measure of the market is now better, the improvement derived from security analysis is smaller:

$$S_P^2 = \left(\frac{.12}{.20}\right)^2 + \left(\frac{.2056}{.8262}\right)^2 = .4219$$

$$S_P = .65; \qquad S_M = .60$$

APPENDIX A

QUANTITATIVE REVIEW

Students in management and investment courses typically come from a variety of backgrounds. Some, who have had strong quantitative training, may feel perfectly comfortable with formal mathematical presentation of material. Others, who have had less technical training, may easily be overwhelmed by mathematical formalism. Most students, however, will benefit from some coaching to make the study of investment easier and more efficient. If you had a good introductory quantitative methods course, and like the text that was used, you may want to refer to it whenever you feel in need of a refresher. If you feel uncomfortable with standard quantitative texts, this reference is for you. Our aim is to present the essential quantitative concepts and methods in a self-contained, nontechnical, and intuitive way. Our approach is structured in line with requirements for the CFA program. The material included is relevant to investment management by the ICFA, the Institute of Chartered Financial Analysts. We hope you find this appendix helpful. Use it to make your venture into investments more enjoyable.

A.1 PROBABILITY DISTRIBUTIONS

Statisticians talk about "experiments," or "trials," and refer to possible outcomes as "events." In a roll of a die, for example, the "elementary events" are the numbers 1 through 6. Turning up one side represents the most disaggregate *mutually exclusive* outcome. Other events are *compound,* that is, they consist of more than one elementary event, such as the result "odd number" or "less than 4." In this case "odd" and "less than 4" are not mutually exclusive. Compound events can be mutually exclusive outcomes, however, such as "less than 4" and "equal to or greater than 4."

In decision making, "experiments" are circumstances in which you contemplate a decision that will affect the set of possible events (outcomes) and their likelihood (probabilities). Decision theory calls for you to identify optimal decisions under various sets of circumstances (experiments), which you may do by determining losses from departures from optimal decisions.

When the outcomes of a decision (experiment) can be quantified, that is, when a numerical value can be assigned to each elementary event, the decision outcome is called a *random variable.* In the context of investment decision making, the random variable (the payoff to the investment decision) is denominated either in dollars or as a percentage rate of return.

The set or list of all possible values of a random variable, *with* their associated probabilities, is called the *probability distribution* of the random variable. Values that are impossible for the random variable to take on are sometimes listed with probabilities of zero. All possible elementary events are assigned values and probabilities, and thus the probabilities have to sum to 1.0.

Sometimes the values of a random variable are *uncountable,* meaning that you cannot make a list of all possible values. For example, suppose you roll a ball on a line and report the distance it rolls before it comes to rest. Any distance is possible, and the precision of the report will depend on the need of the roller and/or the quality of the measuring device. Another uncountable random variable is one that describes the weight of a newborn baby. Any positive weight (with some upper bound) is possible.

We call uncountable probability distributions *continuous,* for the obvious reason that, at least within a range, the possible outcomes (those with positive probabilities) lie anywhere on a continuum of values. Because there is an infinite number of possible values for the random variable in any continuous distribution, such a probability distribution has to be described by a formula that relates the values of the random variable and their associated probabilities, instead of by a simple list of outcomes and probabilities. We discuss continuous distributions later in this section.

Even countable probability distributions can be complicated. For example, on the New York Stock Exchange stock prices are quoted in eighths. This means the price of a stock at some future date is a *countable* random variable. Probability distributions of countable random variables are called *discrete distributions.* Although a stock price cannot dip below zero, it has no upper bound. Therefore a stock price is a random variable that can take on infinitely many values, even though they are countable, and its discrete probability distribution will have to be given by a formula just like a continuous distribution.

There are random variables that are both discrete and finite. When the probability distribution of the relevant random variable is countable and finite, decision making is tractable and relatively easy to analyze. One example is the decision to call a coin toss "heads" or "tails," with a payoff of zero for guessing wrong and 1 for guessing right. The random variable of the decision to guess "heads" has a discrete, finite probability distribution. It can be written as

Event	Value	Probability
Heads	1	.5
Tails	0	.5

This type of analysis usually is referred to as *scenario analysis*. Because scenario analysis is relatively simple, it is used sometimes even when the actual random variable is infinite and uncountable. You can do this by specifying values and probabilities for a set of compound, yet exhaustive and mutually exclusive, events. Because it is simple and has important uses, we handle this case first.

Here is a problem from the 1988 CFA examination.

> Mr. Arnold, an Investment Committee member, has confidence in the forecasting ability of the analysts in the firm's research department. However, he is concerned that analysts may not appreciate risk as an important investment consideration. This is especially true in an increasingly volatile investment environment. In addition, he is conservative and risk averse. He asks for your risk analysis for Anheuser-Busch stock.
>
> **1.** Using Table A.1, calculate the following measures of dispersion of returns for Anheuser-Busch stock under each of the three outcomes displayed. Show calculations.
>
> *a*. Range.
>
> *b*. Variance: $\Sigma \Pr(i)[r_i - E(r)]^2$.
>
> *c*. Standard deviation.
>
> *d*. Coefficient of variation: $CV = \sigma/E(r)$.
>
> **2.** Discuss the usefulness of each of the four measures listed in quantifying risk.

The examination questions require very specific answers. We use the questions as framework for exposition of scenario analysis.

Table A.1 specifies a three-scenario decision problem. The random variable is the rate of return on investing in Anheuser-Busch stock. However, the third column, which specifies the value of the random variable, does not say simply "Return"—it says "Expected Return." This tells us that the scenario description is a compound event consisting of many elementary events, as is almost always the case. We streamline or simplify reality in order to gain tractability.

Analysts who prepare input lists must decide on the number of scenarios with which to describe the entire probability distribution, as well as the rates of return to allocate to each one. This process calls for determining the probability of occurrence of each scenario, *and* the expected rate of return *within* (conditional on) each scenario, which governs the outcome of each scenario. Once you become familiar with scenario analysis, you will be able to build a simple scenario description from any probability distribution.

Table A.1
Anheuser-Busch Companies, Inc., Dispersion of Potential Returns

Outcome	Probability	Expected Return*
Number 1	.20	20%
Number 2	.50	30
Number 3	.30	50

*Assume for the moment that the expected return in each scenario will be realized with certainty. This is the way returns were expressed in the original question.

Expected Returns

The expected value of a random variable is the answer to the question, "What would be the average value of the variable if the 'experiment' (the circumstances and the decision) were repeated infinitely?" In the case of an investment decision, your answer is meant to describe the reward from making the decision.

Note that the question is hypothetical and abstract. It is hypothetical because, practically, the exact circumstances of a decision (the "experiment") often cannot be repeated even once, much less infinitely. It is abstract because, even if the experiment were to be repeated many times (short of infinitely), the *average* rate of return may not be one of the possible outcomes. To demonstrate, suppose that the probability distribution of the rate of return on a proposed investment project is +20% or −20%, with equal probabilities of .5. Intuition indicates that repeating this investment decision will get us ever closer to an average rate of return of zero. But a one-time investment cannot produce a rate of return of zero. Is the "expected" return still a useful concept when the proposed investment represents a one-time decision?

One argument for using expected return to measure the reward from making investment decisions is that, although a specific investment decision may be made only once, the decision maker will be making many (although different) investment decisions over time. Over time, then, the average rate of return will come close to the average of the expected values of all the individual decisions. Another reason for using the expected value is that admittedly we lack a better measure.[1]

The probabilities of the scenarios in Table A.1 predict the relative frequencies of the outcomes. If the current investment in Anheuser-Busch could be replicated many times, a 20% return would occur 20% of the time, a 30% return would occur 50% of the time, and 50% return would occur the remaining 30% of the time. This notion of probabilities and the definition of the expected return tells us how to calculate the expected return.[2]

$$E(r) = .20 \times .20 + .50 \times .30 + .30 \times .50 = .34 \text{ (or 34\%)}$$

Labeling each scenario $i = 1, 2, 3$, and using the summation sign, Σ, we can write the formula for the expected return:

$$\begin{aligned} E(r) &= \Pr(1)r_1 + \Pr(2)r_2 + \Pr(3)r_3 \\ &= \sum_{i=1}^{3} \Pr(i)r_i \end{aligned} \qquad \text{(A.1)}$$

The definition of the expectation in equation A.1 reveals two important properties of random variables. First, if you add a constant to a random variable, its expectation is also increased by the same constant. If, for example, the return in each scenario in Table A.1 were increased by 5%, the expectation would increase to 39%. Try this, using equation A.1. If a random variable is multiplied by a constant, its expectation will change by that same proportion. If you multiply the return in each scenario by 1.5, $E(r)$ would change to $1.5 \times .34 = .51$ (or 51%).

Second, the deviation of a random variable from its expected value is itself a random variable. Take any rate of return r_1 in Table A.1 and define its deviation from the expected value by

$$d_i = r_i - E(r)$$

[1] Another case where we use a less-than-ideal measure is the case of yield to maturity on a bond. The YTM measures the rate of return from investing in a bond *if* it is held to maturity and *if* the coupons can be reinvested at the same yield to maturity over the life of the bond.

[2] We will consistently perform calculations in decimal fractions to avoid confusion.

What is the expected value of d? $E(d)$ is the expected deviation from the expected value, and by equation A.1 it is necessarily zero because

$$\begin{aligned} E(d) &= \Sigma \Pr(i)d_i = \Sigma \Pr(i)[r_i - E(r)] \\ &= \Sigma \Pr(i)r_i - E(r)\Sigma \Pr(i) \\ &= E(r) - E(r) = 0 \end{aligned}$$

Measures of Dispersion

The Range Assume for a moment that the expected return for each scenario in Table A.1 will be realized with certainty in the event that scenario occurs. Then the set of possible return outcomes is unambiguously 20%, 30%, and 50%. The *range* is the difference between the maximum and the minimum values of the random variable, 50% − 20% = 30% in this case. Range is clearly a crude measure of dispersion. Here it is particularly inappropriate because the scenario returns themselves are given as expected values, and therefore the true range is unknown. There is a variant of the range, the *interquartile range,* that we explain in the discussion of descriptive statistics.

The Variance One interpretation of variance is that it measures the "expected surprises." Although that may sound like a contradiction in terms, it really is not. First, think of a surprise as a deviation from expectation. The surprise is not in the *fact* that expectation has not been realized, but rather in the *direction* and *magnitude* of the deviation.

The example in Table A.1 leads us to *expect* a rate of return of 34% from investing in Anheuser-Busch stock. A second look at the scenario returns, however, tells us that we should stand ready to be surprised because the probability of earning exactly 34% is zero. Being sure that our expectation will not be realized does not mean that we can be sure what the realization is going to be. The element of surprise lies in the direction and magnitude of the deviation of the actual return from expectation, and that is the relevant random variable for the measurement of uncertainty. Its probability distribution adds to our understanding of the nature of the uncertainty that we are facing.

We measure the reward by the expected return. Intuition suggests that we measure uncertainty by the expected *deviation* of the rate of return from expectation. We showed in the previous section, however, that the expected deviation from expectation must be zero. Positive deviations, when weighted by probabilities, are exactly offset by negative deviations. To get around this problem, we replace the random variable "deviation from expectations" (denoted earlier by d) with its square, which must be positive even if d itself is negative.

We define the *variance,* our measure of surprise or dispersion, by the *expected squared deviation of the rate of return from its expectation.* With the Greek letter sigma square denoting variance, the formal definition is

$$\sigma^2(r) = E(d^2) = E[r_i - E(r)]^2 = \Sigma \Pr(i)[r_i - E(r)]^2 \quad \text{(A.2)}$$

Squaring each deviation eliminates the sign, which eliminates the offsetting effects of positive and negative deviations.

In the case of Anheuser-Busch, the variance of the rate of return on the stock is

$$\sigma^2(r) = .2(.20 - .34)^2 + .5(.30 - .34)^2 + .3(.50 - .34)^2 = .0124$$

Remember that if you add a constant to a random variable, the variance does not change at all. This is because the expectation also changes by the same constant, and hence deviations from expectation remain unchanged. You can test this by using the data from Table A.1.

Multiplying the random variable by a constant, however, *will* change the variance. Suppose that each return is multiplied by the factor k. The new random variable, kr, has expectation of $E(kr) = kE(r)$. Therefore, the deviation of kr from its expectation is

$$d(kr) = kr - E(kr) = kr - kE(r) = k[r - E(r)] = kd(r)$$

If each deviation is multiplied by k, the squared deviations are multiplied by the square of k:

$$\sigma^2(kr) = k^2\sigma^2(r)$$

To summarize, adding a constant to a random variable does not affect the variance. Multiplying a random variable by a constant, though, will cause the variance to be multiplied by the square of that constant.

The Standard Deviation A closer look at the variance will reveal that its dimension is different from that of the expected return. Recall that we squared deviations from the expected return in order to make all values positive. This alters the *dimension* (units of measure) of the variance to "square percents." To transform the variance into terms of percentage return, we simply take the square root of the variance. This measure is the *standard deviation.* In the case of Anheuser-Busch's stock return, the standard deviation is

$$\sigma = (\sigma^2)^{1/2} = \sqrt{.0124} = .1114 \text{ (or 11.14\%)} \quad \text{(A.3)}$$

Note that you always need to calculate the variance first before you can get the standard deviation. The standard deviation conveys the same information as the variance but in a different form.

We know already that adding a constant to r will not affect its variance, and it will not affect the standard deviation either. We also know that multiplying a random variable by a constant multiplies the variance by the square of that constant. From the definition of the standard deviation in equation A.3, it should be clear that multiplying a random variable by a constant will multiply the standard deviation by the (absolute value of this) constant. The absolute value is needed because the sign of the constant is lost through squaring the deviations in the computation of the variance. Formally,

$$\sigma(kr) = \text{Abs}(k)\,\sigma(r)$$

Try a transformation of your choice using the data in Table A.1.

The Coefficient of Variation To evaluate the magnitude of dispersion of a random variable, it is useful to compare it to the expected value. The ratio of the standard deviation to the expectation is called the *coefficient of variation.* In the case of returns on Anheuser-Busch stock, it is

$$CV = \frac{\sigma}{E(r)} = \frac{.1114}{.3400} = .3285 \quad \text{(A.4)}$$

The standard deviation of the Anheuser-Busch return is about one-third of the expected return (reward). Whether this value for the coefficient of variation represents a big risk depends on what can be obtained with alternative investments.

The coefficient of variation is far from an ideal measure of dispersion. Suppose that a plausible expected value for a random variable is zero. In this case, regardless of the magnitude of the standard deviation, the coefficient of variation will be infinite. Clearly, this measure is not applicable in all cases. Generally, the analyst must choose a measure of dispersion that fits the particular decision at hand. In finance, the standard deviation is the measure of choice in most cases where overall risk is concerned. (For individual assets, the measure β, explained in the text, is the measure used.)

Skewness

So far, we have described the measures of dispersion as indicating the size of the average surprise, loosely speaking. The standard deviation is not exactly equal to the average surprise though, because squaring deviations and then taking the square root of the average square deviation results in greater weight (emphasis) placed on larger deviations. Other than that, it is simply a measure that tells us how big a deviation from expectation can be expected.

Most decision makers agree that the expected value and standard deviation of a random variable are the most important statistics. However, once we calculate them another question about risk (the nature of the random variable describing deviations from expectations) is pertinent: Are the larger deviations (surprises) more likely to be positive? Risk-averse decision makers worry about bad surprises, and the standard deviation does not distinguish good from bad ones. Most risk avoiders are believed to prefer random variables with likely *small negative surprises* and *less* likely *large positive surprises,* to the reverse, likely *small good surprises* and *less* likely *large bad surprises.* More than anything, risk is really defined by the possibility of disaster (large bad surprises).

One measure that distinguishes between the likelihood of large good-vs.-bad surprises is the "third moment." It builds on the behavior of deviations from the expectation, the random variable we have denoted by d. Denoting the *third moment* by M_3, we define it:

$$M_3 = E(d^3) = E[r_i - E(r)]^3 = \Sigma \Pr(i)[r_i - E(r)]^3 \tag{A.5}$$

Cubing each value of d (taking it to the third power) magnifies larger deviations more than smaller ones. Raising values to an odd power causes them to retain their sign. Recall that the sum of all deviations multiplied by their probabilities is zero because positive deviations weighted by their probabilities exactly offset the negative. When *cubed* deviations are multiplied by their probabilities and then added up, however, large deviations will dominate. The sign will tell us in this case whether *large positive* deviations dominate (positive M_3) or whether *large negative* deviations dominate (negative M_3).

Incidentally, it is obvious why this measure of skewness is call the third moment; it refers to cubing. Similarly, the variance is often referred to as the second moment because it requires squaring.

Returning to the investment decision described in Table A.1, with the expected value of 34%, the third moment is

$$M_3 = .2(.20 - .34)^3 + .5(.30 - .34)^3 + .3(.50 - .34)^3 = .000648$$

The sign of the third moment tells us that larger *positive* surprises dominate in this case. You might have guessed this by looking at the deviations from expectation and their probabilities; that is, the most likely event is a return of 30%, which makes for a small negative surprise. The other negative surprise (20% − 34% = −14%) is smaller in magnitude than the positive surprise (50% − 34% = 16%) *and* is also *less* likely (probability .20) relative to the positive surprise, 30% (probability .30). The difference appears small, however, and we do not know whether the third moment may be an important issue for the decision to invest in Anheuser-Busch.

It is difficult to judge the importance of the third moment, here .000648, without a benchmark. Following the same reasoning we applied to the standard deviation, we can take the *third root* of M_3 (which we denote m_3) and compare it to the standard deviation. This yields $m_3 = .0865 = 8.65\%$, which is not trivial compared with the standard deviation (11.14%).

Another Example: Options on Anheuser-Busch Stock

Suppose that the current price of Anheuser-Busch stock is \$30. A call option on the stock is selling for 60 cents, and a put is selling for \$4. Both have an exercise price of \$42 and maturity date to match the scenarios in Table A.1.

