History of Mechanism and Machine Science 1

Marco Ceccarelli
Editor

Distinguished Figures in Mechanism and Machine Science

Their Contributions and Legacies
Part 1

Springer

DISTINGUISHED FIGURES IN MECHANISM AND MACHINE SCIENCE

HISTORY OF MECHANISM AND MACHINE SCIENCE

Volume 1

Series Editor

MARCO CECCARELLI

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This technical approach is an essential characteristic of the series. By discussing technical details and formulations and even reformulating those in terms of modern formalisms the possibility is created not only to track the historical technical developments but also to use past experiences in technical teaching and research today. In order to do so, the emphasis must be on technical aspects rather than a purely historical focus, although the latter has its place too.

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Distinguished Figures in Mechanism and Machine Science

Their Contributions and Legacies
Part 1

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Springer

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PREFACE BY THE SERIES EDITOR, PROFESSOR M. CECCARELLI

This book is part of a book series on the History of Mechanism and Machine Science (HMMS).

This series is novel in its concept of treating historical developments with a technical approach to illustrate the evolution of matters of Mechanical Engineering that are related specifically to mechanism and machine science. Thus, books in the series will describe historical developments by mainly looking at technical details with the aim to give interpretations and insights of past achievements. The attention to technical details is used not only to track the past by giving credit to past efforts and solutions but mainly to learn from the past approaches and procedures that can still be of current interest and use both for teaching and research.

The intended re-interpretation and re-formulation of past studies on machines and mechanisms requires technical expertise more than a merely historical perspective, therefore, the books of the series can be characterized by this emphasis on technical information, although historical development will not be overlooked.

Furthermore, the series will offer the possibility of publishing translations of works not originally written in English, and of reprinting works of historical interest that have gone out of print but are currently of interest again.

I believe that the works published in this series will be of interest to a wide range of readers from professionals to students, and from historians to technical researchers. They will all obtain both satisfaction from and motivation for their work by becoming aware of the historical framework which forms the background of their research.

I would like to take this opportunity to thank the authors and editors of these volumes very much for their efforts and the time they have spent in order to share their accumulated information and understanding of the use of past techniques in the history of mechanism and machine science.

Marco Ceccarelli (Chair of the Scientific Editorial Board)
Cassino, April 2007

PREFACE

This is the first volume of a series of edited books whose aim is to collect contributed papers in a framework that can serve as a dictionary of names of individuals who have made contributions to the discipline of MMS (Mechanism and Machine Science). This dictionary project has the peculiarity that, through descriptions of the ideas and work of these individuals, the papers will illustrate mainly technical developments in the historical evolution of the individual fields that today define the scope of MMS. Thus the core of each contribution will be a survey of biographical notes describing the efforts and experiences of these people.

Finding appropriate technical experts as authors for such papers and encouraging them to write them has been a challenge; it is a demanding and time-consuming effort to produce such in-depth articles that delve deeply into the historical background of their topics of expertise. This first volume of the dictionary project has been possible thanks to the invited authors who have enthusiastically shared the initiative and have spent time and effort in preparing papers that have the novel characteristics of survey and historical notes. The papers in this volume cover the wide field of the History of Mechanical Engineering with specific focus on MMS. I believe that a reader who takes advantage of the papers in this book, as well as future ones, will find further satisfaction and motivation for her or his work (historical or not).

I am grateful to the authors of the articles for their valuable contributions and for preparing their manuscripts on time. A special mention is due to the community of the IFToMM Permanent Commission for History of MMS and particularly to the past Chairperson (1990–1997) Professor Teun Koetsier (Vrije University in Amsterdam, The Netherlands) and the current Chairper-

son (2004–present) Professor Hong-Sen Yan (National Cheng Kung University in Tainan, Taiwan), both of whom supported my idea of the dictionary project, even during my chairmanship in the years 1998–2004. With their work in the IFToMM PC they have fostered both growth of interest in the field of History of MMS and wider participation by the science community at large.

I also wish to acknowledge the professional assistance of the staff of Springer and especially of Miss Anneke Pot and Miss Nathalie Jacobs, who have enthusiastically supported the project by offering their valuable advice through all stages of the organization and writing.

I am grateful to my wife Brunella, my daughters Elisa and Sofia, and my young son Raffaele. Without their patience and comprehension it would not have been possible for me to work on this book and the dictionary project.

Marco Ceccarelli (Editor)
Cassino, March 2007

ARCHIMEDES
(287–212 BC)

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Abstract. Archimedes (ca. 287–212 BC) was born in Syracuse, in the Greek colony of Sicily. He studied mathematics at the Museum in Alexandria. Archimedes systematized the design of simple machines and the study of their functions. He was probably the inventor of the compound pulley and developed a rigorous theory of levers and the kinematics of the screw. He is the founder of statics and of hydrostatics, and his machine designs fascinated subsequent writers. Archimedes was both a great engineer and a great inventor, but his books concentrated on applied mathematics and mechanics and rigorous mathematical proofs. Archimedes was also known as an outstanding astronomer; his observations of solstices were used by other astronomers of the era.

Biographical Notes

Archimedes (ca. 287–212 BC) was born in Syracuse, in the Greek colony of Sicily. His father was the astronomer and mathematician Phidias, and he was

Fig. 1. Archimedes portrait (Courtesy of the MacTutor History of Mathematics Archive run by the School of Mathematics and Statistics at the University of St Andrews, Fife, Scotland).

M. Ceccarelli (ed.), Distinguished Figures in Mechanism and Machine Science, 1–30.

related to King Hieron II (308–216 BC). The name of his father – Pheidias – suggests an origin, at least some generations back, in an artistic background (Stamatis, 1973).

Archimedes went to Alexandria about 250–240 BC to study in the Museum under Conon of Samos, a mathematician and astronomer (the custodian of the Alexandrian library after Euclid's death), Eratosthenes and other mathematicians who had been students of Euclid. The decline of Greek civilization coincides with the rise of Alexandria, founded in honor of Alexander the Great (356–323 BC) in the Nile Delta in Egypt. Alexandria was the greatest city of the ancient world, the capital of Egypt from its founding in 332 BC to AD 642, and became the most important scientific center in the world at that time and a centre of Hellenic scholarship and science. In its University, the Museum (meaning, the house of Muses, the protectresses of the Arts and Sciences) flourished a number of great mathematicians and engineers (Dimarogonas, 2001).

Geometry, in Archimedes time (and almost for the next 2000 years) had been accepted as being the Science of the space in which we live. Euclid was one of the most well known scholars who lived in Alexandria prior to Archimedes' arrival in the city. Euclid's "Elements," written about 300 BC, a comprehensive treatise on geometry, proportions, and the theory of numbers, is the most long-lived of all mathematical works. This elegant logical structure, formulated by Euclid based on a small number of self-evident axioms of the utmost simplicity, undoubtedly influenced the work of Archimedes (Sacheri, 1986). Archimedes later settled in his native city, Syracuse, where he devoted the rest of his life to the study of mathematics and the design of machines.

Archimedes was both a great engineer and a great inventor, although his books concentrated on applied mathematics and mechanics and rigorous mathematical proofs (Heath, 2002). He established the principles of plane and solid geometry. Some of Archimedes' accomplishments were with mathematical principles, such as his calculation of the first reliable value for π to calculate the areas and volumes of curved surfaces and circular forms. In this process, Archimedes used a method similar to integral calculus, which was not to be defined for almost another 2000 years by Newton (1642–1727) and Leibniz (1646–1716). He also created a system of exponential notation to allow him to prove that nothing exists that is too large to be measured (Bell, 1965; Dijksterhuis, 1987; Heath, 2002; Netz, 2004).

In addition to his mathematical studies, Archimedes invented the field of statics, enunciated the law of the lever, the law of equilibrium of fluids, and the law of buoyancy, and he contributed to knowledge concerning at least three of the five simple machines – winch, pulley, lever, wedge, and screw – known to antiquity. He discovered the concept of specific gravity and conducted experiments on buoyancy. He is credited with inventing the compound pulley, the catapult, and the Archimedes Screw, an auger-like device for raising water. He conducted important studies on gravity, balance, and equilibrium that grew out of his work with levers and demonstrated the power of mechanical advantage (Heath, 2001, Archimedes–Apanta (The Works) Vols 1–3, 2002).

"Give me a place to stand," Archimedes is said to have promised, "and I will move the world." Archimedes was referring to the law of the lever, which he had proved in his treatise, *Planes in Equilibrium* (Archimedes–Apanta Vol. 3, 2002). One can say that Archimedes moved the Earth – in principle – without standing anywhere: It is evident that Archimedes was very close to the theory of the force fields for the motion of celestial bodies. Apart from the lever theory, this argument gave rise to a new philosophical problem, that of the particular perspective from which we regard reality (Russell, 1912; Price, 1996).

Archimedes systematized the design of simple machines and the study of their functions and developed a rigorous theory of levers and the kinematics of the screw (Dimarogonas, 2001).

He designed and built *Syracusia* ("The Lady of Syracuse"), the largest ship of his times, 80 m long, 4,000 ton displacement, with three decks. The ship made only its maiden trip to Alexandria because it was too slow and there were no harbour facilities anywhere to handle her (Dimarogonas, 2001; Archimedes–Apanta Vol. 6, 2002).

Archimedes was also known as an outstanding astronomer; his observations of solstices were used by other astronomers of the era. As an astronomer, he developed an incredibly accurate self-moving model of the Sun, Moon, and constellations, which even showed eclipses in a time-lapse manner. The model used a system of screws and pulleys to move the globes at various speeds and on different courses (Archimedes–Apanta Vol. 6, 2002).

At the time of Archimedes, Syracuse was an independent Greek city-state with a 500-year history. The colony of Syracuse was established by Corinthians, led by Archias in 734 BC (Figure 2). The city grew and prospered, and in

Fig. 2. Syracuse, 37° 4′ N and 15° 18′ E, Carthagene and Rome shown in this medieval map.

the course of the 5th century BC the wealth, cultural development, political power and victorious wars against Athenians and Carthaginians ensured for a long time the dominance of Syracuse as the most powerful Greek city over the entire southwestern Mediterranean basin.