The call option allows you to buy the stock at the exercise price. You will choose to do so if the call ends up "in the money," that is, the stock price is above the exercise price. The profit in this case is the difference between the stock price and the exercise price, less cost of the call. Even if you exercise the call, your profit may still be negative if the cash flow from the exercise of the call does not cover the initial cost of the call. If the call ends up "out of the money," that is, the stock price is below the exercise price, you will let the call expire worthless and suffer a loss equal to the cost of the call.

The put option allows you to sell the stock at the exercise price. You will choose to do so if the put ends up "in the money," that is, the stock price is below the exercise price. Your profit is then the difference between the exercise price and the stock price, less the initial cost of the put. Here again, if the cash flow is not sufficient to cover the cost of the put, the investment will show a loss. If the put ends up "out of the money," you again let it expire worthless, taking a loss equal to the initial cost of the put.

The scenario analysis of these alternative investments is described in Table A.2.

The expected rates of return on the call and put are

$$E(r_{\text{call}}) = .2(-1) + .5(-1) + .3(4) = .5 \text{ (or 50\%)}$$
$$E(r_{\text{put}}) = .2(.5) + .5(-.25) + .3(-1) = -.325 \text{ (or } -32.5\%)$$

The negative expected return on the put may be justified by the fact that it is a hedge asset, in this case an insurance policy against losses from holding Anheuser-Busch stock. The variance and standard deviation of the two investments are

$$\sigma^2_{\text{call}} = .2(-1 - .5)^2 + .5(-1 - .5)^2 + .3(4 - .5)^2 = 5.25$$
$$\sigma^2_{\text{put}} = .2[.5 - (-.325)]^2 + .5[-.25 - (-.325)]^2 + .3[-1 - (-.325)]^2 = .2756$$
$$\sigma_{\text{call}} = \sqrt{5.25} = 2.2913 \text{ (or 229.13\%)}$$
$$\sigma_{\text{put}} = \sqrt{.2756} = .525 \text{ (or 52.5\%)}$$

These are very large standard deviations. Comparing the standard deviation of the call's return to its expected value, we get the coefficient of variation:

Table A.2
Scenario Analysis for Investment in Options on Anheuser-Busch Stock

	Scenario 1	Scenario 2	Scenario 3
Probability	.20	.50	.30
Event			
1. Return on stock	20%	30%	50%
Stock price (initial price = \$30)	\$36.00	\$39.00	\$45.00
2. Cash flow from call (exercise price = \$42)	0	0	\$3.00
Call profit (initial price = \$.60)	−\$.60	−\$.60	\$2.40
Call rate of return	−100%	−100%	400%
3. Cash flow from put (exercise price = \$42)	\$6.00	\$3.00	0
Put profit (initial price = \$4)	\$2.00	−\$1.00	−\$4.00
Put rate of return	50%	−25%	−100%

$$CV_{call} = \frac{2.2913}{.5} = 4.5826$$

Refer back to the coefficient of variation for the stock itself, .3275, and it is clear that these instruments have high standard deviations. This is quite common for stock options. The negative expected return of the put illustrates again the problem in interpreting the magnitude of the "surprise" indicated by the coefficient of variation.

Moving to the third moments of the two probability distributions:

$$M_3(\text{call}) = .2(-1 - .5)^3 + .5(-1 - .5)^3 + .3(4 - .5)^3 = 10.5$$
$$M_3(\text{put}) = .2[.5 - (-.325)]^3 + .5[-.25 - (-.325)]^3 + .3[-1 - (-.325)]^3 = .02025$$

Both instruments are positively skewed, which is typical of options and one part of their attractiveness. In this particular circumstance the call is more skewed than the put. To establish this fact, note the third root of the third moment:

$$m_3(\text{call}) = M_3(\text{call})^{1/3} = 2.1898 \text{ (or 218.98\%)}$$
$$m_3(\text{put}) = .02^{1/3} = .2725 \text{ (or 27.25\%)}$$

Compare these figures to the standard deviations, 229.13% for the call and 52.5% for the put, and you can see that a large part of the standard deviation of the option is driven by the possibility of large good surprises instead of by the more likely, yet smaller, bad surprises.[3]

So far we have described discrete probability distributions using scenario analysis. We shall come back to decision making in a scenario analysis framework in Section A.3 on multivariate statistics.

Continuous Distributions: Normal and Lognormal Distributions

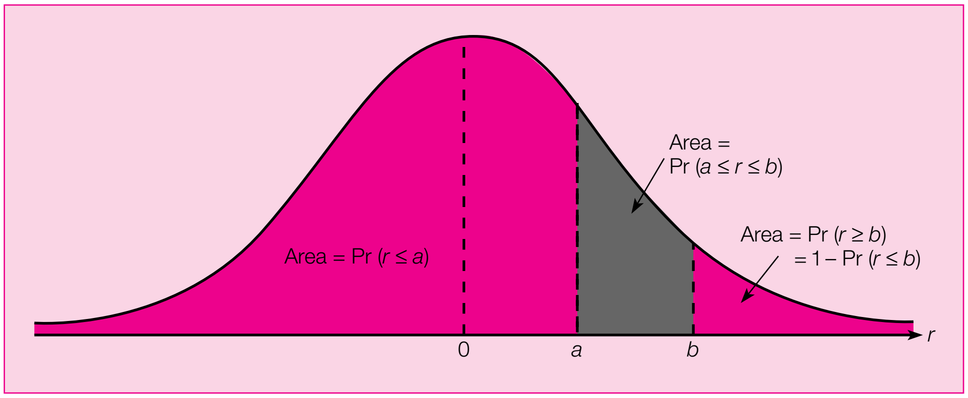
When a compact scenario analysis is possible and acceptable, decisions may be quite simple. Often, however, so many relevant scenarios must be specified that scenario analysis is impossible for practical reasons. Even in the case of Anheuser-Busch, as we were careful to specify, the individual scenarios considered actually represented compound events.

When many possible values of rate of return have to be considered, we must use a formula that describes the probability distribution (relates values to probabilities). As we noted earlier, there are two types of probability distributions: discrete and continuous. Scenario analysis involves a discrete distribution. However, the two most useful distributions in investments, the normal and lognormal, are continuous. At the same time they are often used to approximate variables with distributions that are known to be discrete, such as stock prices. The probability distribution of future prices and returns is discrete—prices are quoted in eighths. Yet the industry norm is to approximate these distributions by the normal or lognormal distribution.

Standard Normal Distribution The normal distribution, also known as Gaussian (after the mathematician Gauss) or bell-shaped, describes random variables with the following properties and is shown in Figure A.1:

[3] Note that the expected return of the put is −32.5%; hence the worst surprise is −67.5%, and the best is 82.5%. The middle scenario is also a positive deviation of 7.5% (with a high probability of .50). These two elements explain the positive skewness of the put.

Figure A.1
Probabilities under the normal density.



- The expected value is the mode (the most frequent elementary event) and also the median (the middle value in the sense that half the elementary events are greater and half smaller). Note that the expected value, unlike the median or mode, requires weighting by probabilities to produce the concept of central value.
- The normal probability distribution is symmetric around the expected value. In other words, the likelihood of equal absolute-positive and negative deviations from expectation is equal. Larger deviations from the expected value are less likely than are smaller deviations. In fact, the essence of the normal distribution is that the probability of deviations decreases exponentially with the magnitude of the deviation (positive and negative alike).
- A normal distribution is identified completely by two parameters, the expected value and the standard deviation. The property of the normal distribution that makes it most convenient for portfolio analysis is that any weighted sum or normally distributed random variables produce a random variable that also is normally distributed. This property is called *stability.* It is also true that if you add a constant to a "normal" random variable (meaning a random variable with a normal probability distribution) or multiply it by a constant, then the transformed random variable also will be normally distributed.

Suppose that n is any random variable (not necessarily normal) with expectation μ and standard deviation σ. As we showed earlier, if you add a constant c to $n,$ the standard deviation is not affected at all, but the mean will change to $\mu + c$. If you multiply n by a constant $b,$ its mean and standard deviation will change by the same proportion to $b\mu$ and $b\sigma$. If n is normal, the transformed variable also will be normal.

Stability, together with the property that a normal variable is completely characterized by its expectation and standard deviation, implies that if we know one normal probability distribution with a given expectation and standard deviation, we know them all.

Subtracting the expected value from each observation and then dividing by the standard deviation we obtain the *standard normal distribution,* which has an expectation of zero and both variance and standard deviation equal to 1.0. Formally, the relationship between the value of the standard normal random variable, $z,$ and its probability, $f,$ is given by

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-z^2}{2}\right) \tag{A.6}$$

where "exp" is the quantity e to the power of the expression in the parentheses. The quantity e is an important number just like the well-known π, which also appears in the function. It is important enough to earn a place on the keyboard of your financial calculator, mostly because it is used also in continuous compounding.

Probability functions of continuous distributions are called *densities* and denoted by f, rather than by the "Pr" of scenario analysis. The reason is that the probability of any of the infinitely many possible values of z is infinitesimally small. Density is a function that allows us to obtain the probability of a *range of values* by integrating it over a desired range. In other words, whenever we want the probability that a standard normal variate (a random variable) will fall in the range from $z = a$ to $z = b$, we have to add up the density values, $f(z)$, for all zs from a to b. There are infinitely many zs in that range, regardless how close a is to b. *Integration* is the mathematical operation that achieves this task.

Consider first the probability that a standard normal variate will take on a value less than or equal to a, that is, z is in the range $[-\infty,a]$. We have to integrate the density from ∞ to a. The result is called the *cumulative (normal) distribution,* denoted by $N(a)$. When a approaches infinity, any value is allowed for z; hence the probability that z will end up in that range approaches 1.0. It is a property of any density that when it is integrated over the entire range of the random variable, the cumulative distribution is 1.0.

In the same way, the probability that a standard normal variate will take on a value less than or equal to b is $N(b)$. The probability that a standard normal variate will take on a value in the range $[a, b]$ is just the difference between $N(b)$ and $N(a)$. Formally,

$$\Pr(a \leq z \leq b) = N(b) - N(a)$$

These concepts are illustrated in Figure A.1. The graph shows the normal density. It demonstrates the symmetry of the normal density around the expected value (zero for the standard normal variate, which is also the mode and the median), and the smaller likelihood of larger deviations from expectation. As is true for any density, the entire area under the density graph adds up to 1.0. The values a and b are chosen to be positive, so they are to the right of the expected value. The leftmost blue shaded area is the proportion of the area under the density for which the value of z is less than or equal to a. Thus this area yields the cumulative distribution for a, the probability that z will be smaller than or equal to a. The gray shaded area is the area under the density graph between a and b. If we add that area to the cumulative distribution of a, we get the entire area up to b, that is, the probability that z will be anywhere to the left of b. Thus the area between a and b has to be the probability that z will fall between a and b.

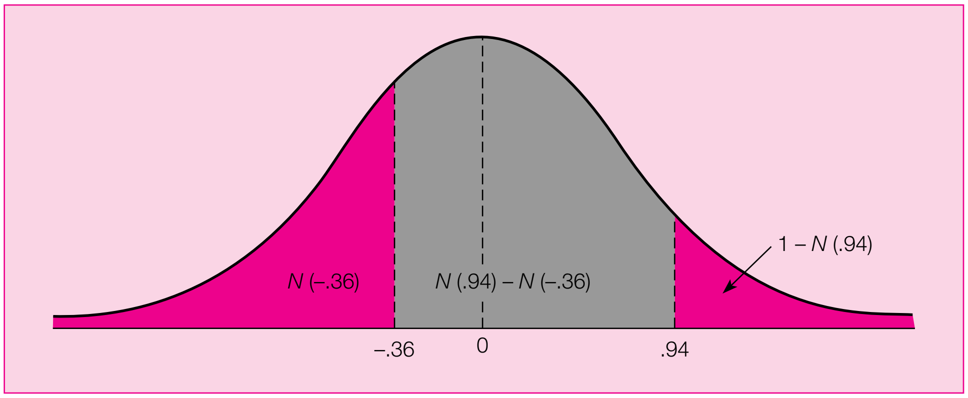
Applying the same logic, we find the probability that z will take on a value greater than b. We know already that the probability that z will be smaller than or equal to b is $N(b)$. The compound events "smaller than or equal to b" and "greater than b" are mutually exclusive *and* "exhaustive," meaning that they include all possible outcomes. Thus their probabilities sum to 1.0, and the probability that z is greater than b is simply equal to one minus the probability that z is less than or equal to b. Formally, $\Pr(z > b) = 1 - N(b)$.

Look again at Figure A.1. The area under the density graph between b and infinity is just the difference between the entire area under the graph (equal to 1.0) and the area between minus infinity and b, that is, $N(b)$.

The normal density is sufficiently complex that its cumulative distribution, its integral, does not have an exact formulaic closed-form solution. It must be obtained by numerical (approximation) methods. These values are produced in tables that give the value $N(z)$ for any z, such as Table 21.2 of this text.

To illustrate, let us find the following probabilities for a standard normal variate:

Figure A.2 **Probabilities and the cumulative normal distribution.**



$$\Pr(z \le -.36) = N(-.36) = \text{Probability that } z \text{ is less than or equal to } -.36$$
$$\Pr(z \le .94) = N(.94) = \text{Probability that } z \text{ is less than or equal to } .94$$
$$\Pr(-.36 \le z \le .94) = N(.94) - N(-.36) = \text{Probability that } z \text{ will be in the range } [-.36, .94]$$
$$\Pr(z > .94) = 1 - N(.94) = \text{Probability that } z \text{ is greater than } .94$$

Use Table 21.2 of the cumulative standard normal (sometimes called the area under the normal density) and Figure A.2. The table shows that

$$N(-.36) = .3594$$
$$N(.94) = .8264$$

In Figure A.2 the area under the graph between −.36 and .94 is the probability that z will fall between −.36 and .94. Hence

$$\Pr(-.36 \le z \le .94) = N(.94) - N(-.36) = .8264 - .3594 = .4670$$

The probability that z is greater than .94 is the area under the graph in Figure A.2, between .94 and infinity. Thus it is equal to the entire area (1.0) less the area from minus infinity to .94. Hence

$$\Pr(z > .94) = 1 - N(.94) = 1 - .8264 = .1736$$

Finally, one can ask, What is the value a for which z will be smaller than or equal to a with probability P? The notation for the function that yields the desired value of a is $\Phi(P)$, so that

$$\text{If } \Phi(P) = a, \text{ then } P = N(a) \qquad \text{(A.7)}$$

For instance, suppose the question is, Which value has a cumulative density of .50? A glance at Figure A.2 reminds us that the area between minus infinity and zero (the expected value) is .5. Thus we can write

$$\Phi(.5) = 0, \text{ because } N(0) = .5$$

Similarly,

$$\Phi(.8264) = .94, \text{ because } N(.94) = .8264$$

and

$$\Phi(.3594) = -.36$$

For practice, confirm with Table 21.2 that $\Phi(.6554) = .40$, meaning that the value of z with a cumulative distribution of .6554 is $z = .40$.

Nonstandard Normal Distributions Suppose that the monthly rate of return on a stock is closely approximated by a normal distribution with a mean of .015 (1.5% per month), and standard deviation of .127 (12.7% per month). What is the probability that the rate of return will fall below zero in a given month? Recall that because the rate is a normal variate, its cumulative density has to be computed by numerical methods. The standard normal table can be used for any normal variate.

Any random variable, x, may be transformed into a new standardized variable, x^*, by the following rule:

$$x^* = \frac{x - E(x)}{\sigma(x)} \tag{A.8}$$

Note that all we have done to x was (1) *subtract* its expectation and (2) *multiply* by one over its standard deviation, $1/[\sigma(x)]$. According to our earlier discussion, the effect of transforming a random variable by adding and multiplying by a constant is such that the expectation and standard deviation of the transformed variable are

$$E(x^*) = \frac{E(x) - E(x)}{\sigma(x)} = 0; \qquad \sigma(x^*) = \frac{\sigma(x)}{\sigma(x)} = 1 \tag{A.9}$$

From the stability property of the normal distribution we also know that if x is normal, so is x^*. A normal variate is characterized completely by two parameters: its expectation and standard deviation. For x^*, these are zero and 1.0, respectively. When we subtract the expectation and then divide a normal variate by its standard deviation, we standardize it; that is, we transform it to a standard normal variate. This trick is used extensively in working with normal (and approximately normal) random variables.

Returning to our stock, we have learned that if we subtract .015 and then divide the monthly returns by .127, the resultant random variable will be standard normal. We can now determine the probability that the rate of return will be zero or less in a given month. We know that

$$z = \frac{r - .015}{.127}$$

where z is standard normal and r the return on our stock. Thus if r is zero, z has to be

$$z(r = 0) = \frac{0 - .015}{.127} = -.1181$$

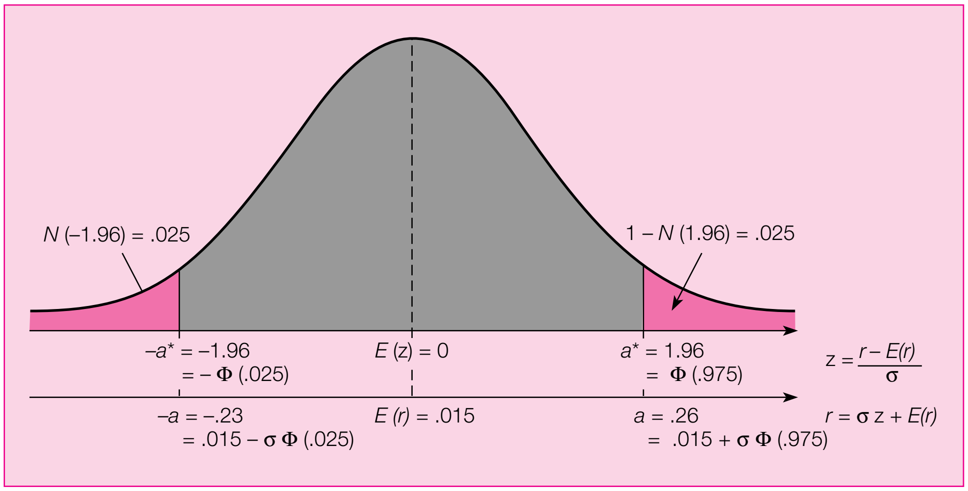
For r to be zero, the corresponding standard normal has to be −11.81%, a negative number. The event "r will be zero or less" is identical to the event "z will be −.1181 or less." Calculating the probability of the latter will solve our problem. That probability is simply $N(-.1181)$. Visit the standard normal table and find that

$$\Pr(r \leq 0) = N(-.1181) = .5 - .047 = .453$$

The answer makes sense. Recall that the expectation of r is 1.5%. Thus, whereas the probability that r will be 1.5% or less is .5, the probability that it will be *zero* or less has to be close, but somewhat lower.

Confidence Intervals Given the large standard deviation of our stock, it is logical to be concerned about the likelihood of extreme values for the monthly rate of return. One

Figure A.3
Confidence intervals and the standard normal density.



way to quantify this concern is to ask: "What is the interval (range) within which the stock return will fall in a given month, with a probability of .95?" Such an interval is called the *95% confidence interval.*

Logic dictates that this interval be centered on the expected value, .015, because r is a normal variate (has a normal distribution) which is symmetric around the expectation. Denote the desired interval by

$$[E(r) - a, E(r) + a] = [.015 - a, .015 + a]$$

which has a length of $2a$. The probability that r will fall within this interval is described by the following expression:

$$\Pr(.015 - a \leq r \leq .015 + a) = .95$$

To find this probability, we start with a simpler problem, involving the *standard normal variate,* that is, a normal with expectation of zero and standard deviation of 1.0.

What is the 95% confidence interval for the standard normal variate, z? The variable will be centered on zero, so the expression is

$$\Pr(-a^* \leq z \leq a^*) = N(a^*) - N(-a^*) = .95$$

You might best understand the substitution of the difference of the appropriate cumulative distributions for the probability with the aid of Figure A.3. The probability of falling outside of the interval is $1 - .95 = .05$. By the symmetry of the normal distribution, z will be equal to or less than $-a^*$ with probability of .025, and with probability .025, z will be greater than a^*. Thus we solve for a^* using

$$-a^* = \Phi(.025), \text{ which is equivalent to } N(-a^*) = .025$$

We can summarize the chain that we have pursued so far as follows. If we seek a $P =$ .95 level confidence interval, we define α as the probability that r will fall outside the confidence interval. Because of the symmetry, α will be split so that half of it is the probability of falling to the right of the confidence interval, while the other half is the probability of falling to the left of the confidence interval. Therefore, the relation between α and P is

$$\alpha = 1 - P = .05; \qquad \frac{\alpha}{2} = \frac{1 - P}{2} = .025$$

We use $\alpha/2$ to indicate that the area that is excluded for r is equally divided between the tails of the distributions. Each tail that is excluded for r has an area of $\alpha/2$. The value $\alpha = 1 - P$ represents the entire value that is excluded for r.

To find $z = \Phi(\alpha/2)$, which is the lower boundary of the confidence interval for the standard normal variate, we have to locate the z value for which the standard normal cumulative distribution is .025, finding $z = -1.96$. Thus we conclude that $-a^* = -1.96$ and $a^* = 1.96$. The confidence interval for z is

$$[E(z) - \Phi(\alpha/2), E(z) + \Phi(\alpha/2)] = [-\Phi(.025), \Phi(.025)]$$
$$= [-1.96, .196]$$

To get the interval boundaries for the nonstandard normal variate r, we transform the boundaries for z by the usual relationship, $r = z\sigma(r) + E(r) = \Phi(\alpha/2)\sigma(r) + E(r)$. Note that all we are doing is setting the expectation at the center of the confidence interval and extending it by a number of standard deviations. The number of standard deviations is determined by the probability that we allow for falling outside the confidence interval (α), or, equivalently, the probability of falling in it (P). Using minus and plus 1.96 for $z = \pm\Phi(\emptyset/.25)$, the distance on each side of the expectation is $\pm 1.96 \times .127 = .249$. Thus we obtain the confidence interval

$$[E(r) - \sigma(r)\Phi(\alpha/2), E(r) + \sigma(r)\Phi(\alpha/2)] = [E(r) - .249, E(r) + .249]$$
$$= [-.234, .264]$$

so that

$$P = 1 - \alpha = \Pr[E(r) - \sigma(r)\Phi(\alpha/2) \leq r \leq E(r) + \sigma(r)\Phi(\alpha/2)]$$

which, for our stock (with expectation .015 and standard deviation .127), amounts to

$$\Pr[-.234 \leq r \leq .264] = .95$$

Note that because of the large standard deviation of the rate of return on the stock, the 95% confidence interval is 49% wide.

To reiterate with a variation on this example, suppose we seek a 90% confidence interval for the annual rate of return on a portfolio, r_p, with a monthly expected return of 1.2% and standard deviation of 5.2%.

The solution is simply

$$\Pr\left[E(r) - \sigma(r)\,\Phi\left(\frac{1-P}{2}\right) \leq r_p \leq E(r) + \sigma(r)\,\Phi\left(\frac{1-P}{2}\right)\right]$$
$$= \Pr[(.012 - .052 \times 1.645) \leq r_p \leq (.012 + .052 \times 1.645)]$$
$$= \Pr[-.0735 \leq r_p \leq .0975] = .90$$

Because the portfolio is of low risk this time (and we require only a 90% rather than a 95% probability of falling within the interval), the 90% confidence interval is only 2.4% wide.

The Lognormal Distribution The normal distribution is not adequate to describe stock prices and returns for two reasons. First, whereas the normal distribution admits any value, including negative values, actual stock prices cannot be negative. Second, the normal distribution does not account for compounding. The lognormal distribution addresses these two problems.

The lognormal distribution describes a random variable that grows, *every instant,* by a rate that is a normal random variable. Thus the progression of a lognormal random variable reflects continuous compounding.

Suppose that the *annual continuously compounded* (ACC) rate of return on a stock is normally distributed with expectation $\mu = .12$ and standard deviation $\sigma = .42$. The stock price at the beginning of the year is $P_0 = \$10$. With continuous compounding (see appendix to Chapter 5), if the ACC rate of return, r_C, turns out to be .23, then the end-of-year price will be

$$P_1 = P_0 \exp(r_C) = 10e^{.23} = \$12.586$$

representing an effective annual rate of return of

$$r = \frac{P_1 - P_0}{P_0} = e^{r_C} - 1 = .2586 \text{ (or 25.86\%)}$$

This is the practical meaning of r, the annual rate on the stock, being lognormally distributed. Note that however negative the ACC rate of return (r_C) is, the price, P_1, cannot become negative.

Two properties of lognormally distributed financial assets are important: their expected return and the allowance for changes in measurement period.

Expected Return of a Lognormally Distributed Asset The expected annual rate of return of a lognormally distributed stock (as in our example) is

$$\begin{aligned} E(r) &= \exp(\mu + \sigma^2/2) - 1 = \exp(.12 + .42^2/2) - 1 = e^{.2082} - 1 \\ &= .2315 \text{ (or 23.15\%)} \end{aligned}$$

This is just a statistical property of the distribution. For this reason, a useful statistic is

$$\mu^* = \mu + \sigma^2/2 = .2082$$

When analysts refer to the expected ACC return on a lognormal asset, frequently they are really referring to μ^*. Often the asset is said to have a normal distribution of the ACC return with expectation μ^* and standard deviation σ.

Change of Frequency of Measured Returns The lognormal distribution allows for easy change of the holding period of returns. Suppose that we want to calculate returns monthly instead of annually. We use the parameter t to indicate the fraction of the year that is desired; in the case of monthly periods, $t = 1/12$. To transform the annual distribution to a t-period (monthly) distribution, it is necessary merely to multiply the expectation and variance of the ACC return by t (in this case, 1/12).

The monthly continuously compounded return on the stock in our example has the expectation and standard deviation of

$$\begin{aligned} \mu(\text{monthly}) &= .12/12 = .01 \text{ (1\% per month)} \\ \sigma(\text{monthly}) &= .42/\sqrt{12} = .1212 \text{ (or 12.12\% per month)} \\ \mu^*(\text{monthly}) &= .2082/12 = .01735 \text{ (or 1.735\% per month)} \end{aligned}$$

Note that we divide variance by 12 when changing from annual to monthly frequency; the standard deviation therefore is divided by the square root of 12.

Similarly, we can convert a nonannual distribution to an annual distribution by following the same routine. For example, suppose that the weekly continuously compounded rate of return on a stock is normally distributed with $\mu^* = .003$ and $\sigma = .07$. Then the ACC return is distributed with

$$\begin{aligned} \mu^* &= 52 \times .003 = .156 \text{ (or 15.6\% per year)} \\ \sigma &= \sqrt{52} \times .07 = .5048 \text{ (or 50.48\% per year)} \end{aligned}$$

In practice, to obtain normally distributed, continuously compounded returns, R, we take the log of 1.0 plus the raw returns:

$$R = \log(1 + r)$$

For short intervals, raw returns are small, and the continuously compounded returns, R, will be practically identical to the raw returns, r. The rule of thumb is that this conversion is not necessary for periods of 1 month or less. That is, approximating stock returns as normal will be accurate enough. For longer intervals, however, the transformation may be necessary.

A.2 DESCRIPTIVE STATISTICS

Our analysis so far has been forward looking, or, as economists like to say, ex ante. We have been concerned with probabilities, expected values, and surprises. We made our analysis more tractable by assuming that decision outcomes are distributed according to relatively simple formulas, and that we know the parameters of these distributions.

Investment managers must satisfy themselves that these assumptions are reasonable, which they do by constantly analyzing observations from relevant random variables that accumulate over time. Distribution of past rates of return on a stock is one element they need to know in order to make optimal decisions. True, the distribution of the rate of return itself changes over time. However, a sample that is not too old does yield information relevant to the next-period probability distribution and its parameters. In this section we explain descriptive statistics, or the organization and analysis of such historic samples.

Histograms, Boxplots, and Time Series Plots

Table A.3 shows the annual excess returns (over the T-bill rate) for two major classes of assets, the S&P 500 index and a portfolio of long-term government bonds, for the period 1926 to 1993.

One way to understand the data is to present it graphically, commonly in a *histogram* or frequency distribution. Histograms of the 68 observations in Table A.3 are shown in Figure A.4. We construct a histogram according to the following principles:

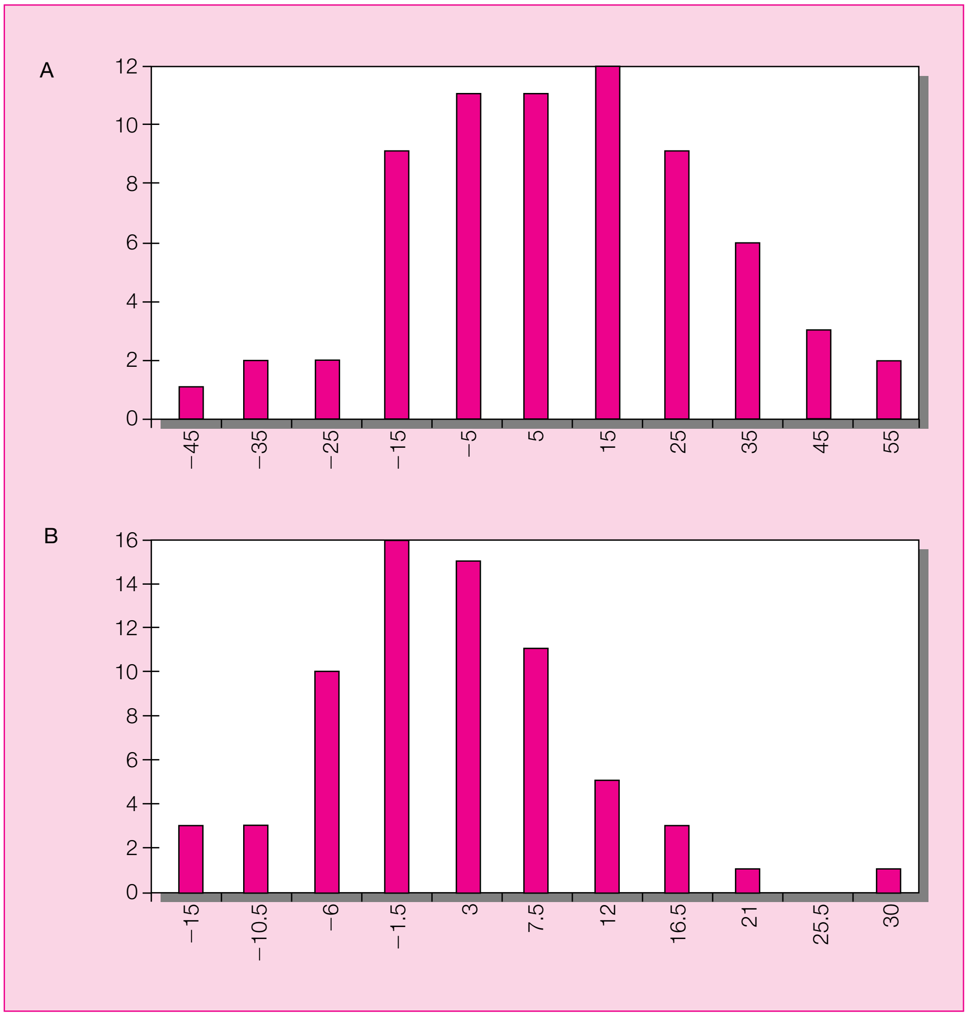
- The range (of values) of the random variable is divided into a relatively small number of equal-sized intervals. The number of intervals that makes sense depends on the number of available observations. The data in Table A.3 provide 68 observations, and thus deciles (10 intervals) seem adequate.
- A rectangle is drawn over each interval. The height of the rectangle represents the frequency of observations for each interval.
- If the observations are concentrated in one part of the range, the range may be divided to unequal intervals. In that case the rectangles are scaled so that their *area* represents the frequency of the observations for each interval. (This is not the case in our samples, however.)
- If the sample is representative, the shape of the histogram will reveal the probability distribution of the random variable. Our total of 68 observations is not a large sample, but a look at the histogram does suggest that the returns may be reasonably approximated by a normal or lognormal distribution.

Table A.3 Excess Return (Risk Premiums) on Stocks and Long-Term Treasury Bonds (Maturity Premiums)

Year	Equity Risk Premium	Maturity Premium	Year	Equity Risk Premium	Maturity Premium
1926	8.35	4.50	1963	19.68	−1.91
1927	34.37	5.81	1964	12.94	−0.03
1928	40.37	−3.14	1965	8.52	−3.22
1929	−13.17	−1.33	1966	−14.82	−1.11
1930	−27.31	2.25	1967	19.77	−13.40
1931	−44.41	−6.38	1968	5.85	−5.47
1932	−9.15	15.88	1969	−15.08	−11.66
1933	53.69	−0.38	1970	−2.52	5.57
1934	−1.60	9.86	1971	9.92	8.84
1935	47.50	4.81	1972	15.14	1.84
1936	33.74	7.33	1973	−21.59	−8.04
1937	−35.34	−0.08	1974	−34.47	−3.65
1938	31.14	5.55	1975	31.40	3.39
1939	−0.43	5.92	1976	18.76	11.67
1940	−9.78	6.09	1977	−12.30	−5.79
1941	−11.65	0.87	1978	−0.62	−8.34
1942	20.07	2.95	1979	8.06	−11.60
1943	25.55	1.73	1980	21.18	−15.19
1944	19.42	2.48	1981	−19.62	−12.86
1945	36.11	10.40	1982	10.87	29.81
1946	−8.42	−0.45	1983	13.71	−8.12
1947	5.21	−3.13	1984	−3.58	5.58
1948	4.69	2.59	1985	24.44	23.25
1949	17.69	5.35	1986	12.31	18.28
1950	30.51	−1.14	1987	−0.24	−8.16
1951	22.53	−5.43	1988	10.46	3.32
1952	16.71	−0.50	1989	23.12	9.74
1953	−2.81	1.81	1990	−10.98	−1.63
1954	51.76	6.33	1991	24.95	13.70
1955	29.99	−2.87	1992	4.16	4.54
1956	4.10	−8.05	1993	7.09	15.34
1957	−13.92	4.31			
1958	41.82	−7.64	Average	8.57	1.62
1959	9.01	−5.21	Standard deviation	20.90	8.50
1960	−3.13	11.12	Minimum	−44.41	−15.19
1961	24.76	−1.16	Maximum	53.69	29.81
1962	−11.46	4.16			

Source: Data from the Center for Research of Security Prices, University of Chicago.

Figure A.4
A. Histogram of the equity risk premium.
B. Histogram of the bond maturity premium.