During Archimedes’ lifetime the first two of the three Punic Wars between the Romans and the Carthaginians were fought. The series of wars between Rome and Carthage were known to the Romans as the “Punic Wars” because of the Latin name for the Carthaginians: Punici, derived from Phoenici, referring to the Carthaginians’ Phoenician ancestry.

During the Second Punic War (218–201 BC) – the great World War of the classical Mediterranean, Syracuse allied itself with Carthage, and when the Roman general Marcellus began a siege on the city in 214 BC, Archimedes was called upon by King Hieron to aid in its defense and later worked as a military engineer for Syracuse (Plutarch AD 45–120).

The historical accounts of Archimedes’ war-faring inventions are vivid and possibly exaggerated. It is claimed that he devised catapult launchers that threw heavy beams and stones at the Roman ships, burning-glasses that reflected the sun’s rays and set ships on fire, and either invented or improved upon a device that would remain one of the most important forms of warfare technology for almost two millennia: the catapult. Plutarchos and Polybios (201–120 BC) describe giant mechanisms for lifting ships from the sea, ship-burning mirrors and a steam gun designed and built by Archimedes. The latter fascinated Leonardo da Vinci, however the validity of these stories is questionable.

Marcellus had given orders that when Syracuse was finally conquered, Archimedes, whose reputation was widely known, should be taken alive. When the Romans finally sacked the city in 212 BC, a soldier found Archimedes quietly etching equations in the sand, absorbed in a mathematical problem. Reportedly, Archimedes ordered the soldier not to disturb the figures in the sand. Enraged, the soldier not knowing who he was (and against the orders of Marcellus), killed him.

What we know of Archimedes' life comes from two radically different lines of tradition. One is his extant writings and the other is the ancient biographical and historical tradition, usually combining the factual with the legendary. The earliest source is Polybius a competent historian writing a couple of generations after Archimedes' death and from the histories authored by Plutarch, Cicero, and other historians several centuries after his death. Due to the length of time between Archimedes' death and his biographers', inconsistencies among their writings may arise.

The translation of many of Archimedes' works in the sixteenth century contributed greatly to the spread of knowledge of them, and influenced the work of the foremost mathematicians and physicists of the next century, including Johannes Kepler, Galileo Galilei, Descartes and Pierre de Fermat (O'Connor and Robertson, 2006). Archimedes together with Isaac Newton (1643–1727) and Carl Friedrich Gauss (1777–1855) is regarded as one of the three greatest mathematicians of all times (Bell, 1965).

His studies greatly enhanced knowledge concerning the way things work, and his practical applications remain vital today; thus Archimedes earned the honorary title "father of experimental science" because he not only discussed and explained many basic scientific principles, but he also tested them in a three-step process of trial and experimentation (Bendick, 1997). The first of these three steps is the idea that principles continue to work even with large changes in size. The second step proposes that mechanical power can be transferred from "toys" and laboratory work to practical applications. The third step states that a rational, step-by-step logic is involved in solving mechanical problems and designing equipment.

The end of the Alexandrian era marked the eclipse of the ancient Greek science, and the systematic study of the design of machines became stagnant for a long period of time. The death of Archimedes by the hands of a Roman soldier is symbolical of a world-change of the first magnitude: the Greeks, with their love of abstract science, were superseded in the leadership of the European world by the practical Romans (Whitehead, 1958).

Archimedes' Works

The attribution of works to Archimedes is a difficult historical question. The extraordinary influence of Archimedes over the scientific revolution was due in the main to Latin and Greek-Latin versions handwritten and then printed from the thirteenth to the seventeenth centuries. Translations into modern European languages came later, and have contributed to an ongoing study in the fields of the History of Greek Mathematics, History and Philosophy of Science and Engineering (Stamatis, 1973; Heath, 2002; Netz, 2004). The Works of Archimedes as well as other extant manuscripts had a difficult path to follow through the ages. A wealth of written scientific heritage has been preserved and a brief discussion on the unique historical significance of this process follows.

Mathematics original texts survive from the earlier era of Babylonia. Babylonians wrote on tablets of unbaked clay, using cuneiform writing. The symbols were pressed into soft clay with the slanted edge of a stylus having a wedge-shaped (hence the name cuneiform) appearance. Many tablets from around 1700 BC have survived and the original text can be read (O'Connor and Robertson, 2006).

Greeks started using papyrus rolls to write their works around 450 BC. Earlier they had only an oral tradition of passing knowledge on (Dimarogonas, 1995). As written records developed, they also used wooden writing boards and wax tablets for works not intended to be permanent. Sometimes writing from this period has survived on inscribed pottery fragments.

Papyrus comes from a grass-like plant grown in the Nile delta region in Egypt and was used as a writing material since 3000 BC. Copies of Archimedes works would have been written on a papyrus roll, about 10 metres long, a typical length of such rolls. These rolls were rather fragile and easily torn, so they tended to become damaged if much used. Even if left untouched they rotted fairly quickly except under particularly dry climatic conditions such as exist in Egypt. The only way that such works could be preserved was by having new copies made fairly frequently and, since this was clearly a major undertaking, it would only be done for texts which were considered of major importance (O'Connor and Robertson, 2006).

No complete Greek mathematics text older than Euclid's Elements has survived, because the Elements was considered such a fine piece of work that it made the older mathematical texts obsolete. From 300 BC until the codex form of book was developed, copies of important mathematics texts must have been copied many times. The codex consisted of flat sheets of material, folded and stitched to produce something much more recognisable as a book. Early codices were made of papyrus but later developments replaced this by vellum. Books from late antiquity very rarely survive, and there is evidence that, during the fifth and sixth centuries – during Byzantium's first period of glory – several such collections containing works by Archimedes were made. At least three codices containing works by Archimedes were produced during the ninth and tenth centuries.

Archimedes published his works in the form of correspondence with the principal mathematicians of his time. How and when this web of correspondence got transformed into collections of "treatises by Archimedes" is not known. Late antiquity was a time of rearrangement, not least of ancient books. Most important, books were transformed from papyrus rolls (typically holding a single treatise in a roll) into parchment codices (typically holding a collection of treatises). Byzantine culture began one of its several renaissances, producing a substantial number of copies of ancient works. It thus appears that a book collecting several treatises by Archimedes was prepared in the sixth century AD by Isidore of Miletus and Anthemios the Tralleus, the architects of Agia-Sofia in Constantinopole. It is believed that this collection of works was a "State-of-the-Art" review for the construction of this huge building. This book was copied by Leo the geometer or his associates, once again in Constantinople, in the ninth century AD (Lazos, 1995; Archimedes–Apanta, 2002; Netz, 2004). At this time Eutocius the Ascalonites (1st century AD), a student of Anthemios, wrote his commentaries on several books of Archimedes that were subsequently lost. Thus, Eutocius commentaries are considered today among the Archimedes books.

William of Moerbeke (1215–1286) archbishop of Corinth and a classical scholar had two Greek manuscripts of the works of Archimedes and made his Latin translations from these manuscripts. The first of the two Greek manuscripts has not been seen since 1311 when presumably it was destroyed. The second manuscript survived longer and was certainly around until the

16th century after which it too vanished. In the years between the time when William of Moerbeke made his Latin translation and its disappearance, this second manuscript was copied several times and some of these copies survive. A good deal of Archimedes' work survived only in Arabic translations of the Greek originals, and was not translated into Latin until 1543. In the early 1450s, Pope Nicholas V commissioned Jacobus de Sancto Cassiano Cremonensis to make a new translation of Archimedes with the commentaries of Eutocius. This became the standard version and was finally printed in 1544.

Heiberg had studied the manuscript tradition of Archimedes for over 35 years, starting with his dissertation, Quaestiones Archimedeae (1879), going to his First Edition (1880–1881) and leading, through numerous articles detailing new discoveries and observations, to the Second Edition (1910–1915). Up until 1899 Heiberg had found no sources of Archimedes' works which were not based on the Latin translations by William of Moerbeke or on the copies of the second Greek manuscript which he used in his translation (Heiberg, 1972).

In 1899 an exceptionally important event occurred as a palimpsest, a prayer book created by a monk on a reused parchment was recognized by Heiberg (Heiberg, 1972; Stamatis, 1973; Netz, 2004) as containing previously unknown works by Archimedes (palimpsest comes from the Greek, meaning "rubbed smooth again"). The Archimedes Palimpsest, copied in the 10th century, contains seven of the Greek mathematician's treatises. Most importantly, it is the only surviving copy of *On Floating Bodies* in the original Greek, and the unique source for the *Method of Mechanical Theorems and Stomachion*. The manuscript was written in Constantinople (today Istanbul) in the 10th century. In the 12th century, the manuscript was taken apart, the original text was scraped off and the Archimedes manuscript then disappeared. In 1906 Heiberg was able to start examining the Archimedes palimpsest in Istanbul.

Originally the pages were about 30 cm by 20 cm but when they were reused the pages were folded in half to make a book 20 cm by 15 cm with 174 pages. Of course this involved writing the new texts at right angles to the Archimedes text and, since it was bound as a book, part of the Archimedes text was in the spine of the "new" 12th century book, and worse, the pages of the Archimedes text had been used in an arbitrary order in making the new book. However, Heiberg reproduced the text successfully and published his reconstruction of the works of Archimedes, while the palimpsest itself remained in the monastery in Istanbul.

Exactly what happened to the Archimedes palimpsest is unclear. It was, it appears, in the hands of an unknown French collector from the 1920s although the palimpsest remained officially lost and most people assumed that it had been destroyed. The French collector may have sold it quite recently, but all we know for certain is that the palimpsest appeared at auction in Christie's in New York in 1998, sold on behalf of an anonymous seller. It was put on display with the spine broken open to reveal all the original text which had been in the spine when it had been examined by Heiberg. It was sold to an anonymous buyer for 2 million dollars on 29 October 1998 but the new owner agreed to make it available for scholarly research. Since 1999, intense efforts have been made to retrieve the Archimedes text. Many techniques have been employed. Multispectral imaging, undertaken by researchers at the Rochester Institute of Technology and Johns Hopkins University, has been successful in retrieving about 80% of the text. More recently the project has focused on experimental techniques to retrieve the remaining 20% (The Archimedes Palimpsest Website, 2005).