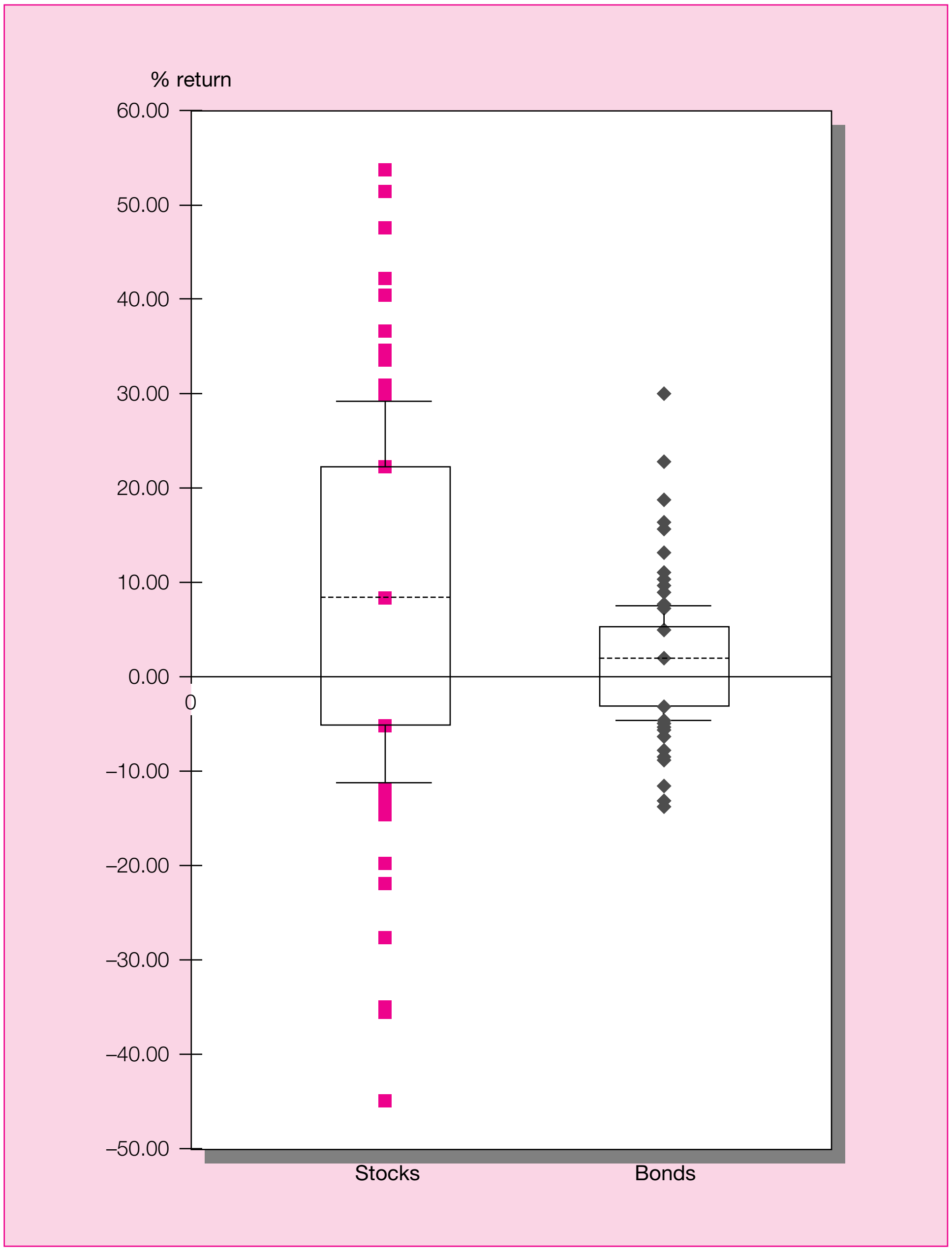


Source: *The Wall Street Journal*. October 15, 1997. Reprinted by permission of *The Wall Street Journal*, © 1997 Dow Jones & Company, Inc. All Rights Reserved Worldwide.

Another way to represent sample information graphically is by *boxplots*. Figure A.5 is an example that uses the same data as in Table A.3. Boxplots are most useful to show the dispersion of the sample distribution. A commonly used measure of dispersion is the *interquartile range.* Recall that the range, a crude measure of dispersion, is defined as the distance between the largest and smallest observations. By its nature, this measure is unreliable because it will be determined by the two most extreme outliers of the sample.

The interquartile range, a more satisfactory variant of the simple range, is defined as the difference between the lower and upper quartiles. Below the *lower* quartile lies 25% of the sample; similarly, above the *upper* quartile lies 25% of the sample. The interquartile range therefore is confined to the central 50% of the sample. The greater the dispersion of a sample, the greater the distance between these two values.

Figure A.5
Boxplots of annual equity risk premium and long-term bond (maturity) risk premium (1926–1993).



In the boxplot the horizontal broken line represents the median, the box the interquartile range, and the vertical lines extending from the box the range. The vertical lines representing the range often are restricted (if necessary) to extend only to 1.5 times the interquartile range, so that the more extreme observations can be shown separately (by points) as outliers.

As a concept check, verify from Table A.3 that the points on the boxplot of Figure A.5 correspond to the following list:

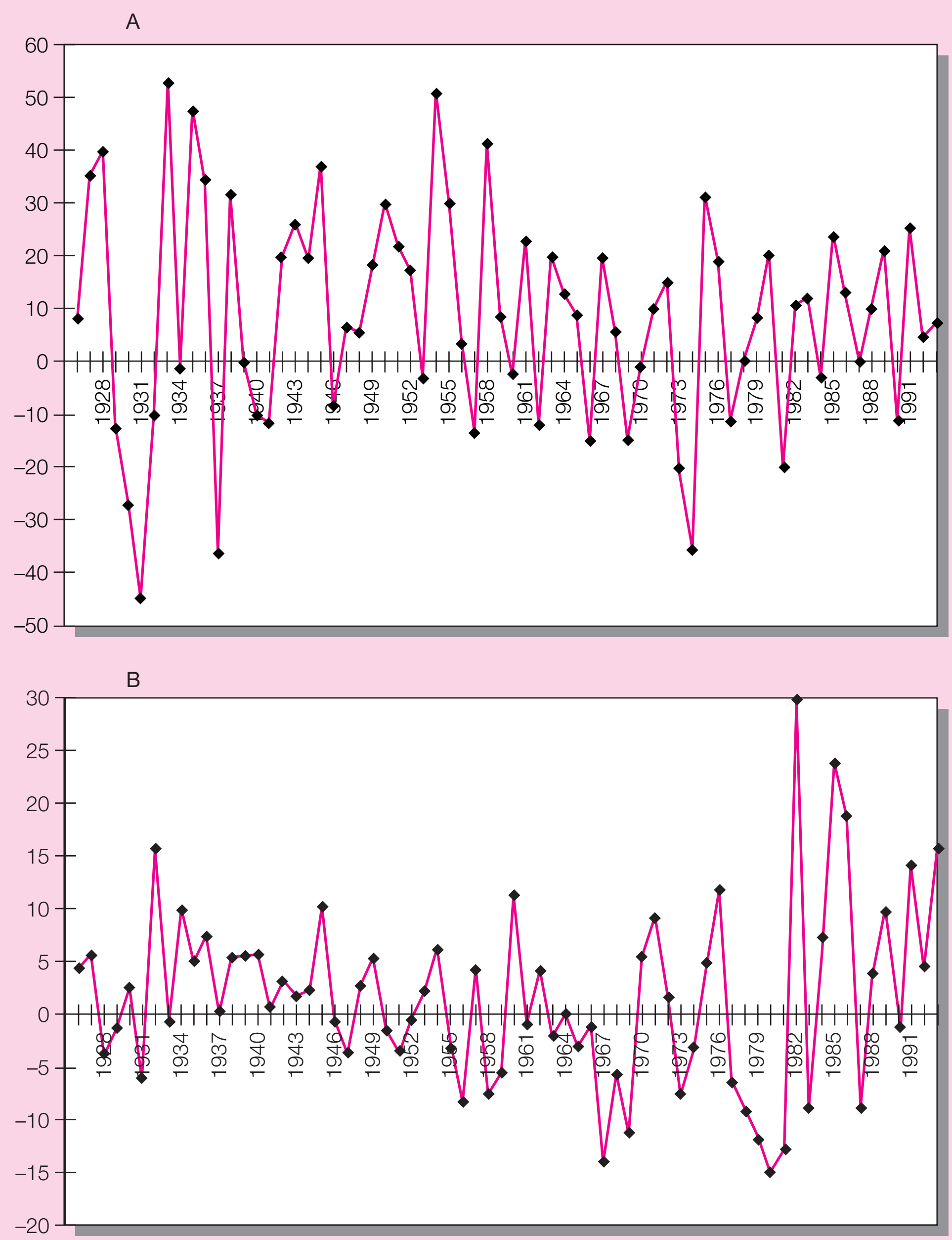
	Equity Risk Premium	Bond Maturity Premium
Lowest extreme points	−44.41	−15.19
	−35.34	−13.40
	−34.47	−12.86
	−27.31	−11.66
	−21.59	−11.60
	−19.62	−8.34
	−15.08	−8.16
	−14.82	−8.12
	−13.92	−8.05
	−13.17	−8.04
	−12.30	−7.64
		−6.38
		−5.79
		−5.47
		−5.43
		−5.21
Lowest quartile	−4.79	−3.33
Median	8.77	1.77
Highest quartile	22.68	5.64
Highest extreme points	29.99	8.84
	30.51	9.74
	31.14	9.86
	31.40	10.40
	33.74	11.12
	34.37	11.67
	36.11	13.70
	40.37	15.34
	41.82	15.88
	47.50	18.28
	51.76	23.25
	53.69	29.81
Interquartile range	27.47	8.97
1.5 times the interquartile range	41.20	13.45
From:	−11.84	−4.95
To:	29.37	8.49

Finally, a third form of graphing is *time series plots,* which are used to convey the behavior of economic variables over time. Figure A.6 shows a time series plot of the excess returns on stocks and bonds from Table A.3. Even though the human eye is apt to see patterns in randomly generated time series, examining time series' evolution over a long period does yield some information. Sometimes, such examination can be as revealing as that provided by formal statistical analysis.

Sample Statistics

Suppose we can assume that the probability distribution of stock returns has not changed over the 68 years from 1926 to 1993. We wish to draw inferences about the probability

Figure A.6 **A. Equity risk premium, 1926–1993. B. Bond maturity premium, 1926–1993.**



distribution of stock returns from the sample of 68 observations of annual stock excess returns in Table A.3.

A central question is whether given observations represent independent observations from the underlying distribution. If they do, statistical analysis is quite straightforward. Our analysis assumes that this is indeed the case. Empiricism in financial markets tends to confirm this assumption in most cases.

Estimating Expected Returns from the Sample Average The definition of expected returns suggests that the sample average be used as an estimate of the expected value. Indeed, one definition of the expected return is the average of a sample when the number of observations tends to infinity.

Denoting the sample returns in Table A.3 by R_t, $t = 1, \ldots, T = 68$, the estimate of the annual expected excess rate of return is

$$\bar{R} = \frac{1}{T}\Sigma R_t = 8.57\%$$

The bar over the R is a common notation for an estimate of the expectation. Intuition suggests that the larger the sample, the greater the reliability of the sample average, and the larger the standard deviation of the measured random variable, the less reliable the average. We discuss this property more fully later.

Estimating Higher Moments The principle of estimating expected values from sample averages applies to higher moments as well. Recall that higher moments are defined as expectations of some power of the deviation from expectation. For example, the variance (second moment) is the expectation of the squared deviation from expectation. Accordingly, the sample average of the squared deviation from the average will serve as the estimate of the variance, denoted by s^2:

$$s^2 = \frac{1}{T-1}\Sigma(R_t - \bar{R})^2 = \frac{1}{67}\Sigma(R_t - .0857)^2 = .04368 \qquad (s = 20.90\%)$$

where $\bar{R}$ is the estimate of the sample average. The average of the squared deviation is taken over $T - 1 = 67$ observations for a technical reason. If we were to divide by T, the estimate of the variance would be downward-biased by the factor $(T - 1)/T$. Here too, the estimate is more reliable the larger the sample and the smaller the true standard deviation.

A.3 MULTIVARIATE STATISTICS

Building portfolios requires combining random variables. The rate of return on a portfolio is the weighted average of the individual returns. Hence understanding and quantifying the interdependence of random variables is essential to portfolio analysis. In the first part of this section we return to scenario analysis. Later we return to making inferences from samples.

The Basic Measure of Association: Covariance

Table A.4 summarizes what we have developed so far for the scenario returns on Anheuser-Busch stock and options. We know already what happens when we add a constant to one of these return variables or multiply by a constant. But what if we combine any two of them? Suppose that we add the return on the stock to the return on the call. We create a new random variable that we denote by $r(s + c) = r(s) + r(c)$, where $r(s)$ is the return on the stock and $r(c)$ is the return on the call.

From the definition, the expected value of the combination variable is

$$E[r(s + c)] = \Sigma \Pr(i) r_i(s + c) \qquad \text{(A.10)}$$

Substituting the definition of $r(s + c)$ into equation A.10 we have

$$\begin{aligned} E[r(s + c)] &= \Sigma \Pr(i)[r_i(s) + r_i(c)] = \Sigma \Pr(i) r_i(s) + \Sigma \Pr(i) r_i(c) \qquad \text{(A.11)} \\ &= E[r(s)] + E[r(c)] \end{aligned}$$

Table A.4
Probability Distribution of Anheuser-Busch Stock and Options

	Scenario 1	Scenario 2	Scenario 3
Probability	.20	.50	.30
Rates of return (%)			
Stock	20	30	50
Call option	−100	−100	400
Put option	50	−25	−100
	***E*(*r*)**	**σ**	**σ^2**
Stock	.340	0.1114	0.0124
Call option	.500	2.2913	5.2500
Put option	−.325	0.5250	0.2756

In words, the expectation of the sum of two random variables is just the sum of the expectations of the component random variables. Can the same be true about the variance? The answer is "no," which is, perhaps, the most important fact in portfolio theory. The reason lies in the statistical association between the combined random variables.

As a first step, we introduce the *covariance,* the basic measure of association. Although the expressions that follow may look intimidating, they are merely squares of sums; that is, $(a + b)^2 = a^2 + b^2 + 2ab$, and $(a - b)^2 = a^2 + b^2 - 2ab$, where the *a*s and *b*s might stand for random variables, their expectations or their deviations from expectations. From the definition of the variance

$$\sigma^2_{s+c} = E[r_{s+c} - E(r_{s+c})]^2 \quad \text{(A.12)}$$

To make equations A.12 through A.20 easier to read, we will identify the variables by subscripts *s* and *c* and drop the subscript *i* for scenarios. Substitute the definition of $r(s + c)$ and its expectation into equation A.12:

$$\sigma^2_{s+c} = E[r_s + r_c - E(r_s) - E(r_c)]^2 \quad \text{(A.13)}$$

Changing the order of variables within the brackets in equation A.13,

$$\sigma^2_{s+c} = E[r_s - E(r_s) + r_c - E(r_c)]^2$$

Within the square brackets we have the sum of the deviations from expectations of the two variables, which we denote by *d*. Writing this out,

$$\sigma^2_{s+c} = E[(d_s + d_c)^2] \quad \text{(A.14)}$$

Equation A.14 is the expectation of a complete square. Taking the square, we find

$$\sigma^2_{s+c} = E(d_s^2 + d_c^2 + 2d_s d_c) \quad \text{(A.15)}$$

The term in parentheses in equation A.15 is the summation of three random variables. Because the expectation of a sum is the sum of the expectations, we can write equation A.15 as

$$\sigma^2_{s+c} = E(d_s^2) + E(d_c^2) + 2E(d_s d_c) \quad \text{(A.16)}$$

In equation A.16 the first two terms on the right-hand side are the variance of the stock (the expectation of its squared deviation from expectation) plus the variance of the call. The third term is twice the expression that is the definition of the covariance discussed in equation A.17. (Note that the expectation is multiplied by 2 because expectation of twice a variable is twice the variable's expectation.)

Table A.5 Deviations, Squared Deviations, and Weighted Products of Deviations from Expectations of Anheuser-Busch Stock and Options

	Scenario 1	Scenario 2	Scenario 3	Probability-Weighted Sum
Probability	0.20	0.50	0.30	
Deviation of stock	−0.14	−0.04	0.16	
Squared deviation	0.0196	0.0016	0.0256	0.0124
Deviation of call	−1.50	−1.50	3.50	
Squared deviation	2.25	2.25	12.25	5.25
Deviation of put	0.825	0.75	−0.675	
Squared deviation	0.680625	0.005625	0.455635	0.275628
Product of deviations ($d_s d_c$)	0.21	0.06	0.56	0.24
Product of deviations ($d_s d_p$)	−0.1155	−0.003	−0.108	−0.057
Product of deviations ($d_c d_p$)	−1.2375	−0.1125	−2.3625	−1.0125

In other words, the variance of a sum of random variables is the sum of the variances *plus* twice the covariance, which we denote by Cov(r_s, r_c), or the covariance between the return on *s* and the return on *c*. Specifically,

$$\text{Cov}(r_s, r_c) = E(d_s d_c) = E\{[r_s - E(r_s)][r_c - E(r_c)]\} \quad \text{(A.17)}$$

The sequence of the variables in the expression for the covariance is of no consequence. Because the order of multiplication makes no difference, the definition of the covariance in equation A.17 shows that it will not affect the covariance either.

We use the data in Table A.4 to set up the input table for the calculation of the covariance, as shown in Table A.5.

First, we analyze the covariance between the stock and the call. In Scenarios 1 and 2, both assets show *negative* deviations from expectation. This is an indication of *positive co-movement.* When these two negative deviations are multiplied, the product, which eventually contributes to the covariance between the returns, is positive. Multiplying deviations leads to positive covariance when the variables move in the same direction and negative covariance when they move in opposite directions. In Scenario 3 both assets show *positive* deviations, reinforcing the inference that the co-movement is positive. The magnitude of the products of the deviations, weighted by the probability of each scenario, when added up, results in a covariance that shows not only the direction of the co-movement (by its sign) but also the degree of the co-movement.

The covariance is a variance-like statistic. Whereas the variance shows the degree of the movement of a random variable about its expectation, the covariance shows the degree of the co-movement of two variables about their expectations. It is important for portfolio analysis that the covariance of a variable with itself is equal to its variance. You can see this by substituting the appropriate deviations in equation A.17; the result is the expectation of the variable's squared deviation from expectation.

The first three values in the last column of Table A.5 are the familiar variances of the three assets, the stock, the call, and the put. The last three are the covariances; two of them are negative. Examine the covariance between the stock and the put, for example. In the first two scenarios the stock realizes negative deviations, while the put realizes positive deviations. When we multiply such deviations, the sign becomes negative. The same happens in the third scenario, except that the stock realizes a positive deviation and the put a negative one. Again, the product is negative, adding to the inference of negative co-movement.

With other assets and scenarios the product of the deviations can be negative in some scenarios, positive in others. The *magnitude* of the products, when *weighted* by the probabilities, determines which co-movements dominate. However, whenever the sign of the products varies from scenario to scenario, the results will offset one another, contributing to a small, close-to-zero covariance. In such cases we may conclude that the returns have either a small, or no, average co-movement.

Covariance between Transformed Variables Because the covariance is the expectation of the product of deviations from expectation of two variables, analyzing the effect of transformations on deviations from expectation will show the effect of the transformation on the covariance.

Suppose that we add a constant to one of the variables. We know already that the expectation of the variable increases by that constant, so deviations from expectation will remain unchanged. Just as adding a constant to a random variable does not affect its variance, it also will not affect its covariance with other variables.

Multiplying a random variable by a constant also multiplies its expectation, as well as its deviation from expectation. Therefore, the covariance with any other variable will also be multiplied by that constant. Using the definition of the covariance, check that this summation of the foregoing discussion is true:

$$\text{Cov}(a_1 + b_1 r_s, a_2 + b_2 r_c) = b_1 b_2 \text{Cov}(r_s, r_c) \tag{A.18}$$

The covariance allows us to calculate the variance of sums of random variables, and eventually the variance of portfolio returns.

A Pure Measure of Association: The Correlation Coefficient

If we tell you that the covariance between the rates of return of the stock and the call is .24 (see Table A.5), what have you learned? Because the sign is positive, you know that the returns generally move in the same direction. However, the number .24 adds nothing to your knowledge of the closeness of co-movement of the stock and the call.

To obtain a measure of association that conveys the degree of intensity of the co-movement, we relate the covariance to the standard deviations of the two variables. Each standard deviation is the square root of the variance. Thus the product of the standard deviations has the dimensions of the variance that is also shared by the covariance. Therefore, we can define the correlation coefficient, denoted by ρ, as

$$\rho_{sc} = \frac{\text{Cov}(r_s, r_c)}{\sigma_s \sigma_c} \tag{A.19}$$

where the subscripts on ρ identify the two variables involved. Because the order of the variables in the expression of the covariance is of no consequence, equation A.19 shows that the order does not affect the correlation coefficient either.

We use the covariances in Table A.5 to show the *correlation matrix* for the three variables:

	Stock	Call	Put
Stock	1.00	0.94	−0.97
Call	0.94	1.00	−0.84
Put	−0.97	−0.84	1.00

The highest (in absolute value) correlation coefficient is between the stock and the put, −.97, although the absolute value of the covariance between them is the lowest by far. The reason is attributable to the effect of the standard deviations. The following properties of the correlation coefficient are important:

- Because the correlation coefficient, just as the covariance, measures only the degree of association, it tells us nothing about causality. The direction of causality has to come from theory and be supported by specialized tests.
- The correlation coefficient is determined completely by deviations from expectations, as are the components in equation A.19. We expect, therefore, that it is not affected by adding constants to the associated random variables. However, the correlation coefficient is invariant also to multiplying the variables by constants. You can verify this property by referring to the effect of multiplication by a constant on the covariance and standard deviation.
- The correlation coefficient can vary from −1.0, perfect negative correlation, to 1.0, perfect positive correlation. This can be seen by calculating the correlation coefficient of a variable with itself. You expect it to be 1.0. Recalling that the covariance of a variable with itself is its own variance, you can verify this using equation A.19. The more ambitious can verify that the correlation between a variable and the negative of itself is equal to −1.0. First, find from equation A.17 that the covariance between a variable and its negative equals the negative of the variance. Then check equation A.19.

Because the correlation between x and y is the same as the correlation between y and x, the *correlation matrix is symmetric about the diagonal.* The diagonal entries are all 1.0 because they represent the correlations of returns with themselves. Therefore, it is customary to present only the lower triangle of the correlation matrix.

Reexamine equation A.19. You can invert it so that the covariance is presented in terms of the correlation coefficient and the standard deviations as in equation A.20:

$$\text{Cov}(r_s r_c) = \rho_{sc}\sigma_s\sigma_c \quad \text{(A.20)}$$

This formulation can be useful, because many think in terms of correlations rather than covariances.

Estimating Correlation Coefficients from Sample Returns Assuming that a sample consists of independent observations, we assign equal weights to all observations and use simple averages to estimate expectations. When estimating variances and covariances, we get an average by dividing by the number of observations minus one.

Suppose that you are interested in estimating the correlation between stock and long-term default-free government bonds. Assume that the sample of 68 annual excess returns for the period 1926 to 1993 in Table A.3 is representative.

Using the definition for the correlation coefficient in equation A.19, you estimate the following statistics (using the subscripts s for stocks, b for bonds, and t for time):

$$\bar{R}_s = \frac{1}{68}\sum_{i=1}^{68} R_{s,t} = .0857; \qquad \bar{R}_b = \frac{1}{68}\sum_t R_{b,t} = 0.162$$

$$\sigma_s = \left[\frac{1}{67}\sum_t (R_{s,t} - \bar{R}_s)^2\right]^{1/2} = .2090$$

$$\sigma_b = \left[\frac{1}{67}\sum_t (R_{b,t} - \bar{R}_b)^2\right]^{1/2} = .0850$$

$$\text{Cov}(R_s, R_b) = \frac{1}{67}\sum_t [(R_{s,t} - \bar{R}_s)(R_{b,t} - \bar{R}_b)] = .00314$$

$$\rho_{sb} = \frac{\text{Cov}(R_s, R_b)}{\sigma_s\sigma_b} = .17916$$

Here is one example of how problematic estimation can be. Recall that we predicate our use of the sample on the assumption that the probability distributions have not changed over the sample period. To see the problem with this assumption, suppose that we reestimate the correlation between stocks and bonds over a more recent period—for example, beginning in 1965, about the time of onset of government debt financing of both the war in Vietnam and the Great Society programs.

Repeating the previous calculations for the period 1965 to 1987, we find

$$\bar{R}_s = .0312; \qquad \bar{R}_b = -.00317$$
$$\sigma_s = .15565; \qquad \sigma_b = .11217$$
$$\text{Cov}(R_s, R_b) = .0057; \qquad \rho_{sb} = .32647$$

A comparison of the two sets of numbers suggests that it is likely, but by no means certain, that the underlying probability distributions have changed. The variance in the rates of return and the size of the sample are why we cannot be sure. We shall return to the issue of testing the sample statistics shortly.

Regression Analysis

We will use a problem from the CFA examination (Level I, 1986) to represent the degree of understanding of regression analysis that is required for the ground level. However, first let us develop some background.

In analyzing measures of association so far, we have ignored the question of causality, identifying simply *independent* and *dependent* variables. Suppose that theory (in its most basic form) tells us that all asset excess returns are driven by the same economic force whose movements are captured by a broad-based market index, such as excess return on the S&P 500 stock index.

Suppose further that our theory predicts a simple, linear relationship between the excess return of any asset and the market index. A linear relationship, one that can be described by a straight line, takes on this form:

$$R_{j,t} = a_j + b_j R_{M,t} + e_{j,t} \tag{A.21}$$

where the subscript j represents any asset, M represents the market index (the S&P 500), and t represents variables that change over time. (In the following discussion we omit subscripts when possible.) On the left-hand side of equation A.21 is the dependent variable, the excess return on asset j. The right-hand side has two parts, the explained and unexplained (by the relationship) components of the dependent variable.

The explained component of R_j is the $a + bR_M$ part. It is plotted in Figure A.7. The quantity a, also called the intercept, gives the value of R_j when the *independent* variable is zero. This relationship assumes that it is a constant. The second term in the explained part of the return represents the driving force, R_M, times the sensitivity coefficient, b, that transmits movements in R_M to movements in R_j. The term b is also assumed to be constant. Figure A.7 shows that b is the slope of the regression line.

The unexplained component of R_j is represented by the *disturbance* term, e_j. The disturbance is assumed to be uncorrelated with the explanatory variable, R_M, and of zero expectation. Such a variable is also called a noise variable because it contributes to the variance but not to the expectation of the dependent variable, R_j.

A relationship such as that shown in equation A.21 applied to data, with coefficients estimated, is called a *regression equation.* A relationship including only one explanatory variable is called *simple regression.* The parameters a and b are called (simple) *regression coefficients.* Because every value of R_j is explained by the regression, the expectation and

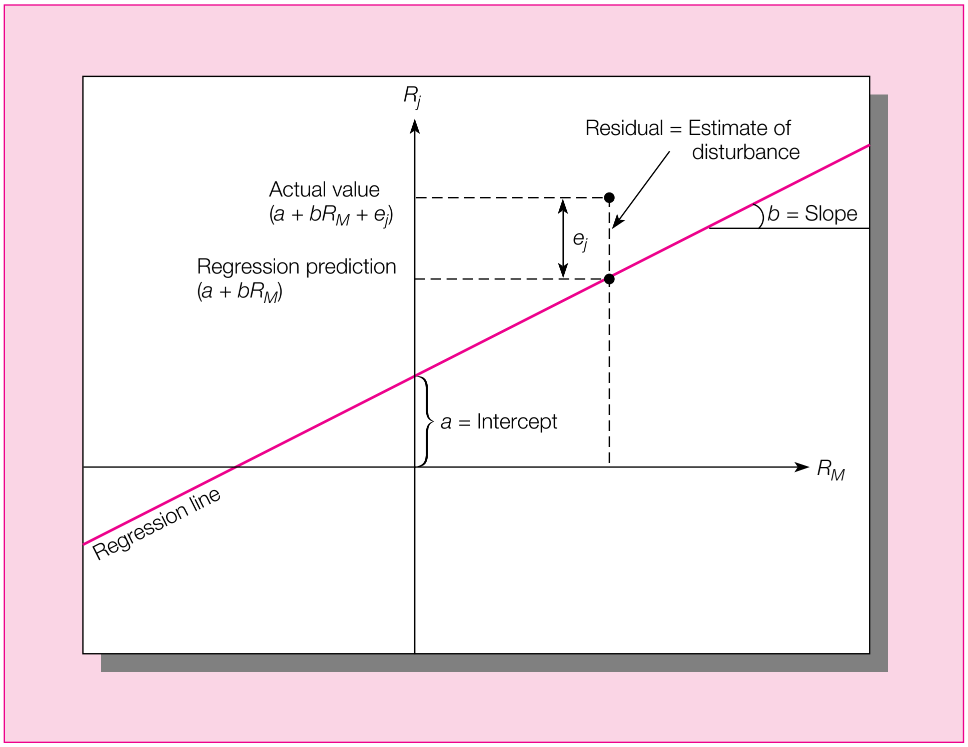


Figure A.7 **Simple regression estimates and residuals. The intercept and slope are chosen so as to minimize the sum of the squared deviations from the regression line.**

variance of R_j are also determined by it. Using the expectation of the expression in equation A.21, we get

$$E(R_j) = a + bE(R_M) \tag{A.22}$$

The constant a has no effect on the variance of R_j. Because the variables r_M and e_j are uncorrelated, the variance of the sum, $bR_M + e$, is the sum of the variances. Accounting for the parameter b multiplying R_M, the variance of R_j will be

$$\sigma_j^2 = b^2\sigma_M^2 + \sigma_e^2 \tag{A.23}$$

Equation A.23 tells us that the contribution of the variance of R_M to that of R_j depends on the regression (slope) coefficient b. The term $(b\sigma_M)^2$ is called the *explained variance.* The variance of the disturbance makes up the *unexplained variance.*

The covariance between R_j and R_M is also given by the regression equation. Setting up the expression, we have

$$\begin{aligned} \text{Cov}(R_j, R_M) &= \text{Cov}(a + bR_M + e, R_M) \\ &= \text{Cov}(bR_M, R_M) = b\text{Cov}(R_M, R_M) = b\sigma_M^2 \end{aligned} \tag{A.24}$$

The intercept, a, is dropped because a constant added to a random variable does not affect the covariance with any other variable. The disturbance term, e, is dropped because it is, by assumption, uncorrelated with the market return.

Equation A.24 shows that the slope coefficient of the regression, b, is equal to

$$b = \frac{\text{Cov}(R_j, R_M)}{\sigma_M^2}$$

The slope thereby measures the co-movements of j and M as a fraction of the movement of the driving force, the explanatory variable M.

One way to measure the explanatory power of the regression is by the fraction of the variance of R_j that it explains. This fraction is called the *coefficient of determination* and denoted by ρ^2.

$$\rho^2_{jM} = \frac{b^2\sigma^2_M}{\sigma^2_j} = \frac{b^2\sigma^2_M}{b^2_M\sigma^2_M + \sigma^2_e} \tag{A.25}$$

Note that the unexplained variance, $\sigma^2{}_e$, has to make up the difference between the coefficient of determination and 1.0. Therefore, another way to represent the coefficient of determination is by

$$\rho^2_{jM} = 1 - \frac{\sigma^2_e}{\sigma^2_j}$$

Some algebra shows that the coefficient of determination is the square of the correlation coefficient. Finally, squaring the correlation coefficient tells us what proportion of the variance of the dependent variable is explained by the independent (the explanatory) variable.

Estimation of the regression coefficients a and b is based on minimizing the sum of the square deviation of the observations from the estimated regression line (see Figure A.7). Your calculator, as well as any spreadsheet program, can compute regression estimates.

The CFA 1986 examination for Level I included this question:

Question.

Pension plan sponsors place a great deal of emphasis on universe rankings when evaluating money managers. In fact, it appears that sponsors assume implicitly that managers who rank in the top quartile of a representative sample of peer managers are more likely to generate superior relative performance in the future than managers who rank in the bottom quartile.

The validity of this assumption can be tested by regressing percentile rankings of managers in one period on their percentile rankings from the prior period.

1. Given that the implicit assumption of plan sponsors is true to the extent that there is perfect correlation in percentile rankings from one period to the next, list the numerical values you would expect to observe for the slope of the regression, and the R-squared of the regression.
2. Given that there is no correlation in percentile rankings from period to period, list the numerical values you would expect to observe for the intercept of the regression, the slope of the regression, and the R-squared of the regression.
3. Upon performing such a regression, you observe an intercept of .51, a slope of −.05, and an R-squared of .01. Based on this regression, state your best estimate of a manager's percentile ranking next period if his percentile ranking this period were .15.
4. Some pension plan sponsors have agreed that a good practice is to terminate managers who are in the top quartile and to hire those who are in the bottom quartile. State what those who advocate such a practice expect implicitly about the correlation and slope from a regression of the managers' subsequent ranking on their current ranking.

Answer.

1. Intercept = 0
   Slope = 1
   R-squared = 1
2. Intercept = .50
   Slope = 0.0
   R-squared = 0.0

3. 50th percentile, derived as follows:

$y = a + bx$
$= .51 - 0.05(.15)$
$= .51 - .0075$
$= .5025$

Given the very low R-squared, it would be difficult to estimate what the manager's rank would be.

4. Sponsors who advocate firing top-performance managers and hiring the poorest implicitly expect that both the correlation and slope would be significantly negative.

Multiple Regression Analysis

In many cases, theory suggests that a number of independent, explanatory variables drive a dependent variable. This concept becomes clear enough when demonstrated by a two-variable case. A real estate analyst offers the following regression equation to explain the return on a nationally diversified real estate portfolio:

$$\text{RE}_t = a + b_1\text{RE}_{t-1} + b_2\text{NVR}_t + e_t \quad \text{(A.26)}$$

The dependent variable is the period t real estate portfolio, RE_t. The model specifies that the explained part of that return is driven by two independent variables. The first is the previous period return, RE_{t-1}, representing persistence of momentum. The second explanatory variable is the current national vacancy rate, NVR_t.

As in the simple regression, a is the intercept, representing the value that RE is expected to take when the explanatory variables are zero. The (slope) regression coefficients, b_1 and b_2, represent the *marginal* effect of the explanatory variables.

The coefficient of determination is defined exactly as before. The ratio of the variance of the disturbance, e, to the total variance of RE is 1.0 *minus* the coefficient of determination. The regression coefficients are estimated here, too, by finding coefficients that minimize the sum of squared deviations of the observations from the prediction of the regression.

A.4 HYPOTHESIS TESTING

The central hypothesis of investment theory is that nondiversifiable (systematic) risk is rewarded by a higher *expected* return. But do the data support the theory? Consider the data on the excess return on stocks in Table A.3. The estimate of the expected excess return (the sample average) is 8.57%. This appears to be a hefty risk premium, but so is the risk—the estimate of the standard deviation for the same sample is 20.9%. Could it be that the positive average is just the luck of the draw? Hypothesis testing supplies probabilistic answers to such concerns.

The first step in hypothesis testing is to state the claim that is to be tested. This is called the *null hypothesis* (or simply the *null*), denoted by H_0. Against the null, an alternative claim (hypothesis) is stated, which is denoted by H_1. The objective of hypothesis testing is to decide whether to reject the null in favor of the alternative while identifying the probabilities of the possible errors in the determination.

A hypothesis is *specified* if it assigns a value to a variable. A claim that the risk premium on stocks is zero is one example of a specified hypothesis. Often, however, a hypothesis is general. A claim that the risk premium on stocks is not zero would be a completely general alternative against the specified hypothesis that the risk premium is zero. It amounts to "anything but the null." The alternative that the risk premium is *positive,* although not completely general, is still unspecified. Although it is sometimes desirable to test two unspecified hypotheses (e.g., the claim that the risk premium is zero or negative, against the claim that it is positive), unspecified hypotheses complicate the task of determining the probabilities of errors in judgment.

What are the possible errors? There are two, called Type I and Type II errors. Type I is the event that we will *reject* the null when it is *true.* The probability of Type I error is called the *significance level.* Type II is the event that we will *accept* the null when it is *false.*

Suppose we set a criterion for acceptance of H_0 that is so lax that we know for certain we will accept the null. In doing so we will drive the significance level to zero (which is good). If we will never reject the null, we will also never reject it when it is true. At the same time the probability of Type II error will become 1 (which is bad). If we will accept the null for certain, we must also do so when it is false.

The reverse is to set a criterion for acceptance of the null that is so stringent that we know for certain that we will reject it. This drives the probability of Type II error to zero (which is good). By never accepting the null, we avoid accepting it when it is false. Now, however, the significance level will go to 1 (which is bad). If we always reject the null, we will reject it even when it is true.

To compromise between the two evils, hypothesis testing fixes the significance level; that is, it limits the probability of Type I error. Then, subject to this present constraint, the ideal test will minimize the probability of Type II error. If we *avoid* Type II error (accepting the null when it is false) we actually *reject* the null when it is indeed *false.* The probability of doing so is one minus the probability of Type II error, which is called the *power of the test.* Minimizing the probability of Type II error maximizes the power of the test.

Testing the claim that stocks earn a risk premium, we set the hypotheses as

$$H_0{:}E(R) = 0 \qquad \text{The expected excess return is zero}$$
$$H_1{:}E(R) > 0 \qquad \text{The expected excess return is positive}$$

H_1 is an *unspecified alternative.* When a null is tested against a completely general alternative, it is called a *two-tailed test* because you may reject the null in favor of both greater or smaller values.