The best sources of the Archimedes works are those of Heiberg in 1915 (Heiberg, 1972), Heath's translation into English of Archimedes' collected works in 1912, Dijksterhuis' republished translation of the 1938 study of Archimedes and his works (Dijksterhuis, 1987), Stamatis (1970) (in Greek) with the addition of the Archimedes work on the hydraulic clock (in Arabic), and the most recent from Netz in 2004 with a collection of Archimedes' works translated into English based on the best sources and a comprehensive analysis of the existing resources for the Archimedes works.

According to Netz (2004) in the most expansive sense, bringing in the Arabic tradition in its entirety, 31 works may be ascribed to Archimedes. The corpus surviving in Greek – where Eutocius' commentaries are considered as well – includes the following works:

1. *On the Sphere and the Cylinder. The First Book*
2. *Eutocius' commentary on the First Book*
3. *On the Sphere and the Cylinder. The Second Book*
4. *Eutocius' commentary on the Second Book*
5. *Spiral Lines*
6. *Conoids and Spheroids*
7. *Measurement of the Circle (Dimensio Circuli)*

8. *Eutocius' commentary to the above*
9. *The Sand Reckoner (Arenarius)*
10. *Planes in Equilibrium*
11. *Eutocius' commentary to the above*
12. *Quadrature of the Parabola*
13. *The Method*
14. *The first book On Floating Bodies (de Corporibus Fluztantzbus)*
15. *The second book On Floating Bodies (de Corporibus Fluztantzbus)*
16. *The Cattle Problem (Problems Bovinum)*
17. *Stomachion*

Additionally, there are 13 works ascribed to Archimedes by Arabic sources, five are paraphrases or extracts of 1, 3, 7, 14 and 17, four are either no longer extant or, when extant, can be proved to have no relation to Archimedes, while four may have some roots in an Archimedean original (Heiberg, 1972; Stamatis, 1973; Netz, 2004). These four are:

- *Construction of the Regular Heptagon*
- *On Tangent Circles*
- *On Lemmas*
- *On Assumptions*

None of these works seems to be in such textual shape that we can consider them, as they stand, as works by Archimedes, even though some of the results there may have been discovered by him. Finally, several works by Archimedes are mentioned in ancient sources but are no longer extant. These are listed by Heiberg as "fragments," collected at the end of the second volume of the second edition:

- *On Polyhedra*
- *On the Measure of a Circle*
- *On Plynths and Cylinders*
- *On Surfaces and Irregular Bodies*
- *Mechanics*
- *Catoptrics*
- *On Sphere-Making*
- *On the Length of the Year*

From extant works whose present state seems to be essentially that intended by Archimedes, ten works mentioned above as: 1, 2, 5, 6, 9, 10, 12, 13, 14, and 15 are attributed in great probability to Archimedes. A short description of the contents of each manuscript follows.

1. *On the Sphere and the Cylinder. The First Book*. In this treatise Archimedes obtains the result he was most proud of: that the area and volume of a sphere are in the same relationship to the area and volume of the circumscribed straight cylinder. Archimedes built on previous work of Euclid to reach conclusions about spheres, cones, and cylinders. If these three figures have the same base and height (a cone inscribed in a hemisphere which itself is inscribed within a cylinder) then the ratio of their volumes will be 1:2:3. In addition, the surface of the sphere is equivalent to two-thirds of the surface of the cylinder which encloses it.

2. *Eutocius' commentary on the First Book*. Commentaries in the form of a collection of minimal glosses, selectively explicating mathematical details in the argument.

3. *On the Sphere and the Cylinder. The Second Book*. Archimedes provides the ratios for the surface area and volume of a sphere and then solves a series of problems concerning spheres. Archimedes built on the previous work of Euclid to reach conclusions about spheres, cones, and cylinders. If those three figures have the same base and height (a cone inscribed in a hemisphere which itself is inscribed within a cylinder) the ratio of their volumes will be 1:2:3. In addition, the surface of the sphere is equivalent to two-thirds of the surface of the cylinder which encloses it. The book contains several famous proofs, including his demonstration that the surface of the sphere is equivalent to two-thirds of the surface of the cylinder which encloses it and the volume of a sphere is equal to $4/3r^3$.

4. *Eutocius' commentary on the Second Book*. A very thorough work, commenting upon a very substantial proportion of the assertions made by Archimedes, sometimes proceeding further into separate mathematical and historical discussions with a less direct bearing on Archimedes' text.

5. Spiral Lines. In this scroll, spirals are first defined and Archimedes develops many properties of tangents to, and areas associated with, the spiral of Archimedes – i.e., the locus of a point moving with uniform speed along a straight line that itself is rotating with uniform speed about a fixed point. It was one of only a few curves beyond the straight line and the conic sections known in antiquity. This is the first mechanical curve (i.e., traced by a moving point) ever considered by a Greek mathematician.

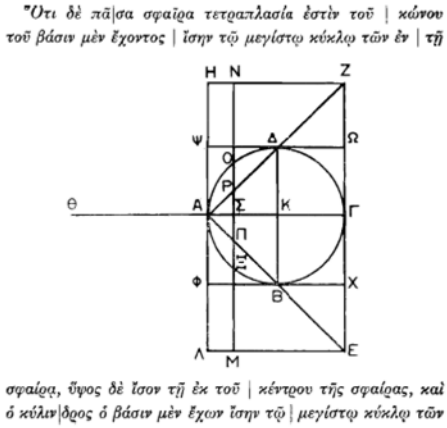

Fig. 3. Proof that the volume of a sphere equals two-thirds of the volume of a cylinder surrounding it, as long as the cylinder's height and width are equal to the sphere's diameter if these figures have the same base and height – imagine a cone inscribed in a hemisphere which itself is inscribed within a cylinder – the ratio of their volumes will be 1:2:3 (Stamatis, 1973, B p. 397).

6. *Conoids and Spheroids*. In this scroll Archimedes calculates the areas and volumes of sections of solids formed by the revolution of a conic section (circle, ellipse, parabola, or hyperbola) about its axis (cones, spheres and paraboloids. In modern terms, these are problems of integration.

7. *Measurement of the Circle (Dimensio Circuli)*. By a method that involved measuring the perimeter of inscribed and circumscribed polygons, Archimedes correctly determined with a method that can, in principle, be extended indefinitely, that the ratio of a circle's perimeter to its diameter is the same as the ratio of the circle's area to the square of the radius. He did not call this ratio ã but he gave a procedure similar to integral calculus to approximate it to arbitrary accuracy and gave an approximation of it as between $3+10/71$ (approximately 3.1408) and $3+1/7$ (approximately 3.1429) which was not to be defined for almost another 2000 years. Archimedes used this method to figure the areas and volumes of curved surfaces and circular forms. The use of this technique, elaborated upon in another volume, *The Method*, anticipated the development of integral calculus by 2000 years.

8. *Eutocius' commentary to the above*. Commentaries selectively explicating mathematical details in the argument.

9. *The Sand Reckoner (Arenarius).* In this work Archimedes created a system of exponential notation to allow him to prove that nothing exists that is too large to be measured. He also counts the number of grains of sand fitting inside the universe. This book mentions Aristarchus of Samos' theory of the solar system (concluding that "this is impossible"), contemporary ideas about the size of the Earth and the distance between various celestial bodies. From the introductory letter we also learn that Archimedes' father was an astronomer.

10. *Planes in Equilibrium.* The mechanics of levers is used for the calculation of the areas and centers of gravity of various geometric figures. The importance of the center of gravity in balancing equal weights is analysed in detail.

11. *Eutocius' commentary to the above.*

12. *Quadrature of the Parabola.* In this work, Archimedes calculates the area of a segment of a parabola (the figure delimited by a parabola and a secant line not necessarily perpendicular to the axis). The final answer is obtained by triangulating the area and summing the geometric series with a ratio 1/4.

13. *The Method.* In this work, which was unknown in the Middle Ages, came to the attention of modern readers only in 1906 AD, following the initial discovery of the Archimedes Palimpsest. Archimedes pioneered the use of infinitesimals, showing how breaking up a figure in an infinite number of infinitely small parts could be used to determine its area or volume. In this work Archimedes most explicitly connects the mathematical and the physical. He claims here that he has invented a procedure that allows him to use physics – in particular, mechanics – to derive mathematical results. Archimedes derives a wide range of results, including such highlights of his mathematical achievement as the volume of the sphere and the volumes of segments of solids of revolution.

14. *The first book On Floating Bodies (de Corporibus Fluztantzbus).* In the first part of this treatise, Archimedes spells out the law of equilibrium of fluids, and proves that water around a center of gravity will adopt a spherical form. This is probably an attempt at explaining the observation made by Greek astronomers that the Earth is round. Note that his fluids are not self-gravitating: he assumes the existence of a point towards which all things fall and he derives the spherical shape. In this book, he demonstrated that the surface of a liquid of constant density at rest, is spherical with the center of the

sphere located at the earth's center. One legend of Archimedes holds that he first understood this connection between the weight of a floating object and the resulting increase in water level while watching bath water rise as he sunk his body into a tub.

15. *The second book On Floating Bodies (de Corporibus Fluitantibus).* In the second part, a veritable tour-de-force, he calculates the equilibrium positions of sections of paraboloids. This was probably an idealization of the shapes of ships' hulls. Some of his sections float with the base under water and the summit above water, which is reminiscent of the way icebergs float, although Archimedes probably was not thinking of this application.

16. *The Cattle Problem (Problems Bovinum).* Archimedes wrote a letter to the scholars in the Library of Alexandria, who apparently had underestimated the importance of Archimedes' works. In these letters, he dares them to count the numbers of cattle in the Herd of the Sun by solving a number of simultaneous Diophantine equations, some of them quadratic (in the more complicated version). This problem is one of the famous problems solved with the aid of a computer. The solution is a very large number, approximately 7.760271×10206544.

17. *Stomachion.* Only a fragment survives. Apparently, this is a study in a tangram-like game, where areas are covered by given geometrical figures.In this scroll, Archimedes calculates the areas of the various pieces. This may be the first reference we have to this game. Recent discoveries indicate that Archimedes was attempting to determine how many ways the strips of paper could be assembled into the shape of a square. This is possibly the first use of combinatorics to solve a problem.