When both hypotheses are unspecified, the test is difficult because the calculation of the probabilities of Type I and II errors is complicated. Usually, at least one hypothesis is simple (specified) and set as the null. In that case it is relatively easy to calculate the significance level of the test. Calculating the power of the test that assumes the *unspecified* alternative is true remains complicated; often it is left unsolved.

As we will show, setting the hypothesis that we wish to reject, $E(R) = 0$ as the null (the "straw man"), makes it harder to accept the alternative that we favor, our theoretical bias, which is appropriate.

In testing $E(R) = 0$, suppose we fix the significance level at 5%. This means that we will reject the null (and accept that there is a positive premium) *only* when the data suggest that the probability the null is true is 5% or less. To do so, we must find a critical value, denoted z_α (or critical values in the case of two-tailed tests), that corresponds to $\alpha = .05$, which will create two regions, an acceptance region and a rejection region. Look at Figure A.8 as an illustration.

If the sample average is to the right of the critical value (in the rejection region), the null is rejected; otherwise, it is accepted. In the latter case it is too likely (i.e., the probability is greater than 5%) that the sample average is positive simply because of sampling error. If the sample average is greater than the critical value, we will reject the null in favor of the

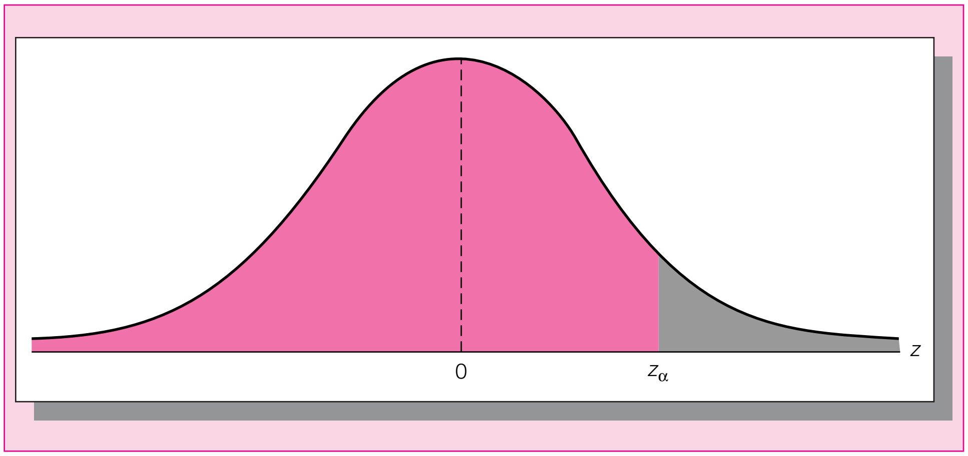


Figure A.8
Under the null hypothesis, the sample average excess return should be distributed around zero. If the actual average exceeds z_α, we conclude that the null hypothesis is false.

alternative. The probability that the positive value of the sample average results from sampling error is 5% or less.

If the alternative is one-sided (one-tailed), as in our case, the acceptance region covers the entire area from minus infinity to a positive value, above which lies 5% of the distribution (see Figure A.8). The critical value is z_α in Figure A.8. When the alternative is two-tailed, the area of 5% lies at both extremes of the distribution and is equally divided between them, 2.5% on each side. A two-tailed test is more stringent (it is harder to reject the null). In a one-tailed test the fact that our theory predicts the direction in which the average will deviate from the value under the null is weighted in favor of the alternative. The upshot is that for a significance level of 5%, with a one-tailed test, we use a confidence interval of $\alpha = .05$, instead of $\alpha/2 = .025$ as with a two-tailed test.

Hypothesis testing requires assessment of the probabilities of the test statistics, such as the sample average and variance. Therefore, it calls for some assumption about the probability distribution of the underlying variable. Such an assumption becomes an integral part of the null hypothesis, often an implicit one.

In this case we assume that the stock portfolio excess return is normally distributed. The distribution of the test statistic is derived from its mathematical definition and the assumption of the underlying distribution for the random variable. In our case the test statistic is the sample average.

The sample average is obtained by summing all observations ($T = 68$) and then multiplying by $1/T = 1/68$. Each observation is a random variable, drawn independently from the same underlying distribution, with an unknown expectation μ, and standard deviation σ. The expectation of the sum of all observations is the sum of the T expectations (all equal to μ) divided by T, therefore equal to the population expectation. The result is 8.57%, which is equal to the true expectation *plus* sampling errors. Under the null hypothesis, the expectation is zero, and the entire 8.57% constitutes sampling error.

To calculate the variance of the sample average, recall that we assumed that all observations were independent, or uncorrelated. Hence the variance of the sum is the sum of the variances, that is, T times the population variance. However, we also transform the sum, multiplying it by $1/T$; therefore, we have to divide the variance of the sum $T\sigma^2$ by T^2. We end up with the variance of the sample average as the population variance divided by T. The standard deviation of the sample average, which is called the *standard error,* is

$$\sigma(\text{average}) = \left(\frac{1}{T^2}\Sigma\sigma^2\right)^{1/2} = \left(\frac{1}{T^2}T\sigma^2\right)^{1/2} = \frac{\sigma}{\sqrt{T}} = \frac{.2090}{\sqrt{68}} = .0253 \quad \text{(A.27)}$$

Our test statistic has a standard error of 2.53%. It makes sense that the larger the number of observations, the *smaller* the *standard error* of the estimate of the expectation. However, note that it is the variance that goes down by the proportion $T = 68$. The standard error goes down by a much smaller proportion, $\sqrt{T} = 8.25$.

Now that we have the sample mean, 8.57%, its standard deviation, 2.53%, and know that the distribution under the null is normal, we are ready to perform the test. We want to determine whether 8.57% is significantly positive. We achieve this by standardizing our statistic, which means that we subtract from it its expected value under the null hypothesis and divide by its standard deviation. This standardized statistic can now be compared to z values from the standard normal tables. We ask whether

$$\frac{\bar{R} - E(R)}{\sigma} > z_\alpha$$

We would be finished except for another caveat. The assumption of normality is all right in that the test statistic is a weighted sum of normals (according to our assumption about returns). Therefore, it is also normally distributed. However, the analysis also requires that we *know* the variance. Here we are using a sample variance that is only an *estimate* of the true variance.

The solution to this problem turns out to be quite simple. The normal distribution is replaced with *Student-t* (or *t,* for short) *distribution.* Like the normal, the t distribution is symmetric. It depends on degrees of freedom, that is, the number of observations less one. Thus, here we replace z_α with $t_{\alpha,T-1}$.

The test is then

$$\frac{\bar{R} - E(R)}{\sigma} > t_{\alpha,T-1}$$

When we substitute in sample results, the left-hand side is a standardized statistic and the right-hand side is a t-value derived from t tables for $\alpha = .05$ and $T - 1 = 68 - 1 = 67$. We ask whether the inequality holds. If it does, we *reject* the null hypothesis with a 5% significance level; if it does not, we *cannot reject* the null hypothesis. (In this example, $t_{.05,67} = 1.67$.) Proceeding, we find that

$$\frac{.0857 - 0}{.0253} = 3.39 > 1.67$$

In our sample the inequality holds, and we reject the null hypothesis in favor of the alternative that the risk premium is positive.

A repeat of the test of this hypothesis for the 1965-to-1987 period may make a skeptic out of you. For that period the sample average is 3.12%, the sample standard deviation is 15.57%, and there are $23 - 1 = 22$ degrees of freedom. Does that give you second thoughts?

The *t*-Test of Regression Coefficients

Suppose that we apply the simple regression model (equation A.21) to the relationship between the long-term government bond portfolio and the stock market index, using the sample in Table A.3. The estimation result (% per year) is

$$a = .9913, \qquad b = .0729, \qquad R\text{-squared} = .0321$$

We interpret these coefficients as follows. For periods when the excess return on the market index is zero, we expect the bonds to earn an excess return of 99.13 basis points. This is the role of the intercept. As for the slope, for each percentage return of the stock portfolio in any year, the bond portfolio is expected to earn, *additionally,* 7.29 basis points. With the average equity risk premium for the sample period of 8.57%, the sample average for bonds is .9913 + (.0729 × 8.57) = 1.62%. From the squared correlation coefficient you know that the variation in stocks explains 3.21% of the variation in bonds.

Can we rely on these statistics? One way to find out is to set up a hypothesis test, presented here for the regression coefficient *b*.

H_0: $b = 0$ The regression slope coefficient is zero, meaning that changes in the independent variable do not explain changes in the dependent variable

H_1: $b > 0$ The dependent variable is sensitive to changes in the independent variable (with a *positive* covariance)

Any decent regression software supplies the statistics to test this hypothesis. The regression customarily assumes that the dependent variable and the disturbance are normally distributed, with an unknown variance that is estimated from the sample. Thus the regression coefficient *b* is normally distributed. Because once again the null is that $b = 0$, all we need is an estimate of the standard error of this statistic.

The estimated standard error of the regression coefficient is computed from the estimated standard deviation of the disturbance and the standard deviation of the explanatory variable. For the regression at hand, that estimate is $s(b) = .0493$. Just as in the previous exercise, the critical value of the test is

$$s(b)t_{\alpha,T-1}$$

Compare this value to the value of the estimated coefficient *b*. We will reject the null in favor of $b > 0$ if

$$b > s(b)t_{\alpha,T-1}$$

which, because the standard deviation $s(b)$ is positive, is equivalent to the following condition:

$$\frac{b}{s(b)} > t_{\alpha,T-1}$$

The *t*-test reports the ratio of the estimated coefficient to its estimated standard deviation. Armed with this *t-ratio,* the number of observations, *T,* and a table of the *Student-t* distribution, you can perform the test at the desired significance level.

The *t*-ratio for our example is .0729/.0493 = 1.4787. The *t*-table for 68 degrees of freedom shows we cannot reject the null at a significance level of 5%, for which the critical value is 1.67.

A question from the CFA 1987 exam calls for understanding of regression analysis and hypothesis testing.

Question.

An academic suggests to you that the returns on common stocks differ based on a company's market capitalization, its historical earnings growth, the stock's current yield, and whether or not the company's employees are unionized. You are skeptical that there are any attributes other than market exposure as measured by beta that explain differences in returns across a sample of securities.

Nonetheless, you decide to test whether or not these other attributes account for the differences in returns. You select the S&P 500 stocks as your sample, and regress their returns each month for the past five years against the company's market capitalization at the beginning of each month, the company's growth in earnings throughout the previous 12 months, the prior year's dividend divided by the stock price at the beginning of each month, and a dummy variable that has a value of 1 if employees are unionized and 0 if not.

1. The average *R*-squared from the regression is .15, and it varies very little from month to month. Discuss the significance of this result.
2. You note that all of the coefficients of the attributes have *t*-statistics greater than 2 in most of the months in which the regressions were run. Discuss the significance of these attributes in terms of explaining differences in common stock returns.
3. You observe in most of the regressions that the coefficient of the dummy variable is −.14 and the *t*-statistic is −4.74. Discuss the implication of the coefficient regarding the relationship between unionization and the return on a company's common stock.

Answer.

1. Differences in the attributes' values together explain about 15% of the differences in return among the stocks in the S&P 500 index. The remaining unexplained differences in return may be attributable to omitted attributes, industry affiliations, or stock-specific factors. This information by itself is not sufficient to form any qualitative conclusions. The fact that *R*-squared varied little from month to month implies that the relationship is stable and the observed results are not sample specific.
2. Given a *t*-statistic greater than 2 in most of the months, one would regard the attribute coefficients as statistically significant. If the attribute coefficients were not significantly different from zero, one would expect *t*-statistics greater than 2 in fewer than 5% of the regressions for each attribute coefficient. Because the *t*-statistics are greater than 2 much more frequently, one should conclude that they are definitely significant in terms of explaining differences in stock returns.
3. Because the coefficient for the dummy variable representing unionization has persistently been negative and since it persistently has been statistically significant, one would conclude that disregarding all other factors, unionization lowers a company's common stock return. That is, everything else being equal, nonunionized companies will have higher returns than companies whose employees are unionized. Of course, one would want to test the model further to see if there are omitted variables of other problems that might account for this apparent relationship.

APPENDIX B

REFERENCES TO CFA QUESTIONS

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GLOSSARY

Abnormal return. Return on a stock beyond what would be predicted by market movements alone. Cumulative abnormal return (CAR) is the total abnormal return for the period surrounding an announcement or the release of information.

Accounting earnings. Earnings of a firm as reported on its income statement.

Acid test ratio. See quick ratio.

Active management. Attempts to achieve portfolio returns more than commensurate with risk, either by forecasting broad market trends or by identifying particular mispriced sectors of a market or securities in a market.

Active portfolio. In the context of the Treynor-Black model, the portfolio formed by mixing analyzed stocks of perceived nonzero alpha values. This portfolio is ultimately mixed with the passive market index portfolio.

Adjustable-rate mortgage. A mortgage whose interest rate varies according to some specified measure of the current market interest rate.

Adjusted forecast. A (micro or macro) forecast that has been adjusted for the imprecision of the forecast.

Agency problem. Conflicts of interest among stockholders, bondholders, and managers.

Alpha. The abnormal rate of return on a security in excess of what would be predicted by an equilibrium model like CAPM or APT.

American depository receipts (ADRs). Domestically traded securities representing claims to shares of foreign stocks.

American option, European option. An American option can be exercised before and up to its expiration date. Compare with a *European option,* which can be exercised only on the expiration date.

Announcement date. Date on which particular news concerning a given company is announced to the public. Used in *event studies,* which researchers use to evaluate the economic impact of events of interest.

Appraisal ratio. The signal-to-noise ratio of an analyst's forecasts. The ratio of alpha to residual standard deviation.

Arbitrage. A zero-risk, zero-net investment strategy that still generates profits.

Arbitrage pricing theory. An asset pricing theory that is derived from a factor model, using diversification and arbitrage arguments. The theory describes the relationship between expected returns on securities, given that there are no opportunities to create wealth through risk-free arbitrage investments.

Asked price. The price at which a dealer will sell a security.

Asset allocation decision. Choosing among broad asset classes such as stocks versus bonds.

Asset turnover (ATO). The annual sales generated by each dollar of assets (sales/assets).

Auction market. A market where all traders in a good meet at one place to buy or sell an asset. The NYSE is an example.

Average collection period, or days' receivables. The ratio of accounts receivable to sales, or the total amount of credit extended per dollar of daily sales (average AR/sales × 365).

Balance sheet. An accounting statement of a firm's financial position at a specified time.

Bank discount yield. An annualized interest rate assuming simple interest, a 360-day year, and using the face value of the security rather than purchase price to compute return per dollar invested.

Banker's acceptance. A money market asset consisting of an order to a bank by a customer to pay a sum of money at a future date.

Basis. The difference between the futures price and the spot price.

Basis risk. Risk attributable to uncertain movements in the spread between a futures price and a spot price.

Benchmark error. Use of an inappropriate proxy for the true market portfolio.

Beta. The measure of the systematic risk of a security. The tendency of a security's returns to respond to swings in the broad market.

Bid–asked spread. The difference between a dealer's bid and asked price.

Bid price. The price at which a dealer is willing to purchase a security.

Binomial model. An option valuation model predicated on the assumption that stock prices can move to only two values over any short time period.

Black-Scholes formula. An equation to value a call option that uses the stock price, the exercise price, the risk-free interest rate, the time to maturity, and the standard deviation of the stock return.

Block house. Brokerage firms that help to find potential buyers or sellers of large block trades.

Block sale. A transaction of more than 10,000 shares of stock.

Block transactions. Large transactions in which at least 10,000 shares of stock are bought or sold. Brokers or "block houses" often search directly for other large traders rather than bringing the trade to the stock exchange.

Bogey. The return an investment manager is compared to for performance evaluation.

Bond. A security issued by a borrower that obligates the issuer to make specified payments to the holder over a specific period. A *coupon bond* obligates the issuer to make interest payments called coupon payments over the life of the bond, then to repay the *face value* at maturity.

Bond equivalent yield. Bond yield calculated on an annual percentage rate method. Differs from effective annual yield.

Book value. An accounting measure describing the net worth of common equity according to a firm's balance sheet.

Brokered market. A market where an intermediary (a broker) offers search services to buyers and sellers.

Budget deficit. The amount by which government spending exceeds government revenues.

Bull CD, bear CD. A *bull CD* pays its holder a specified percentage of the increase in return on a specified market index while guaranteeing a minimum rate of return. A *bear CD* pays the holder a fraction of any fall in a given market index.

Bullish, bearish. Words used to describe investor attitudes. *Bullish* means optimistic; *bearish* means pessimistic. Also used in bull market and bear market.

Bundling, unbundling. A trend allowing creation of securities either by combining primitive and derivative securities into one composite hybrid or by separating returns on an asset into classes.

Business cycle. Repetitive cycles of recession and recovery.

Callable bond. A bond that the issuer may repurchase at a given price in some specified period.

Call option. The right to buy an asset at a specified exercise price on or before a specified expiration date.

Call protection. An initial period during which a callable bond may not be called.

Capital allocation decision. Allocation of invested funds between risk-free assets versus the risky portfolio.

Capital allocation line (CAL). A graph showing all feasible risk–return combinations of a risky and risk-free asset.

Capital gains. The amount by which the sale price of a security exceeds the purchase price.

Capital market line (CML). A capital allocation line provided by the market index portfolio.

Capital markets. Includes longer-term, relatively riskier securities.

Cash/bond selection. Asset allocation in which the choice is between short-term cash equivalents and longer-term bonds.

Cash delivery. The provision of some futures contracts that requires not delivery of the underlying assets (as in agricultural futures) but settlement according to the cash value of the asset.

Cash equivalents. Short-term money-market securities.

Cash flow matching. A form of immunization, matching cash flows from a bond portfolio with an obligation.

Certainty equivalent. The certain return providing the same utility as a risky portfolio.

Certificate of deposit. A bank time deposit.

Clearinghouse. Established by exchanges to facilitate transfer of securities resulting from trades. For options and futures contracts, the clearinghouse may interpose itself as a middleman between two traders.

Closed-end (mutual) fund. A fund whose shares are traded through brokers at market prices; the fund will not redeem shares at their net asset value. The market price of the fund can differ from the net asset value.