Review of Main Works on Mechanism Design

Archimedes' mechanical skill, together with his theoretical knowledge, enabled him to construct many ingenious machines. Archimedes contributed greatly to the theory of the lever, screw, and pulley, although he did not invent any of these machines. Of these three, the lever is perhaps the oldest, having been used in various forms for centuries prior to Archimedes. Levers appeared as early as 5000 BC in the form of a simple balance scale (steelyard), and within a few thousand years workers in the Near East and India were using a crane-like lever, called the shaduf, to lift containers of water.

Archimedes' contribution lay in his explanation of the lever's properties, and in his broadened application of the device.

The shaduf, first used in Mesopotamia in about 3000 BC, consisted of a long wooden lever that pivoted on two upright posts. At one end of the lever was a counterweight, and at the other a pole with a bucket attached. The operator pushed down on the pole to fill the bucket with water, then used the counterweight to assist in lifting the bucket. By about 500 BC, other water-lifting devices, such as the water wheel, had come into use.

Where the lever was concerned, Archimedes provided a law governing the use of levers. In this formulation, the effort arm was equal to the distance from the pivot (fulcrum) to the point of applied effort, and the load arm equal to the distance from the fulcrum to the center of the load weight. Thus established, effort multiplied by the length of the effort arm is equal to the load multiplied by the length of the load arm – meaning that the longer the effort end, the less the force required to raise the load.

Archimedes' work with levers and pulleys led to the inventions of the compound pulley. This mechanism was crucial for the development of large cranes and artillery machines. Figure 4 shows a lifting machine (trispas-

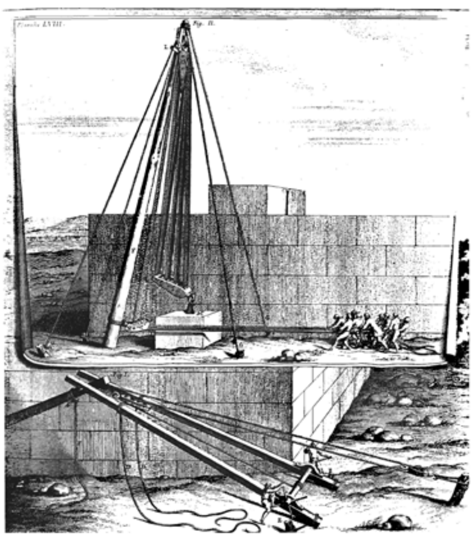

Fig. 4. Lifting machine (trispaston) from Archimedes times, drawing from Vitruvius book (Lazos, 1995).

ton) from a Vitruvius book (Lazos, 1995). Again, in the case of the pulley, Archimedes improved on an established form of technology by providing a theoretical explanation considering that a pulley operates according to much the same principle as a lever and the principle of the mechanical advantage was introduced. This he demonstrated, according to one story, by moving the *Syracusia* ship fully loaded, single-handedly while remaining seated some distance away. In the late modern era, compound pulley systems would find application in such everyday devices as elevators and escalators.

Some three centuries after Archimedes, Hero of Alexandria (first century AD) expanded on his laws concerning levers. Then in 1743 John Wyatt (1700–1766) introduced the idea of the compound lever, in which two or more levers work together to further reduce effort.

During his time in Egypt, he invented a hand-cranked manual pump, known as Archimedes' screw, that is still used in many parts of the world. He used the screw principle to improve on the shaduf and other rudimentary pumping devices. Its open structure is capable of lifting fluids even if they contain large amounts of debris.

With regard to the screw, Figure 5, Archimedes provided the theory for the screw geometry and construction, in this case with a formula for a simple spiral, and translated this into the practical Archimedes screw, a device for lifting water. The invention consists of a metal pipe in a corkscrew shape that draws water upward as it revolves. It proved particularly useful for lifting water from the hold of a ship, though in many countries today it remains in use as a simple pump for drawing water out of the ground.

Vitruvius in his book *De Architectura* (Book X, Chapter VI, The Water Screw) provides details for the construction and the operation of the water screw (see Figure 6).

This idea of enclosing a screw inside a cylinder is in essence the first water pump (Figure 7). This device soon gained application throughout the ancient world. Archaeologists discovered a screw-driven olive press in the ruins of Pompeii, destroyed by the eruption of Mount Vesuvius in 79 AD, and Hero later mentioned the use of a screw-type machine in his *Mechanica*. The Archimedean screw has been the basis for the creation of many other tools, such as the combine and auger drills.

The Greeks from Syracuse developed the first catapults, a result of engineering research financed by the tyrant Dionysius. Early catapults probably fired arrows from a bow not much stronger than one a man could draw. By

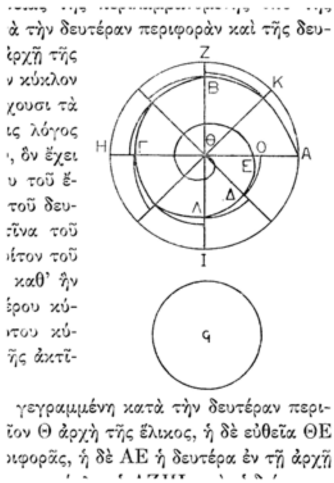

Fig. 5. Archimedes' spiral (Stamatis, 1973).

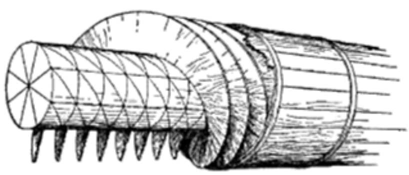

Fig. 6. The Water Screw, Vitruvius *De Architectura* (Book X, Chapter VI).

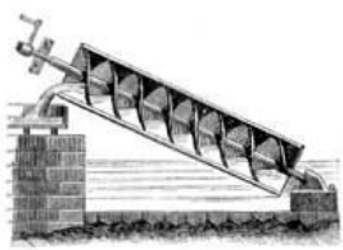

Fig. 7. Archimedes' screw machine from Archimedes times (Lazos, 1995).

mechanizing the drawing and releasing of the arrow, however, the catapult inventors made possible the construction of much more powerful bows. To mechanize the archer's motions the catapult engineers incorporated a number of appropriate design features (Soedel and Foley, 1979; Dimarogonas, 1993; Dimarogonas, 1995).

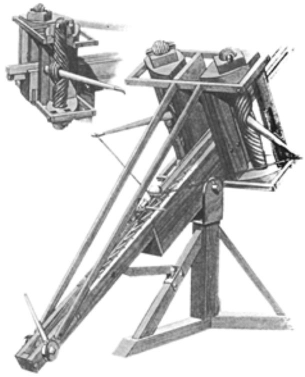

Fig. 8. Reconstruction of Catapult from Alexander the Great times (Lazos, 1995).

Fig. 9. Archimedes and his huge Catapult (Lazos, 1995).

The basic piece in the catapult was the stock, a compound beam that formed the main axis of the weapon (Figures 8 and 9). Along the top of the stock was a dovetail groove, in which another beam, the slider, could move back and forth. The slider carried at its rear surface a claw-and-trigger arrangement for grasping and releasing the bowstring. In front of the claw on top of the slider was a trough in which the arrow lay and from which it was launched. In operation the slider was run forward until the claw could seize the bowstring. Then the slider was forced to the rear taking the string with it until the bow was fully drawn. In the earlier versions linear ratchets alongside the stock engaged pawls on the slider to resist the force of the bow. Later a circular ratchet at the rear of the stock was adopted. Forcing back the slider on the first catapults was probably done by hand, but before long the size and power of the machines called for a winch.

A flexible bow was mounted at the end of a long wood framework enclosing a dovetail slider in this early arrow-firing catapult, based on a design originally devised by technicians working for Dionysius the Elder of Syracuse in the fourth century BC. The movable slider, carrying the bowstring with it by means of a claw-and-trigger arrangement, was held to the rear of the stock against the force of the bow by a linear ratchet, after being pulled back with the aid of a circular winch. The piece connecting the catapult to its pedestal appears to have been an ancient version of the universal joint. The bow itself probably consisted of three different materials glued together: a wood core, a front layer of animal sinew and a back layer of horn. Since sinew is so strong in tension and horn in compression, such bows would have been much more powerful than the ordinary kind carved out a single piece of wood. The arrow was roughly two meters long.

Dimensions of a board forming the top piece of one of the torsion-spring frames from a large stone throwing catapult were specified by the catapult designers in terms of the dimensions of the vertical sides of the frame, which in turn were determined by the diameter of the cord bundle forming the torsion spring. The thickness of the top board is not known for certain, but was probably equal to the diameter of the cord bundle. The catapult builders appear to have proceeded by first laying out a rectangle with one side equal to the depths of the vertical framepiece and the other side equal to twice this length.

With the onset of specialized military engines the equality of arms was lost. Special mathematical and technical skills were necessary to build and maintain a catapult, and the risks involved in operating it were less than those of the arms carried by the rank and file. One of the crucial steps in designing the torsion springs was establishing a ratio between the diameter and the length of the cylindrical bundle of elastic cords. All the surviving catapult specifications imply that an optimum cylindrical configuration was indeed reached, and it could not be departed from except in special circumstances, such as the exclusively short range machines Archimedes built at Syracuse.

This optimization of the cord bundle was completed by roughly 270 BC, perhaps by the group of Greek engineers working for the Ptolemaic dynasty in Egypt. There and at Rhodes the experiments of the catapult researchers were, according to Philo, “heavily subsidized because they had ambitious kings who fostered craftsmanship.” This phase of the investigations culminated in quantified results of a distinctly modern kind. The results were summarized in two formulas. For the arrow shooter the diameter of the cord bundle was set simply as 1/9 of the arrow length. The more complex stone thrower formula stated, in modern terms, that the diameter of the cord bundle in dactyls (about 19.8 millimeters) is equal to 1.1 times the cube root of 100 times the weight of the ball in minas (about 437 grams). $d = 1.13\sqrt{100}$ m. The stone-thrower formula has two remarkable features. First, it gives a true and accurate solution for optimal design. To do this they had to maximize the potential energy stored in the torsion springs. Modern elasticity theory applied to the design of these springs tells us that the stored energy available will be proportional to the amount of initial tension given the bundle in stringing it through the frame, the additional tension caused by the pre-twisting of the bundle, the square of the angle indicating the amount of additional twisting by the pulling back of the bow arm, and the cube of the bundle’s diameter. The cubing of the bundle’s diameter means that to express the diameter in terms of the mass of the projectile one would have to extract a cube-root.

It is the utilization of a cube-root extractor that constitutes the second remarkable feature of the stone-thrower formula, because it was written at a time when Greek mathematics was not yet capable of dealing fully with third-degree equations. Archytas of Tarentum and Eudoxus of Cnidus had devised elegant theoretical solutions, but they were three-dimensional, very awkward physically and of no use in performing calculations. There the matter stood until the advent of the torsion bow. Most of the next group of solvers of the cube-root problem had either a direct or an indirect connection with catapults.

The next solver of the cube-root problem was Eratosthenes, a friend of Archimedes and a native of Alexandria, which was then a center of catapult research. Eratosthenes stated explicitly that the catapult was the chief practical reason for working on cube-root problems. We can assume he was interested in engineering problems, since Archimedes dedicated his book *On Method* to him.

All of the next group of cube-root investigators, including Philo of Byzantium, Archimedes of Syracuse and Hero of Alexandria, were famous for their work on catapults. It is interesting to note that the largest stone-thrower on record, a three-talent (78 kilogram) machine, was built by Archimedes. Archimedes was also forced to depart from normal catapult pro-

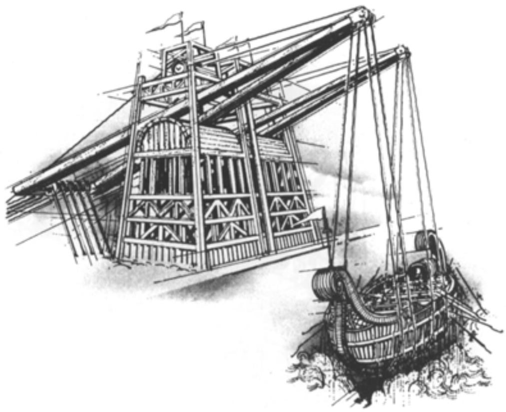

Fig. 10. An illustration of a rather elaborate claw (Lazos, 1995).

portions in building his short-range machines. Their effectiveness testifies to his skill as a mathematical engineer.

Having arrived at an optimal volume and configuration for the torsion-spring bundle, the catapult engineers continued their experiments until they had optimized the dimensions for the remaining pieces of the machine. Eventually the catapult engineers wrote their text in such a way that the dimensions of the major parts were given as multiples of the diameter of the spring. Once this diameter had been calculated for the size of the projectile desired, the rest of the machine was automatically brought to the proper scale. The surviving texts that contain this information testify to a level of engineering rationality that was not achieved again until the time of the Industrial Revolution.

The last major improvement in catapult design came in later Roman times, when the basic material of the frame was changed from wood to iron. This innovation made possible a reduction in size, an increase in stress levels and a greater freedom of travel for the bow arms. The new open frame also simplified aiming, which with the wood construction of the earlier machines had been limited, particularly for close moving targets.

As mentioned earlier, Plutarch and Polyvius describe giant mechanisms capturing Roman ships for the defence of Syracuse. An illustration of a rather elaborate claw of such a machine is shown in Figure 10.

Cicero (106–43 BC) writes that the Roman consul Marcellus brought two devices back to Rome from the sacked city of Syracuse. One device mapped the sky on a sphere and the other predicted the motions of the sun and the

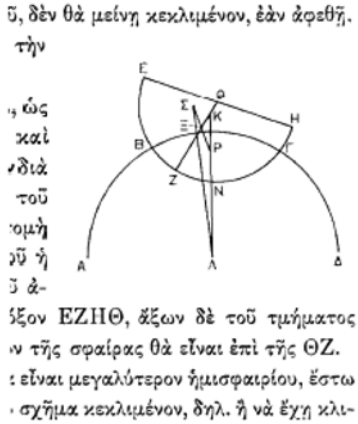

Fig. 11a. *The first book On Floating Bodies ΟΧΟΥΜΕΝΩΝ Α*. Proof of the vertical equilibrium of a spherical body lighter than the liquid immersed in (Stamatis, 1973 B, p. 287).

moon and the planets (i.e., an orrery). He credits Thales and Eudoxus for constructing these devices. For some time this was assumed to be a legend of doubtful nature, but the discovery of the Antikythera mechanism (De Solla Price, 1975; Dimarogonas, 2001) has changed the view of this issue, and it is indeed probable that Archimedes possessed and constructed such devices. Also, Pappus of Alexandria and Sextus Empiricus (Archimedes–Apanta Vol. 6, 2002) write that Archimedes had written a practical book on the construction of such spheres entitled *On Sphere-Making*. Some of those references may be based on confusions with other, extant works, while others may be pure legend. The reference to the work *On Polyhedra*, however, made by Pappus in his *Mathematical Collections* is very detailed and convincing.

Archimedes' discoveries in catoptrics are reported (Lazos, 1995; Archimedes–Apanta, 2002). It is said that Archimedes prevented one Roman attack on Syracuse by using a large array of mirrors (speculated to have been highly polished shields) to reflect sunlight onto the attacking ships causing them to catch fire. This popular legend has been tested many times since the Renaissance and often discredited as it seemed the ships would have had to have been virtually motionless and very close to shore for them to ignite, an unlikely scenario during a battle. Tests were performed in Greece by engineer Sakas in 1974 (Lazos, 1995) and by another group at MIT (MIT experiments) in 2004 and concluded that the mirror weapon was a possibility.

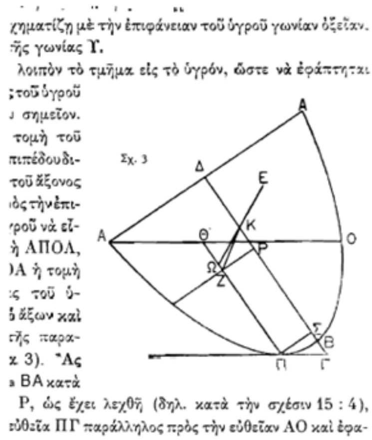

Fig. 11b. *The second book On Floating Bodies OXOYMENΩN B*. Proof of the vertical equilibrium of a spherical body lighter than the liquid immersed in (Stamatis E. B p. 345).

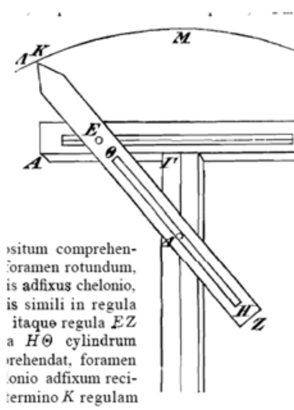

Fig. 12. Design of a curve, *The Method* (Heiberg, 1972, p. 99).

Archimedes discovered fundamental theorems concerning the center of gravity of plane geometric shapes and solids. His most famous theorem, which traditionally became known as Archimedes' Principle, was used to determine the weight of a body immersed in a liquid. Based on this Archimedes principle, shipbuilders understood that a boat should have a large enough volume to displace enough water to balance its weight (Figure 11).

Archimedes' studies in fluid mechanics gave rise to the most famous story associated with him. It was said that while trying to weigh the gold in the king's crown, Archimedes discovered the principle of buoyancy: when an object is placed in water, it loses exactly as much weight as the weight of the water it has displaced. The result that Archimedes discovered was the first law of hydrostatics, better known as Archimedes' Principle.

Archimedes was the first mathematician to introduce mechanical curves (those traced by a moving point) as legitimate objects of study (Figure 12).

Modern Interpretation of Main Contributions in Machines and Mechanisms Design

The philosophical foundation of knowledge, aesthetics and ethics are discussed in the works of Dimarogonas[1] (Dimarogonas, 1976, 1990, 1992, 1993, 1995, 1997) in order to identify their implications in engineering design. Dimarogonas scrutinized many major scientific libraries in the United States and Europe for source material. He documented that the fundamental axioms of design were discovered during the middle of the last century in Europe and traced the origin of vibration theory to Archimedes and others of that period by unearthing obscure documents in continental libraries. He brought to light certain important historical developments in the field of dynamics and vibrations that were either glossed over or not fully understood.

In the Ancient Great Empires of Mesopotamia and Egypt and the feudal societies of the East, India and China, there were parallel developments of crafts and technology without the concurrent development of sciences. The technical advances were arrived at by long evolution or invention and not by a conscious search for the solution of a problem of society. Moreover, the political and social system did not allow for liberal thinking, necessary for the development of scientific thought, and the knowledge was confined to the clergy or to the ruling cast.

In Greek Society, there was a production surplus which allowed members of the society to be employed in tasks which were not of immediate use, such as arts and philosophy (Dimarogonas, 2001). The general use of steel in agriculture and war, the popularization of the alphabet and the general

[1] Professor Andrew D. Dimarogonas (1938–2000) was widely recognized as a distinguished authority in various specialties of mechanical engineering. He made important contributions to the study of mechanical design and vibrations and received the 1999 ASME Engineer-Historian Award for his many works on integrating the history of mechanical engineering.

use of papyrus paper for book writing, were among the reasons for the rapid advancement of learning and science in ancient Greece. Moreover, the colonization of the shores of the east Mediterranean created small but dynamic and enterprising Greek communities with little or no dependence on the Metropolis. This helped the development of science in two ways: It allowed for exchange of ideas and information with the Eastern Civilizations and for the development of new ideas. The most important contribution of the pioneers of Greek (and Western) science, who were philosophers of the Ionian School, was the idea that Nature can be explained.

Rapid advancement in natural sciences was followed by systematic attempts to organize knowledge in engineering and, in particular, in machine design, developing this body of knowledge beyond the level of a mere craft. Kinematics and machine design have a distinct place in the history of engineering because they comprised the very first of its divisions to receive a mathematical foundation. Heraclitus of Ephesus (ca. 550–475 BC) appears to have been the first to separate the study of motion itself from dynamics, the forces causing the motion, and introduced the principle of *retribution*, or change, in the motion of celestial bodies. The first known written record of the word "machine" appears in Homer and Herodotus to describe political manipulation (Dimarogonas, 1997). The word was not used with its modern meaning until Aeschylus (4th century BC) used it to describe the theatrical device used to bring the gods or the heroes of the drama on stage; whence the Latin term *deus ex machina*. Mechanema (mechanism), in turn, as used by Aristophanes, means "an assemblage of machines." None of these theatrical machines, made of perishable materials, is extant. However, there are numerous references to such machines in extant Greek plays and also in vase paintings, from which they can be reconstructed (Chondros, 2004).