Collateral. A specific asset pledged against possible default on a bond. *Mortgage bonds* are backed by claims on property. *Collateral trust bonds* are backed by claims on other securities. *Equipment obligation bonds* are backed by claims on equipment.

Collateralized mortgage obligation (CMO). A mortgage pass-through security that partitions cash flows from underlying mortgages into classes called *tranches,* that receive principal payments according to stipulated rules.

Commercial paper. Short-term unsecured debt issued by large corporations.

Commission broker. A broker on the floor of the exchange who executes orders for other members.

Common stock. Equities, or equity securities, issued as ownership shares in a publicly held corporation. Shareholders have voting rights and may receive dividends based on their proportionate ownership.

Comparison universe. The collection of money managers of similar investment style used for assessing relative performance of a portfolio manager.

Complete portfolio. The entire portfolio, including risky and risk-free assets.

Constant growth model. A form of the dividend discount model that assumes dividends will grow at a constant rate.

Contango theory. Holds that the futures price must exceed the expected future spot price.

Contingent claim. Claim whose value is directly dependent on or is contingent on the value of some underlying assets.

Contingent immunization. A mixed passive-active strategy that immunizes a portfolio if necessary to guarantee a minimum acceptable return but otherwise allows active management.

Convergence property. The convergence of futures prices and spot prices at the maturity of the futures contract.

Convertible bond. A bond with an option allowing the bondholder to exchange the bond for a specified number of shares of common stock in the firm. A *conversion ratio* specifies the number of shares. The *market conversion price* is the current value of the shares for which the bond may be exchanged. The *conversion premium* is the excess of the bond's value over the conversion price.

Corporate bonds. Long-term debt issued by private corporations typically paying semiannual coupons and returning the face value of the bond at maturity.

Correlation coefficient. A statistic in which the covariance is scaled to a value between minus one (perfect negative correlation) and plus one (perfect positive correlation).

Cost-of-carry relationship. See spot-futures parity theorem.

Country selection. A type of active international management that measures the contribution to performance attributable to investing in the better-performing stock markets of the world.

Coupon rate. A bond's interest payments per dollar of par value.

Covariance. A measure of the degree to which returns on two risky assets move in tandem. A positive covariance means that asset returns move together. A negative covariance means they vary inversely.

Covered call. A combination of selling a call on a stock together with buying the stock.

Covered interest arbitrage relationship. See interest rate parity theorem.

Credit enhancement. Purchase of the financial guarantee of a large insurance company to raise funds.

Cross hedge. Hedging a position in one asset using futures on another commodity.

Cross holdings. One corporation holds shares in another firm.

Cumulative abnormal return. See abnormal return.

Currency selection. Asset allocation in which the investor chooses among investments denominated in different currencies.

Current account. The difference between imports and exports, including merchandise, services, and transfers such as foreign aid.

Current ratio. A ratio representing the ability of the firm to pay off its current liabilities by liquidating current assets (current assets/current liabilities).

Current yield. A bond's annual coupon payment divided by its price. Differs from yield to maturity.

Day order. A buy order or a sell order expiring at the close of the trading day.

Days' receivables. See average collection period.

Dealer market. A market where traders specializing in particular commodities buy and sell assets for their own accounts. The OTC market is an example.

Debenture or unsecured bond. A bond not backed by specific collateral.

Dedication strategy. Refers to multiperiod cash flow matching.

Default premium. A differential in promised yield that compensates the investor for the risk inherent in purchasing a corporate bond that entails some risk of default.

Deferred annuities. Tax-advantaged life insurance product. Deferred annuities offer deferral of taxes with the option of withdrawing one's funds in the form of a life annuity.

Defined benefit plans. Pension plans in which retirement benefits are set according to a fixed formula.

Defined contribution plans. Pension plans in which the employer is committed to making contributions according to a fixed formula.

Delta (of option). See hedge ratio.

Delta neutral. The value of the portfolio is not affected by changes in the value of the asset on which the options are written.

Demand shock. An event that affects the demand for goods and services in the economy.

Derivative asset/contingent claim. Securities providing payoffs that depend on or are contingent on the values of other assets such as commodity prices, bond and stock prices, or market index values. Examples are futures and options.

Derivative security. See primitive security.

Detachable warrant. A warrant entitles the holder to buy a given number of shares of stock at a stipulated price. A detachable warrant is one that may be sold separately from the package it may have originally been issued with (usually a bond).

Direct search market. Buyers and sellers seek each other directly and transact directly.

Discounted dividend model (DDM). A formula to estimate the intrinsic value of a firm by figuring the present value of all expected future dividends.

Discount function. The discounted value of $1 as a function of time until payment.

Discretionary account. An account of a customer who gives a broker the authority to make buy and sell decisions on the customer's behalf.

Diversifiable risk. Risk attributable to firm-specific risk, or nonmarket risk. *Nondiversifiable* risk refers to systematic or market risk.

Diversification. Spreading a portfolio over many investments to avoid excessive exposure to any one source of risk.

Dividend payout ratio. Percentage of earnings paid out as dividends.

Dollar-weighted return. The internal rate of return on an investment.

Doubling option. A sinking fund provision that may allow repurchase of twice the required number of bonds at the sinking fund call price.

Dow theory. A technique that attempts to discern long- and short-term trends in stock market prices.

Dual funds. Funds in which income and capital shares on a portfolio of stocks are sold separately.

Duration. A measure of the average life of a bond, defined as the weighted average of the times until each payment is made, with weights proportional to the present value of the payment.

Dynamic hedging. Constant updating of hedge positions as market conditions change.

EAFE index. The European, Australian, Far East index, computed by Morgan Stanley, is a widely used index of non-U.S. stocks.

Earnings retention ratio. Plowback ratio.

Earnings yield. The ratio of earnings to price, E/P.

Economic earnings. The real flow of cash that a firm could pay out forever in the absence of any change in the firm's productive capacity.

Effective annual yield. Annualized interest rate on a security computed using compound interest techniques.

Efficient diversification. The organizing principle of modern portfolio theory, which maintains that any risk-averse investor will search for the highest expected return for any level of portfolio risk.

Efficient frontier. Graph representing a set of portfolios that maximize expected return at each level of portfolio risk.

Efficient market hypothesis. The prices of securities fully reflect available information. Investors buying securities in an efficient market should expect to obtain an equilibrium rate of return. Weak-form EMH asserts that stock prices already reflect all information contained in the history of past prices. The semistrong-form hypothesis asserts that stock prices already reflect all publicly available information. The strong-form hypothesis asserts that stock prices reflect all relevant information including insider information.

Elasticity (of an option). Percentage change in the value of an option accompanying a 1 percent change in the value of a stock.

Endowment funds. Organizations chartered to invest money for specific purposes.

Equivalent taxable yield. The pretax yield on a taxable bond providing an after-tax yield equal to the rate on a tax-exempt municipal bond.

Eurodollars. Dollar-denominated deposits at foreign banks or foreign branches of American banks.

European, Australian, Far East (EAFE) index. A widely used index of non-U.S. stocks computed by Morgan Stanley.

European option. A European option can be exercised only on the expiration date. Compare with an American option, which can be exercised before, up to, and including its expiration date.

Event study. Research methodology designed to measure the impact of an event of interest on stock returns.

Excess return. Rate of return in excess of the risk-free rate.

Exchange rate. Price of a unit of one country's currency in terms of another country's currency.

Exchange rate risk. The uncertainty in asset returns due to movements in the exchange rates between the dollar and foreign currencies.

Exchanges. National or regional auction markets providing a facility for members to trade securities. A seat is a membership on an exchange.

Exercise or strike price. Price set for calling (buying) an asset or putting (selling) an asset.

Expectations hypothesis (of interest rates). Theory that forward interest rates are unbiased estimates of expected future interest rates.

Expected return. The probability-weighted average of the possible outcomes.

Expected return–beta relationship. Implication of the CAPM that security risk premiums (expected excess returns) will be proportional to beta.

Face value. The maturity value of a bond.

Factor model. A way of decomposing the factors that influence a security's rate of return into common and firm-specific influences.

Factor portfolio. A well-diversified portfolio constructed to have a beta of 1.0 on one factor and a beta of zero on any other factor.

Fair game. An investment prospect that has a zero risk premium.

FIFO. The first-in first-out accounting method of inventory valuation.

Filter rule. A technical analysis technique stated as a rule for buying or selling stock according to past price movements.

Financial assets. Financial assets such as stocks and bonds are claims to the income generated by real assets or claims on income from the government.

Financial intermediary. An institution such as a bank, mutual fund, investment company, or insurance company that serves to connect the household and business sectors so households can invest and businesses can finance production.

Firm-specific risk. See diversifiable risk.

First-pass regression. A time series regression to estimate the betas of securities or portfolios.

Fiscal policy. The use of government spending and taxing for the specific purpose of stabilizing the economy.

Fixed annuities. Annuity contracts in which the insurance company pays a fixed dollar amount of money per period.

Fixed-charge coverage ratio. Ratio of earnings to all fixed cash obligations, including lease payments and sinking fund payments.

Fixed-income security. A security such as a bond that pays a specified cash flow over a specific period.

Flight to quality. Describes the tendency of investors to require larger default premiums on investments under uncertain economic conditions.

Floating-rate bond. A bond whose interest rate is reset periodically according to a specified market rate.

Floor broker. A member of the exchange who can execute orders for commission brokers.

Flower bond. Special Treasury bond (no longer issued) that may be used to settle federal estate taxes at par value under certain conditions.

Forced conversion. Use of a firm's call option on a callable convertible bond when the firm knows that bondholders will exercise their option to convert.

Foreign exchange market. An informal network of banks and brokers that allows customers to enter forward contracts to purchase or sell currencies in the future at a rate of exchange agreed upon now.

Foreign exchange swap. An agreement to exchange stipulated amounts of one currency for another at one or more future dates.

Forward contract. An agreement calling for future delivery of an asset at an agreed-upon price. Also see futures contract.

Forward interest rate. Rate of interest for a future period that would equate the total return of a long-term bond with that of a strategy of rolling over shorter-term bonds. The forward rate is inferred from the term structure.

Fourth market. Direct trading in exchange-listed securities between one investor and another without the benefit of a broker.

Fully diluted earnings per share. Earnings per share expressed as if all outstanding convertible securities and warrants have been exercised.

Fundamental analysis. Research to predict stock value that focuses on such determinants as earnings and dividends prospects, expectations for future interest rates, and risk evaluation of the firm.

Futures contract. Obliges traders to purchase or sell an asset at an agreed-upon price on a specified future date. The long position is held by the trader who commits to purchase. The short position is held by the trader who commits to sell. Futures differ from forward contracts in their standardization, exchange trading, margin requirements, and daily settling (marking to market).

Futures option. The right to enter a specified futures contract at a futures price equal to the stipulated exercise price.

Futures price. The price at which a futures trader commits to make or take delivery of the underlying asset.

Geometric average. The *n*th root of the product of *n* numbers. It is used to measure the compound rate of return over time.

Globalization. Tendency toward a worldwide investment environment, and the integration of national capital markets.

Gross domestic product (GDP). The market value of goods and services produced over time including the income of foreign corporations and foreign residents working in the United States, but excluding the income of U.S. residents and corporations overseas.

Guaranteed insurance contract. A contract promising a stated nominal rate of interest over some specific time period, usually several years.

Hedge ratio (for an option). The number of stocks required to hedge against the price risk of holding one option. Also called the option's delta.

Hedging. Investing in an asset to reduce the overall risk of a portfolio.

Hedging demands. Demands for securities to hedge particular sources of consumption risk, beyond the usual mean-variance diversification motivation.

Holding-period return. The rate of return over a given period.

Homogenous expectations. The assumption that all investors use the same expected returns and covariance matrix of security returns as inputs in security analysis.

Horizon analysis. Interest rate forecasting that uses a forecast yield curve to predict bond prices.

Immunization. A strategy that matches durations of assets and liabilities so as to make net worth unaffected by interest rate movements.

Implied volatility. The standard deviation of stock returns that is consistent with an option's market value.

Income beneficiary. One who receives income from a trust.

Income fund. A mutual fund providing for liberal current income from investments.

Income statement. A financial statement showing a firm's revenues and expenses during a specified period.

Indenture. The document defining the contract between the bond issuer and the bondholder.

Index arbitrage. An investment strategy that exploits divergences between actual futures prices and their theoretically correct parity values to make a profit.

Index fund. A mutual fund holding shares in proportion to their representation in a market index such as the S&P 500.

Index model. A model of stock returns using a market index such as the S&P 500 to represent common or systematic risk factors.

Index option. A call or put option based on a stock market index.

Indifference curve. A curve connecting all portfolios with the same utility according to their means and standard deviations.

Inflation. The rate at which the general level of prices for goods and services is rising.

Initial public offering. Stock issued to the public for the first time by a formerly privately owned company.

Input list. List of parameters such as expected returns, variances, and covariances necessary to determine the optimal risky portfolio.

Inside information. Nonpublic knowledge about a corporation possessed by corporate officers, major owners, or other individuals with privileged access to information about a firm.

Insider trading. Trading by officers, directors, major stockholders, or others who hold private inside information allowing them to benefit from buying or selling stock.

Insurance principle. The law of averages. The average outcome for many independent trials of an experiment will approach the expected value of the experiment.

Interest coverage ratio, or times interest earned. A financial leverage measure (EBIT divided by interest expense).

Interest rate. The number of dollars earned per dollar invested per period.

Interest rate parity theorem. The spot-futures exchange rate relationship that prevails in well-functioning markets.

Interest rate swaps. A method to manage interest rate risk where parties trade the cash flows corresponding to different securities without actually exchanging securities directly.

Intermarket spread swap. Switching from one segment of the bond market to another (from Treasuries to corporates, for example).

In the money. In the money describes an option whose exercise would produce profits. Out of the money describes an option where exercise would not be profitable.

Intrinsic value (of a firm). The present value of a firm's expected future net cash flows discounted by the required rate of return.

Intrinsic value of an option. Stock price minus exercise price, or the profit that could be attained by immediate exercise of an in-the-money option.

Investment bankers. Firms specializing in the sale of new securities to the public, typically by underwriting the issue.

Investment company. Firm managing funds for investors. An investment company may manage several mutual funds.

Investment-grade bond. Bond rated BBB and above or Baa and above. Lower-rated bonds are classified as speculative-grade or junk bonds.

Investment portfolio. Set of securities chosen by an investor.

Jensen's measure. The alpha of an investment.

Junk bond. See speculative-grade bond.

Law of one price. The rule stipulating that equivalent securities or bundles of securities must sell at equal prices to preclude arbitrage opportunities.

Leading economic indicators. Economic series that tend to risk or fall in advance of the rest of the economy.

Leakage. Release of information to some persons before official public announcement.

Leverage ratio. Ratio of debt to total capitalization of a firm.

LIFO. The last-in first-out accounting method of valuing inventories.

Limited liability. The fact that shareholders have no personal liability to the creditors of the corporation in the event of bankruptcy.

Limit order. An order specifying a price at which an investor is willing to buy or sell a security.

Liquidation value. Net amount that could be realized by selling the assets of a firm after paying the debt.

Liquidity. Liquidity refers to the speed and ease with which an asset can be converted to cash.

Liquidity preference theory. Theory that the forward rate exceeds expected future interest rates.

Liquidity premium. Forward rate minus expected future short interest rate.

Load fund. A mutual fund with a sales commission, or load.

London Interbank Offered Rate (LIBOR). Rate that most creditworthy banks charge one another for large loans of Eurodollars in the London market.

Long hedge. Protecting the future cost of a purchase by taking a long futures position to protect against changes in the price of the asset.

Maintenance, or variation, margin. An established value below which a trader's margin cannot fall. Reaching the maintenance margin triggers a margin call.

Margin. Describes securities purchased with money borrowed from a broker. Current maximum margin is 50 percent.

Market-book ratio. Market price of a share divided by book value per share.

Market capitalization rate. The market-consensus estimate of the appropriate discount rate for a firm's cash flows.

Market model. Another version of the index model that breaks down return uncertainty into systematic and nonsystematic components.

Market or systematic risk, firm-specific risk. Market risk is risk attributable to common macroeconomic factors. Firm-specific risk reflects risk peculiar to an individual firm that is independent of market risk.

Market order. A buy or sell order to be executed immediately at current market prices.

Market portfolio. The portfolio for which each security is held in proportion to its market value.

Market price of risk. A measure of the extra return, or risk premium, that investors demand to bear risk. The reward-to-risk ratio of the market portfolio.

Market segmentation or preferred habitat theory. The theory that long- and short-maturity bonds are traded in essentially distinct or segmented markets and that prices in one market do not affect those in the other.

Market timer. An investor who speculates on broad market moves rather than on specific securities.

Market timing. Asset allocation in which the investment in the market is increased if one forecasts that the market will outperform T-bills.

Market-value-weighted index. An index of a group of securities computed by calculating a weighted average of the returns of each security in the index, with weights proportional to outstanding market value.

Marking to market. Describes the daily settlement of obligations on futures positions.

Mean-variance analysis. Evaluation of risky prospects based on the expected value and variance of possible outcomes.

Mean-variance criterion. The selection of portfolios based on the means and variances of their returns. The choice of the higher expected return portfolio for a given level of variance or the lower variance portfolio for a given expected return.

Measurement error. Errors in measuring an explanatory variable in a regression that leads to biases in estimated parameters.

Membership or seat on an exchange. A limited number of exchange positions that enable the holder to trade for the holder's own accounts and charge clients for the execution of trades for their accounts.

Minimum-variance frontier. Graph of the lowest possible portfolio variance that is attainable for a given portfolio expected return.

Minimum-variance portfolio. The portfolio of risky assets with lowest variance.

Modern portfolio theory (MPT). Principles underlying analysis and evaluation of rational portfolio choices based on risk–return trade-offs and efficient diversification.

Monetary policy. Actions taken by the Board of Governors of the Federal Reserve System to influence the money supply or interest rates.

Money market. Includes short-term, highly liquid, and relatively low-risk debt instruments.

Mortality tables. Tables of probability that individuals of various ages will die within a year.

Mortgage-backed security. Ownership claim in a pool of mortgages or an obligation that is secured by such a pool. Also called a *pass-through,* because payments are passed along from the mortgage originator to the purchaser of the mortgage-backed security.