During this period Archimedes designed machines and mechanisms in a systematic way using a mathematical axiomatic foundation and experiments. This is a proccess not arrived at empirically through long evolution, and this point separates engineering science from technology and crafts. According to Dimarogonas (Dimarogonas, 2001) the first design theory was part of aesthetic theory. The "beautiful" included also functional (useful) and ethical (good) implications. Their development and the relation of function with form and ethical dimension prevailed, forming the intuitive knowledge required for machine implementation with methods of systematic design. Function and ethics were inseparable from form. This society simply could not afford spending resources only for aesthetic pleasure. It was able however to afford a pleasing appearance for the useful goods of everyday life and to pay attention to the more general societal needs.

As the slavery-based society of Rome reached maturity, productivity fell and once again utility prevailed over form in design considerations. The Romans were great engineers and designers. Aesthetic and ethical dimensions were not important. Romans were mostly unaware of Greek mathematics until the 2nd Century AD, when Greek mathematical works started being translated into Latin. A substantial number of treatises in architectural and mechanical design exists, mainly encyclopedic in nature: thc one by Vitruvius is the most notable. The Romans further gave the world sophisticated legal and administrative systems and separated the professions of civil and mechanical engineering (Dimarogonas, 2001).

It is among the Eleatic philosophers that we can find important beginnings of logic which was developed by Platon and Aristoteles into a science and served as an instrument for the parallel development of the natural sciences, especially mathematics and physics, by such pioneers as Pythagoras, Aristoteles, Eucledes and Archimedes. Experimentation was established as a method for scientific reasoning.

"Give me a place to stand, and I shall move the Earth," Archimedes is said to have promised (Dijksterhuis, 1987). Archimedes was referring to the law of the lever, which (in the variant form of the law of the balance) he had proved in his treatise, *Planes in Equilibrium*. One can say that Archimedes moved the Earth – in principle – *without standing anywhere*. Also, Archimedes figured out that the Earth and a pebble are the same kind of thing, differing only in size. This revolutionary idea yields to imagine a vantage point from which the earth and the pebble can both be seen for what they are. Archimedes went one better, and offered to move the Earth, if someone would supply him with this vantage point, and a suitable lever. Despite the familiar lever law this concept gave rise to the question of the need to think about time from a new viewpoint *outside* time. This atemporal perspective of time has been distinguished by modern philosophers as the "Archimedean view from nowhere" leading to the four-dimensional "block universe," of which time is simply a part (Price, 1996).

Infinity is central to the history of Western mathematics and was influenced by the work of Archimedes. The theorem of the wedge, introduced for calculation of the properties of curved objects, thus actually producing an argument using infinity, is a very Greek problem, the problem to which Archimedes contributed more than anyone else. To determine the area of sections bounded by geometric figures such as parabolas and ellipses, Archimedes broke the sections into an infinite number of rectangles and added the areas together. This is known as integration. He also anticipated the invention of differential calculus as he devised ways to approximate the slope of the tangent lines to his figures. In *The Method*, Archimedes was trying to work out the volume of an unusual shape by dividing it into an infinite number of slices. Archimedes had drawn a diagram of a triangular prism. Inside this he drew a circular wedge. This was the volume that he wanted to calculate. He then drew a second curve inside the wedge. Modern mathematicians already understood that Archimedes had used some very complex ideas to work out that a slice through the wedge equals a slice through the curve times a slice through the prism divided by a slice through the rectangle. But what no-one knew was how Archimedes had added up an infinite number of these slices to work out the volume of the wedge. His approximation of π between 3-1/2 and 3-10/71 was the most accurate of his time and he devised a new way to approximate square roots. Unhappy with the unwieldy Greek number system, he devised his own that could accommodate larger numbers more easily. However, his greatest invention was integral calculus.

Archimedes used mechanics as a tool to think about abstract problems, rather than as a field of study itself. Archimedes systematized the design of simple machines and the study of their functions. He invented the entire field of hydrostatics with the discovery of the Archimedes' Principle. Archimedes studied fluids at rest (Dijksterhuis, 1987; Stamatis, 1973), hydrostatics, and it was nearly 2000 years before Daniel Bernoulli took the next step when he combined Archimedes' idea of pressure with Newton's laws of motion to develop the subject of fluid dynamics. He made many other discoveries in geometry, mechanics and other fields and introduced step-by-step logic combined with analysis and experiments in solving mechanical problems and design of machinery procedures.

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AGUSTÍN DE BETANCOURT Y MOLINA (1758–1824)

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Abstract. Agustín de Betancourt, together with José María de Lanz, is known as co-author of "Essai sur la composition des machines", considered to be the first modern treatise on machines and the first book that contains a proposal for the classification of mechanisms based on criteria of transformation of motion. Two periods can be distinguished in his biographical trajectory: the first one is in Spain at the service of the Spanish Crown, from his birth in 1758 until 1808, and the second one is in Russia at the service of the Russian Empire from 1808 until his death in 1824. This paper is focused on the works and contributions developed in the Spanish period.

Biographical Notes

Agustín José Pedro del Carmen Domingo de Candelaria de Betancourt y Molina was born on the 1st of February, 1758 in Puerto de la Cruz (Canary Islands, Spain), in the bosom of an enlightened family.

Fig. 1. A portrait of Agustín de Betancourt.

M. Ceccarelli (ed.), Distinguished Figures in Mechanism and Machine Science, 31–60.

His primary education was carried out in the Dominican Monastery of La Orotava. Agustín de Betancourt himself will say later that, from all he had learned throughout his life, nothing was as useful as the development during his first years in the Canary Islands of some textile machines, such as the thread covering machine, which were made as a hobby and had been the origin of his attraction to the mechanical arts.

In his biography, several periods can be distinguished: from 1778 to 1784 there is a first formative period in Madrid; from 1785 to 1791 there is a second formative period in Paris in which the future Royal Laboratory of Machines was developed; from 1792 to 1793 there is a period in Madrid as Director of the Royal Laboratory of Machines; from 1793 to 1796 he visited England where he had the opportunity to learn about Watt's works on the steam machine; from 1797 to 1798 he visited Paris where he published two important essays on the steam engine; for the period 1799 to 1807 he returned to Spain and created the School of Civil Engineering; for the period 1807 to 1808 he returned to Paris and published "Essai sur la composition des machines"; in 1808 he moved to Russia, remaining there until his death in 1824. Next, each one of these periods will be analyzed with greater detail.

Under a grant of the Secretary of Industry, D. José Gálvez, Betancourt moved to Madrid in October of 1778. From 1778 to 1784 he studied in Madrid at the Reales Estudios de San Isidro, directed by his cousin Estanislao Lugo-Viña Molina, where he learned Arithmetic, Algebra, Geometry, Trigonometry, Mathematical Analysis, Theory of Curved Lines, Differential and Integral Calculus and Mechanics (static and dynamic) and at the Real Academia de Bellas Artes de San Fernando where he studied Physics and Drawing. In 1783, D. José Moñino, Count of Floridablanca, first Secretary of State, put him in charge of a visit to inspect the mines of Almadén, given his recognized expertise. Betancourt wrote three important Reports about this inspection.

In 1784 he received a grant from the Secretariat of the Indies to study underground architecture (mine engineering). Simultaneously he received an order to visit the Channel of Aragón for an inspection. In April, on the way to France, he elaborated a new Report, but until today it remains lost. In Paris he participated in the activity of the École des Ponts et Chaussées.

In 1785 he went back to Madrid. After an interview with the Secretary of State, D. José Moñino, Count of Floridablanca, he was asked to establish in Spain a new school, namely, the Escuela de Caminos y Canales (School of Roads and Channels, today's School of Civil Engineering). However, the agreements for this assignment were: Betancourt and a selected group of grant holders were to enroll at the École des Ponts et Chaussées in Paris in order to obtain the degree of Hydraulic Engineer; in this school, and other similar institutions, they would acquire expertise and mechanical specialization; and they were to collect models of machines of general utility in public works and industry.

On September 10, 1785, Betancourt went again to Paris, where he was well accepted from the Director of the School Jean Rodolphe Perronet and professor Gaspard François de Prony. In April 1788 the Spanish ambassador, the Count of Fernán-Núñez, visited Betancourt's home and workshop and was very impressed with the many scale models, so that in his letter to the Secretary of State, dated 23 April, he proposed the creation of a Laboratory of Machines in Madrid. In 1791, due to the situation in France, the King of Spain, Carlos IV, decided that Betancourt should return to Spain, bringing with him his collection of drawings and scale models. The whole collection (including 42 drawings) was received in Spain between July and September. In April 1792 the Laboratory, located in the Palacio del Buen Retiro, was opened to the public. On the 14th of October, 1792, Betancourt was officially appointed Director of the Laboratory. The whole collection was composed of 270 models, 359 drawings and 99 reports.

Due to scarce interest shown in the Laboratory, evidenced by the low number of visitors and because most of them were either curious or unemployed people instead of people interested in applying the models to public works or to industry, he asked for permission to go abroad to study, first in England, from 1793 to 1796, and later to France, from 1797 to 1798.

In England he visited factories and saw different types of machines, which attracted, without a doubt, Betancourt's interest in research on the theory of machines. While in France, he submitted two important reports to the Académie des Sciences of Paris. In the first, "Mémoire sur une machine à vapeur à double effet", he revealed to the Continent the double-action steam engine, which he had observed in action in England between 1793 and 1796. This report led Jacque-Constantin Périer to construct the first double-action steam engine in France. In the second report, "Mémoire sur la force expansive de la vapeur de l'eau", he published the results of a series of measurements establishing the relation between temperature and steam pressure.

From 1797 to 1798, in Paris, he received a grant to improve the optical telegraph. In 1799 the Secretary of State, Don Mariano Luis de Urquijo, created the General Inspectorate of Roads and the Body of Engineers of Roads, which Betancourt joined with the category of Commissioner.

In 1799 he returned to Spain. From 1799 to 1800 he dedicated himself fundamentally to the tasks of installing a line of optical telegraphs between Madrid and Cádiz, building 70 turrets at intervals of ten to twelve kilometers. In 1802 Betancourt became the Chief Inspector, and from that position he founded the School of Roads and Channels, locating it in the Real Gabinete de Máquinas (Royal Laboratory of Machines). In that same year the basic textbooks for the education of the students were printed: "Geometría descriptiva" by Monge and "Tratado de Mecánica" by Francoeur. By that time, Lanz had joined the faculty of the Institution. From 1802 to 1807, Betancourt remained in charge of the School of Roads and Channels, the Royal Laboratory of Machines and the General Inspectorate of the Corps of Road Engineers. From 1802 to 1805 Lanz and Betancourt both taught in the classrooms of the School.

From May to October 1807, Lanz and Betancourt were together again in Paris. It was then when the work, developed previously by Lanz and Betancourt, was reconstructed, reviewed and presented in the Ecole Politecnique with the title of "Essai sur la composition des machines". In October 1808, due to the state of war created in Spain,[1] Betancourt left for Russia and worked for Czar Alexander I. In Russia he spent great efforts to successfully make new inroads for engineering through several activities in design, teaching, and organization. This activity was fully attributed to Betancourt and to this date he is remembered in the Russian history of Mechanical Engineering. In 1809 he organized the Institute of Engineers of Communication Routes in Russia, of which he was inspector and advisor. In 1816 he was President of the Committee of Constructions and Hydraulic Works and also of the newly created committee for the construction of the fair at Nizhni Nóvgorod. In

[1] His definitive departure from Spain to Russia was due to the difficult situation through which Spain passed after the Napoleonic invasion and to the warm welcome on the part of Czar Alexander I. In a letter addressed to his brother Jose in 1814, he commented on the matter: "Since I observed the enmity that reigned in Spain between the Prince of Asturias and Godoy, I supposed that a revolution should arise in Spain and that in such case it was necessary, in order not to perish with my family, to look for asylum in a foreign kingdom in which to put it out of danger, and it seemed to me that Russia would be the most appropriate".

1818 he was named Chief of a main directorate of the Department of Communication Routes. In Russia he improved the arms industry and constructed bridges using a new system of arches. He built, in collaboration with Carbonnier, the hall of the riding school of Moscow, which was by then the largest hall without inner supports; the span of its roof was said to be forty meters long. He also constructed the aqueduct of Taïtzy and set up a state paper industry. In 1822 he was removed from this position and saw his authority diminished. In 1824 Betancourt died in Saint Petersburg.

List of Main Works

Manuscripts

Catálogo de la colección de Modelos, Planos y Manuscritos que, de orden del Primer Secretario de Estado, ha recogido en Francia Don Agustín de Betancourt y Molina. 1792. (Catalog of the collection of Models, Plans and Manuscripts that, by order of the First Secretary of State, Don Agustín de Betancourt and Molina were collected in France.)

Descripción del establecimiento de Yndrid donde se funden y barrenan los cañones de hierro para la Marina Real de Francia. 1791. (Description of the factory at Yndrid where iron cannons are melted and drilled for the Royal Navy of France.)

Description d'une machine à couper les roseaux et les autres plantes aquatiques qui obstruent beaucoup de canaux et de rivières navigables. (Description of a machine to cut the water reeds and other plants which block many channels and navigable rivers.)

Dessin de la machine pour faire monter et descendre les bateaux d'un canal inférieur et réciproquement, sur deux plans inclinés, exécutée en Angleterre, dans le comté de Shropshire, sur le bord de la rivière de Severn, près du pont de fer à Coalbrookdale, à 4 lieus environ à l'ouest de Shefnal: Levé et dessiné sur les lieux par M. de Betancourt. (Drawing of a machine to lift and take down boats from a lower channel and reciprocally, on two inclined levels, carried out in England, in the county of Shropshire, on the river bank of Severn, close to the iron bridge of Coalbrookdale, 4 lieus approximately to the west of Shefnal: Surveyed and drawn on the spot by Mr. Betancourt.)

Explication des principales parties du moulin pour moudre le silex. 1796. (Explanation of the main parts of a mill to grind flint.)

Informe dirigido a Mariano Luis de Urquijo sobre el método de transmitir noticias a distancia por medio de señales inventado por José Fornell. 1799. (Report directed to Mariano Luis de Urquijo on a method for long-distance transmission of news by means of signals invented by Jose Fornell.)

Informe dirigido al Duque de Alcudia sobre la bomba hidráulica diseñada por Francisco Zacarías. 1793. (Report directed to the Duque of Alcudia on the hydraulic pump designed by Francisco Zacarías.)

Machine à curer proposée pour le port de Venise. (Cleaning machine proposed for the port of Venice.)

Mémoire sur une machine à vapeur à double effet. 1789. (Report on a double-acting engine.)

Memoria sobre la purificación del carbón de piedra, y modo de aprovechar las materias que contiene. 1785. (Report on the purification of stone coal, and the way to take advantage of the materials that it contains.)

Primera memoria sobre las aguas existentes en las Reales Minas de Almadén, en el mes de julio de 1783: y sobre las máquinas y demás concerniente a su extracción. 1783. (First report on the water found in the Royal Mines of Almadén, the month of July 1783: and on the machines and others affairs concerning its extraction.)

Segunda memoria sobre las máquinas que usan en las minas de Almadén, en que se expresan sus ventajas, y defectos, y algunos medios de remediarlos. 1783. (Second report on the machines used in the mines of Almadén, in which their advantages are expressed, and defects, and some means to remedy them.)

Tercera memoria sobre todas las operaciones que se hacen dentro del Cerco en que están los hornos de fundición de Almadén. 1783. (Third report on all the operations that are made in the surroundings of the smelting furnaces at Almadén.)

Printed

Mémoire sur la force expansive de la vapeur de l'eau, lu a l'Académie Royale des Sciences. 1790. (Report on the expansive force of steam, read at the Royal Academy of Sciences.)

Essai sur la composition des machines: Programme du cours élémentaire des machines pour l'an 1808 par M. Hachette. 1808. (Essay on the composition of machines: Program of the elementary course of machines for the year 1808 by Mr. Hachette.)

Essai sur la composition des machines. 2 éd, rev., corr. y augm. 1819. (Essay on the composition of machines. 2nd version reviewed, corrected and augmented.)

Analytical essay on the construction of machines: translated from French. 1820.

Versuch über die Zusammenstzung der Machinens: aus dem Französischen. 1829.

Essai sur la composition des machines. 3 éd, rev., corr. y augm. 1840. (Essay on the composition of machines: 3rd version reviewed, corrected and augmented.)

Review of Main Works on the Design of Mechanisms

There are two main works of Betancourt that are related to the Theory of Machines. The first one is "Essai sur la composition des machines" which was co-authored with José Maria de Lanz and was published in 1808, constituting the first modern treatise on machines in which a classification of mechanisms appears based on the transformation of motion. The second one is reflected in "Mémoire sur une machine à vapeur à double effet", presented to the Royal French Academy of Sciences in 1789, which contains a description of Watt's double-acting steam engine together with the development of a method of path-generating synthesis applied to dimensioning of Watt's mechanism.

The "Essai sur la composition des machines"

In 1808, as it has been already mentioned, the treatise written by Lanz and Betancourt was published. The title itself marked a substantial difference with respect to previous works: it deals with the composition of the machines, that is to say, it focuses the analysis not on the machine itself but on the mechanisms that constitute it.

The book comes accompanied by the first program of the Elementary Course of Machines, developed in l'École Polytechnique, that was given at that time by Jean Nicolas Pierre Hachette (1769–1834), disciple of Monge. In this program the initial steps that were taken in the creation of this first Course of Machines are reported. Monge proposed to dedicate two months of the first year of studies to the description of the elements of the machines and also to the machines used in public works, with which a new approach to teaching on machines appears: the machines are combinations of elements, whose purpose is to transform motion.

In the *Journal of l'École*, Monge exposed his ideas:

> The forces of Nature at man's disposal have three different elements: mass, speed and direction of motion. Hardly ever do the three elements of the forces in question have the qualities that agree with the proposed target; and the main object of the machines is to turn the effective forces into others in which these elements are of such nature so as to produce the desired effect. Each machine is made-up of several elementary parts, each one with a particular target that can be reached in several different ways according to the circumstances. The enumeration of all the forms in which it is possible to change the elements of the forces and the description of the different means to produce the same change in different circumstances, must offer to the workers the greater resources for all classes of jobs.[2]

The program shows ten types of elemental transformations of motion:

Rectilinear continuous in:

1. Rectilinear continuous.
2. Rectilinear alternative.
3. Circular continuous.
4. Circular alternative.

Circular continuous in:

5. Rectilinear alternative.
6. Circular continuous.
7. Circular alternative.

Rectilinear alternative in:

8. Rectilinear alternative.
9. Circular alternative.

Circular alternative in:

10. Circular alternative.

[2] J.N.P. Hachette: "Sur le cours des machines de l'École Polytechnique", introduction to the 1808 edition of "Essai sur la composition des machines" of J.M. Lanz and A. Betancourt, p. VI.

The program contains an attached picture of elementary machines in which eighty nine mechanisms are classified according to the ten types of transformations of motion previously described.

The Council of Instruction of l'École, as recorded in the introduction of the Essai, affirmed with respect to the publication of the book:

> So it was the system according to which Mr. Hachette had begun the attached picture of the elementary machines, when he had knowledge that Mr. Lanz and Betancourt had elaborated a similar work in agreement with the same plan. The Council of Instruction, based on Mr. Monge and Hachette's report, proposed to the Governor that the result of Mr. Lanz and Betancourt's work (both commissioned by the Spanish government) should be put in print, the School paying for it. This work, transferred by its authors to the Polytechnic School, now appears under the title of "Essai sur la composition des machines". (The Council of Instruction of the School, which heard the reading of this article in its meeting of the 12th of August of 1808, has ordered to print it).[3]

The book served as support material to the Course of Machines. In the development of the text, the motion by means of a given curve is added together with the rectilinear and circular motion. This is a first important contribution because the two last motions predominated in the old machines. The curvilinear motion represents an advance in the possibilities of the machines.

The introductory commentary to the general table is very interesting because in it the authors explain the principles of classification and the utility of the table itself.