Multifactor CAPM. Generalization of the basic CAPM that accounts for extra-market hedging demands.

Municipal bonds. Tax-exempt bonds issued by state and local governments, generally to finance capital improvement projects. General obligation bonds are backed by the general taxing power of the issuer. Revenue bonds are backed by the proceeds from the project or agency they are issued to finance.

Mutual fund. A firm pooling and managing funds of investors.

Mutual fund theorem. A result associated with the CAPM, asserting that investors will choose to invest their entire risky portfolio in a market-index mutual fund.

Naked option writing. Writing an option without an offsetting stock position.

Nasdaq. The automated quotation system for the OTC market, showing current bid–asked prices for thousands of stocks.

Neglected-firm effect. That investments in stock of less well-known firms have generated abnormal returns.

Nominal interest rate. The interest rate in terms of nominal (not adjusted for purchasing power) dollars.

Nonsystematic risk. Nonmarket or firm-specific risk factors that can be eliminated by diversification. Also called unique risk or diversifiable risk. Systematic risk refers to risk factors common to the entire economy.

Normal backwardation theory. Holds that the futures price will be bid down to a level below the expected spot price.

Open-end (mutual) fund. A fund that issues or redeems its own shares at their net asset value (NAV).

Open (good-till-canceled) order. A buy or sell order remaining in force for up to six months unless canceled.

Open interest. The number of futures contracts outstanding.

Optimal risky portfolio. An investor's best combination of risky assets to be mixed with safe assets to form the complete portfolio.

Option elasticity. The percentage increase in an option's value given a 1 percent change in the value of the underlying security.

Original issue discount bond. A bond issued with a low coupon rate that sells at a discount from par value.

Out of the money. Out of the money describes an option where exercise would not be profitable. In the money describes an option where exercise would produce profits.

Over-the-counter market. An informal network of brokers and dealers who negotiate sales of securities (not a formal exchange).

Par value. The face value of the bond.

Passive investment strategy. See passive management.

Passive management. Buying a well-diversified portfolio to represent a broad-based market index without attempting to search out mispriced securities.

Passive portfolio. A market index portfolio.

Passive strategy. See passive management.

Pass-through security. Pools of loans (such as home mortgage loans) sold in one package. Owners of pass-throughs receive all principal and interest payments made by the borrowers.

Peak. The transition from the end of an expansion to the start of a contraction.

P/E effect. That portfolios of low P/E stocks have exhibited higher average risk-adjusted returns than high P/E stocks.

Personal trust. An interest in an asset held by a trustee for the benefit of another person.

Plowback ratio. The proportion of the firm's earnings that is reinvested in the business (and not paid out as dividends). The plowback ratio equals 1 minus the dividend payout ratio.

Political risk. Possibility of the expropriation of assets, changes in tax policy, restrictions on the exchange of foreign currency for domestic currency, or other changes in the business climate of a country.

Portfolio insurance. The practice of using options or dynamic hedge strategies to provide protection against investment losses while maintaining upside potential.

Portfolio management. Process of combining securities in a portfolio tailored to the investor's preferences and needs, monitoring that portfolio, and evaluating its performance.

Portfolio opportunity set. The expected return–standard deviation pairs of all portfolios that can be constructed from a given set of assets.

Preferred habitat theory. Holds that investors prefer specific maturity ranges but can be induced to switch if risk premiums are sufficient.

Preferred stock. Nonvoting shares in a corporation, paying a fixed or variable stream of dividends.

Premium. The purchase price of an option.

Price value of a basis point. The change in the value of a fixed-income asset resulting from a one basis point change in the asset's yield to maturity.

Price–earnings multiple. See price–earnings ratio.

Price–earnings ratio. The ratio of a stock's price to its earnings per share. Also referred to as the P/E multiple.

Primary market. New issues of securities are offered to the public here.

Primitive security, derivative security. A *primitive security* is an instrument such as a stock or bond for which payments depend only on the financial status of its issuer. A *derivative security* is created from the set of primitive securities to yield returns that depend on factors beyond the characteristics of the issuer and that may be related to prices of other assets.

Principal. The outstanding balance on a loan.

Profit margin. See return on sales.

Program trading. Coordinated buy orders and sell orders of entire portfolios, usually with the aid of computers, often to achieve index arbitrage objectives.

Prospectus. A final and approved registration statement including the price at which the security issue is offered.

Protective covenant. A provision specifying requirements of collateral, sinking fund, dividend policy, etc., designed to protect the interests of bondholders.

Protective put. Purchase of stock combined with a put option that guarantees minimum proceeds equal to the put's exercise price.

Proxy. An instrument empowering an agent to vote in the name of the shareholder.

Public offering, private placement. A *public offering* consists of bonds sold in the primary market to the general public; a *private placement* is sold directly to a limited number of institutional investors.

Pure yield pickup swap. Moving to higher yield bonds.

Put bond. A bond that the holder may choose either to exchange for par value at some date or to extend for a given number of years.

Put-call parity theorem. An equation representing the proper relationship between put and call prices. Violation of parity allows arbitrage opportunities.

Put option. The right to sell an asset at a specified exercise price on or before a specified expiration date.

Quick ratio. A measure of liquidity similar to the current ratio except for exclusion of inventories (cash plus receivables divided by current liabilities).

Random walk. Describes the notion that stock price changes are random and unpredictable.

Rate anticipation swap. A switch made in response to forecasts of interest rates.

Real assets, financial assets. *Real assets* are land, buildings, and equipment that are used to produce goods and services. *Financial assets* are claims such as securities to the income generated by real assets.

Real interest rate. The excess of the interest rate over the inflation rate. The growth rate of purchasing power derived from an investment.

Realized compound yield. Yield assuming that coupon payments are invested at the going market interest rate at the time of their receipt and rolled over until the bond matures.

Rebalancing. Realigning the proportions of assets in a portfolio as needed.

Registered bond. A bond whose issuer records ownership and interest payments. Differs from a bearer bond, which is traded without record of ownership and whose possession is its only evidence of ownership.

Registered trader. A member of the exchange who executes frequent trades for his or her own account.

Registration statement. Required to be filed with the SEC to describe the issue of a new security.

Regression equation. An equation that describes the average relationship between a dependent variable and a set of explanatory variables.

REIT. Real estate investment trust, which is similar to a closed-end mutual fund. REITs invest in real estate or loans secured by real estate and issue shares in such investments.

Remainderman. One who receives the principal of a trust when it is dissolved.

Replacement cost. Cost to replace a firm's assets. "Reproduction" cost.

Repurchase agreements (repos). Short-term, often overnight, sales of government securities with an agreement to repurchase the securities at a slightly higher price. A *reverse repo* is a purchase with an agreement to resell at a specified price on a future date.

Residual claim. Refers to the fact that shareholders are at the bottom of the list of claimants to assets of a corporation in the event of failure or bankruptcy.

Residuals. Parts of stock returns not explained by the explanatory variable (the market-index return). They measure the impact of firm-specific events during a particular period.

Resistance level. A price level above which it is supposedly difficult for a stock or stock index to rise.

Return on assets (ROA). A profitability ratio; earnings before interest and taxes divided by total assets.

Return on equity (ROE). An accounting ratio of net profits divided by equity.

Return on sales (ROS), or profit margin. The ratio of operating profits per dollar of sales (EBIT divided by sales).

Reversing trade. Entering the opposite side of a currently held futures position to close out the position.

Reward-to-volatility ratio. Ratio of excess return to portfolio standard deviation.

Riding the yield curve. Buying long-term bonds in anticipation of capital gains as yields fall with the declining maturity of the bonds.

Risk arbitrage. Speculation on perceived mispriced securities, usually in connection with merger and acquisition targets.

Risk-averse, risk-neutral, risk lover. A *risk-averse* investor will consider risky portfolios only if they provide compensation for risk via a risk premium. A *risk-neutral* investor finds the level of risk irrelevant and considers only the expected return of risk prospects. A *risk lover* is willing to accept lower expected returns on prospects with higher amounts of risk.

Risk-free asset. An asset with a certain rate of return; often taken to be short-term T-bills.

Risk-free rate. The interest rate that can be earned with certainty.

Risk lover. See risk-averse.

Risk-neutral. See risk-averse.

Risk premium. An expected return in excess of that on risk-free securities. The premium provides compensation for the risk of an investment.

Risk–return trade-off. If an investor is willing to take on risk, there is the reward of higher expected returns.

Risky asset. An asset with an uncertain rate of return.

Seasoned new issue. Stock issued by companies that already have stock on the market.

Secondary market. Already-existing securities are bought and sold on the exchanges or in the OTC market.

Second-pass regression. A cross-sectional regression of portfolio returns on betas. The estimated slope is the measurement of the reward for bearing systematic risk during the period.

Securitization. Pooling loans for various purposes into standardized securities backed by those loans, which can then be traded like any other security.

Security analysis. Determining correct value of a security in the marketplace.

Security characteristic line. A plot of the excess return on a security over the risk-free rate as a function of the excess return on the market.

Security market line. Graphical representation of the expected return–beta relationship of the CAPM.

Security selection. See security selection decision.

Security selection decision. Choosing the particular securities to include in a portfolio.

Semistrong-form EMH. See efficient market hypothesis.

Separation property. The property that portfolio choice can be separated into two independent tasks: (1) determination of the optimal risky portfolio, which is a purely technical problem, and (2) the personal choice of the best mix of the risky portfolio and the risk-free asset.

Serial bond issue. An issue of bonds with staggered maturity dates that spreads out the principal repayment burden over time.

Sharpe's measure. Reward-to-volatility ratio; ratio of portfolio excess return to standard deviation.

Shelf registration. Advance registration of securities with the SEC for sale up to two years following initial registration.

Short interest rate. A one-period interest rate.

Short position or hedge. Protecting the value of an asset held by taking a short position in a futures contract.

Short sale. The sale of shares not owned by the investor but borrowed through a broker and later repurchased to replace the loan. Profit is earned if the initial sale is at a higher price than the repurchase price.

Simple prospect. An investment opportunity where a certain initial wealth is placed at risk and only two outcomes are possible.

Single-country funds. Mutual funds that invest in securities of only one country.

Single-factor model. A model of security returns that acknowledges only one common factor. See factor model.

Single index model. A model of stock returns that decomposes influences on returns into a systematic factor, as measured by the return on a broad market index, and firm-specific factors.

Sinking fund. A procedure that allows for the repayment of principal at maturity by calling for the bond issuer to repurchase some proportion of the outstanding bonds either in the open market or at a special call price associated with the sinking fund provision.

Skip-day settlement. A convention for calculating yield that assumes a T-bill sale is not settled until two days after quotation of the T-bill price.

Small-firm effect. That investments in stocks of small firms appear to have earned abnormal returns.

Soft dollars. The value of research services that brokerage houses supply to investment managers "free of charge" in exchange for the investment managers' business.

Specialist. A trader who makes a market in the shares of one or more firms and who maintains a "fair and orderly market" by dealing personally in the stock.

Speculation. Undertaking a risky investment with the objective of earning a greater profit than an investment in a risk-free alternative (a risk premium).

Speculative-grade bond. Bond rated Ba or lower by Moody's, or BB or lower by Standard & Poor's, or an unrated bond.

Spot-futures parity theorem, or cost-of-carry relationship. Describes the theoretically correct relationship between spot and futures prices. Violation of the parity relationship gives rise to arbitrage opportunities.

Spot rate. The current interest rate appropriate for discounting a cash flow of some given maturity.

Spread (futures). Taking a long position in a futures contract of one maturity and a short position in a contract of different maturity, both on the same commodity.

Spread (options). A combination of two or more call options or put options on the same stock with differing exercise prices or times to expiration. A money spread refers to a spread with different exercise price; a time spread refers to differing expiration date.

Squeeze. The possibility that enough long positions hold their contracts to maturity that supplies of the commodity are not adequate to cover all contracts. A *short squeeze* describes the reverse: short positions threaten to deliver an expensive-to-store commodity.

Standard deviation. Square root of the variance.

Statement of cash flows. A financial statement showing a firm's cash receipts and cash payments during a specified period.

Stock exchanges. Secondary markets where already-issued securities are bought and sold by members.

Stock selection. An active portfolio management technique that focuses on advantageous selection of particular stocks rather than on broad asset allocation choices.

Stock split. Issue by a corporation of a given number of shares in exchange for the current number of shares held by stockholders. Splits may go in either direction, either increasing or decreasing the number of shares outstanding. A *reverse split* decreases the number outstanding.

Stop-loss order. A sell order to be executed if the price of the stock falls below a stipulated level.

Straddle. A combination of buying both a call and a put on the same asset, each with the same exercise price and expiration date. The purpose is to profit from expected volatility.

Straight bond. A bond with no option features such as callability or convertibility.

Street name. Describes securities held by a broker on behalf of a client but registered in the name of the firm.

Strike price. See exercise price.

Strip, strap. Variants of a straddle. A *strip* is two puts and one call on a stock; a *strap* is two calls and one put, both with the same exercise price and expiration date.

Stripped of coupons. Describes the practice of some investment banks that sell "synthetic" zero coupon bonds by marketing the rights to a single payment backed by a coupon-paying Treasury bond.

Strong-form EMH. See efficient market hypothesis.

Subordination clause. A provision in a bond indenture that restricts the issuer's future borrowing by subordinating the new leaders' claims on the firm to those of the existing bond holders. Claims of *subordinated* or *junior* debtholders are not paid until the prior debt is paid.

Substitution swap. Exchange of one bond for a bond with similar attributes but more attractively priced.

Supply shock. An event that influences production capacity and costs in the economy.

Support level. A price level below which it is supposedly difficult for a stock or stock index to fall.

Swaption. An option on a swap.

Systematic risk. Risk factors common to the whole economy, for example, nondiversifiable risk; see market risk.

Tax anticipation notes. Short-term municipal debt to raise funds to pay for expenses before actual collection of taxes.

Tax deferral option. The feature of the U.S. Internal Revenue Code that the capital gains tax on an asset is payable only when the gain is realized by selling the asset.

Tax-deferred retirement plans. Employer-sponsored and other plans that allow contributions and earnings to be made and accumulate tax free until they are paid out as benefits.

Tax-timing option. Describes the investor's ability to shift the realization of investment gains or losses and their tax implications from one period to another.

Tax swap. Swapping two similar bonds to receive a tax benefit.

Technical analysis. Research to identify mispriced securities that focuses on recurrent and predictable stock price patterns and on proxies for buy or sell pressure in the market.

Tender offer. An offer from an outside investor to shareholders of a company to purchase their shares at a stipulated price, usually substantially above the market price, so that the investor may amass enough shares to obtain control of the company.

Term insurance. Provides a death benefit only, no build-up of cash value.

Term premiums. Excess of the yields to maturity on long-term bonds over those of short-term bonds.

Term structure of interest rates. The pattern of interest rates appropriate for discounting cash flows of various maturities.

Third market. Trading of exchange-listed securities on the OTC market.

Times interest earned. See interest coverage ratio.

Time value (of an option). The part of the value of an option that is due to its positive time to expiration. Not to be confused with present value or the time value of money.

Time-weighted return. An average of the period-by-period holding-period returns of an investment.

Tobin's *q*. Ratio of market value of the firm to replacement cost.

Tranche. See collateralized mortgage obligation.

Treasury bill. Short-term, highly liquid government securities issued at a discount from the face value and returning the face amount at maturity.

Treasury bond or note. Debt obligations of the federal government that make semiannual coupon payments and are issued at or near par value.

Treynor's measure. Ratio of excess return to beta.

Triple-witching hour. The four times a year that the S&P 500 futures contract expires at the same time as the S&P 100 index option contract and option contracts on individual stocks.

Trough. The transition point between recession and recovery.

Unbundling. See bundling.

Underwriting, underwriting syndicate. Underwriters (investment bankers) purchase securities from the issuing company and resell them. Usually a syndicate of investment bankers is organized behind a lead firm.

Unemployment rate. The ratio of the number of people classified as unemployed to the total labor force.

Unique risk. See diversifiable risk.

Unit investment trust. Money invested in a portfolio whose composition is fixed for the life of the fund. Shares in a unit trust are called redeemable trust certificates, and they are sold at a premium above net asset value.

Universal life policy. An insurance policy that allows for a varying death benefit and premium level over the term of the policy, with an interest rate on the cash value that changes with market interest rates.

Uptick, or zero-plus tick. A trade resulting in a positive change in a stock price, or a trade at a constant price following a preceding price increase.

Utility. The measure of the welfare or satisfaction of an investor.

Utility value. The welfare a given investor assigns to an investment with a particular return and risk.

Variable annuities. Annuity contracts in which the insurance company pays a periodic amount linked to the investment performance of an underlying portfolio.

Variable life policy. An insurance policy that provides a fixed death benefit plus a cash value that can be invested in a variety of funds from which the policyholder can choose.

Variance. A measure of the dispersion of a random variable. Equals the expected value of the squared deviation from the mean.

Variation margin. See maintenance margin.

Volatility risk. The risk in the value of options portfolios due to unpredictable changes in the volatility of the underlying asset.

Warrant. An option issued by the firm to purchase shares of the firm's stock.

Weak-form EMH. See efficient market hypothesis.

Weekend effect. The common recurrent negative average return from Friday to Monday in the stock market.

Well-diversified portfolio. A portfolio spread out over many securities in such a way that the weight in any security is close to zero.

Whole-life insurance policy. Provides a death benefit and a kind of savings plan that builds up cash value for possible future withdrawal.

Workout period. Realignment period of a temporary misaligned yield relationship.

World investable wealth. The part of world wealth that is traded and is therefore accessible to investors.

Writing a call. Selling a call option.

Yield curve. A graph of yield to maturity as a function of time to maturity.

Yield to maturity. A measure of the average rate of return that will be earned on a bond if held to maturity.

Zero-beta portfolio. The minimum-variance portfolio uncorrelated with a chosen efficient portfolio.

Zero coupon bond. A bond paying no coupons that sells at a discount and provides payment of face value only at maturity.

Zero-investment portfolio. A portfolio of zero net value, established by buying and shorting component securities, usually in the context of an arbitrage strategy.

NAME INDEX

SUBJECT INDEX

A

C

F

I

Q

R

S

T

U

V