> Motions used in the mechanical arts are rectilinear, circular or are determined according to given curves; they can be continuous or alternative (backward and forward motion or swinging) and can consequently be combined in pairs resulting in fifteen different options or twenty one if each one of these motions is combined with another one of the same class. All machines have the aim of performing one or several of these twenty one combinations. This picture includes the exhibition of these different combinations of motions with all the

[3] J.N.P. Hachette (1808): "Sur le cours des machines de l'ÉŘcole Polytechnique", introduction to the 1808 version of "Essai sur la composition des machines" by J.M. Lanz and A. Betancourt, p. VIII.

> examples that we have been able to find; the examples will be represented aside, in greater size, adding the explanation and the uses to which each can be put. By means of this picture, great ease will be acquired in choosing and creating all classes of machines and in inventing new ones according to need.[4]

The combination of three trajectories (rectilinear, circular and curved) with two types of motion (continuous and alternative) gives rise to six combinations.

The combination of these six types of input motion with six types of output motion would give rise, theoretically, to thirty-six combinations, but they are reduced to twenty-one because some combinations do not appear.[5] In contrast to the ten combinations of Hachette's table twenty-one types of transformations appear. The general table contains one hundred and thirty-four mechanisms, as opposed to the eighty-nine of Hachette.

It is very important to emphasize the usefulness of the table and of the book given by the authors because of the novelty of its methodology, based on choosing mechanisms, to create and to invent new machines according to need.

With the purpose of helping in the identification of each one of these solutions, a classifying system was developed. Each figure is designated by a letter and a number that indicates its position in the table. In the same treatise the authors affirm that:

> Each horizontal row will be the object of a section designated with the same number; in it, the proposed target will be announced; the general solution of analogous problems to the transformation that needs to be carried out will be given; the particular cases or the different means of execution which we know will be developed, indicating the sources from where we have extracted this knowledge, and finally, thoughts

[4] Lanz (1808), illustration AK6.

[5] As for the circular continuous input there is no rectilinear continuous output; for the input according to a continuous curve there is no rectilinear continuous output nor circular continuous; for the rectilinear alternative input there is no rectilinear continuous output, circular continuous and by continuous curve; for circular alternative input there is neither rectilinear continuous nor rectilinear alternative output, there is no circular continuous and by continuous curve output; for the input according to an alternative curve there is only output according to an alternative curve. Altogether they make fifteen combinations that do not appear.

on the usefulness of these means and the diverse machines to which they have been applied will be added.[6]

The book contains a large classifying table in which all the mechanisms that are distributed according to the criteria of transformation of motion appear, such as it has been commented previously. Such mechanisms appear depicted on a greater scale throughout ten tables.

The tables come preceded by explanatory texts for each one of the mechanisms proposed. Sometimes, if the mechanisms are very elementary or very known, the explanation is concise; nevertheless, in most of the mechanisms, the authors explain their operation, they cite the sources from where they have obtained the data and they include the applications to which they have reference.[7]

As for the types of mechanisms, it can be indicated that the one hundred and thirty-four proposals contain mechanisms of all types: by frequency of appearance, gears predominate appearing thirty-nine times (wheels, racks, cylindrical, conical and worm crown gears), pulleys and cables appear thirty-three times; articulated mechanisms appear twenty-seven times; cams and escapements appear both eighteen times; worm gears appear six times; and chains appear five times. Seventeen mechanisms are operated by weights and eleven mechanisms are operated by springs.

Although the explanations might suggest that some of the solutions have been developed or improved by the authors, in two of them their applications are explicitly mentioned, and in another three, their design was made by Betancourt.[8] From these, mechanism I.17 seems to us especially interest-

[6] Lanz (1808), p. 2.

[7] As an example of the extensive research into sources, this commentary to the Archimedes' screw is included: "About the theory of Archimedes, Hydrodynamique by Daniel Bernouilli can be consulted; a Report by M. Pitot, printed in the Reports of l'Académie des Sciences in 1736; another one by Mr. Euler in the Reports of l'Académie imperial of Pétersbourg, volume V, year 1754; the work done by P. Belgrado, whose title is 'Theorie cochleae Archimedis, ab observationibus, experimentis et analyticis rationibus ducta', year 1767; the prize given in 1765 to M. Jean-Frédéric Hennert, by the Academy of Prusia; and the work done by M. Paucton, on the theory of the screw of Archimedes".

[8] Figure H1 corresponds to a wedge mechanism that is said that was used by Betancourt in England to raise the lower cylinder of a great rolling mill. In section S. II it is said that in "L'Architecture Hydraulique" by Prony, vol. II, there is the description of a procedure invented by Betancourt, to regulate the velocity of a steam engine by means of a floater provided with a siphon; in M7 an elevating water device, also attributed to Betancourt; in O8 a universal joint appears of which an application by Betancourt and Breguet

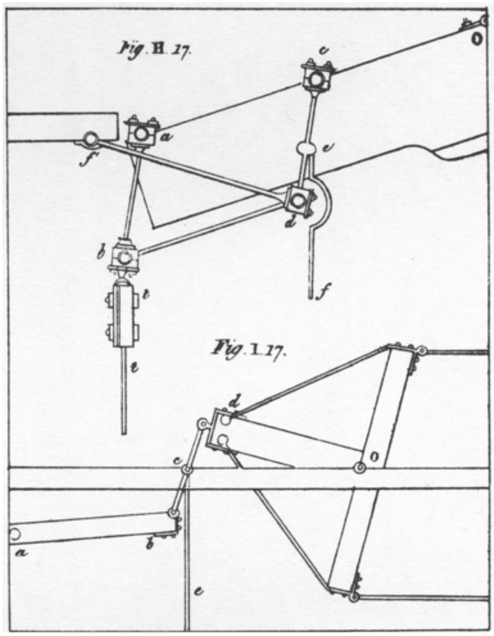

Fig. 2. Watt's straight-line linkages: (Lanz 1808) (Fig. H 17) Watt's extended linkage. (Fig. I 17) Watt's singular linkage, attributed in the Essai to Betancourt.

ing (Figure 2) as it is none other than Watt's straight-line linkage applied to the steam engine. In the book it is attributed to Betancourt. Since the authors always made reference to the known designers of the mechanism, it is quite possible that Betancourt did not know of Watt's design and arrived at the same conclusion through the parallelogram mechanism applied to the steam engine.

In some mechanisms the explanation is very long. For example in the one that talks about the diverse hydraulic wheels using bowls or buckets, or also on the different solutions to the escapement mechanisms used in clockmaking and in part dedicated to mechanisms for plotting curves.

In certain cases, the description of the operation of the mechanism comes accompanied by the explanation of a method for its dimensioning. It is interesting to point out one referring to the design of a cam and another one,

to the optical telegraph is indicated; in I17 Watt's four-link mechanism appears, attributed to Betancourt.

that we will mention later with greater amplitude, related to the design of a mechanism of rectilinear guidance alternative to Watt's.

Another very interesting aspect of the work is that, in most of the mechanisms, their well-known applications are mentioned. In contrast with the extended vision of which the book professes, fundamental mechanisms applied to civil engineering, it is possible to observe that the more related application is in machine tools which appear nineteen times; followed by clockmaking[9] which appears fifteen times; next, mills and the steam engine, with each appearing ten times; which are followed by drop hammers and other similar machines that appear nine times; drawing tools also appear nine times; textile machinery appears seven times; pumps for water elevation appear six times; lifting and dragging machines appear five times; and, hydraulic wheels and polishing machines, with each appearing three times. Some few common applications stand out, such as for example a machine to fillet fish, a mechanism applied to the optical telegraph, another used in the pedal of a piano or several used in fair attractions. Also the use of measurement and control mechanisms is interesting: balls for opening and closing of valves, an instrument for measuring the speed of a ship, and stress control in transmissions.

From the commentaries included in the book it is possible to deduce that, aside from the machines that could be known by the authors in their trips, many mechanisms are selected after exhaustive bibliographical review. Some famous authors of Theatrum Machinarum and machine collection books are mentioned in the Essai: Besson (ca. 1540–1573), Branca (1571–1645), Leupold (1674–1727), Diderot (1713–1784), Berthelot and Belidor (1693–1761). The more referenced sources are diverse collections of inventions and patents published throughout the XVIII century: The "Machines approuvées par l'Académie", the "Annales des arts et manufactures" by O'Reylli and "The repertory of arts and manufactures". Along with them, reports and books by well-known authors such as Daniel Bernouilli, Pitot, Euler, Hut, Coulomb, De la Hire, Hachette and Prony appear.

The importance of this book and its diffusion throughout the first half of the XIX century is authenticated by its numerous editions: three in French in the years of 1808, 1819 and 1840; two in English with the title "Analytical essay on the construction of machines" in the years 1820 and 1822; one in German with the title "Versuch über Zusammensetzung der Maschinen von Lanz und Betancourt" in 1829. Surprisingly there was no edition in Spanish.

[9] Possibly it was due to the great friendship and collaboration with L. Breguet.

In 1875, Reuleaux, talking about the Lanz and Betancourt book, writes: “Without great changes, it has remained of general use until our times and, therefore, it has obtained the approval of general recognition”.[10]

The “Mémoire sur une machine à vapeur à double effet”

At this point, Betancourt’s prominence returns. Betancourt’s description of how he arrived at the knowledge of Watt’s machine is very interesting:

> Being in charge of gathering a collection of models relative to hydraulics as ordered by the Spanish Court, I wished to see a steam engine that had all the discoveries made until the moment. So I arranged to move to England with the purpose of acquiring all the necessary knowledge on the perfection of this machine; I did not ignore that in this country, in which most of the applications of the steam engines have been made, is where greater number of opportunities you can have to recognize the defects and therefore the corrections to apply.
>
> Hardly had I arrived at London, I spoke to several mechanics and physicists; all they had done was to explain to me the effect of steam in the old machines; and they did not say anything to me that was not already known in France. But knowing that the gentlemen Wast and Bolton (*sic*) had made recent discoveries on the steam engine, by means of which they produced the same effects with less combustible, I made the decision to go to Birmingham to meet these famous artists. When I met them they received me with the greatest honesty and as a sign of esteem they showed me their button and silver-clad factories; but they did not show me any of their steam engines, all they did was to tell me, that those they manufactured at that moment were superior to any, because their velocities were controlled voluntarily and they consumed much less combustible than those that they had manufactured previously; they did not let me suspect where did so great advantages came from.
>
> Back to London, a friend got me a permission to see the mills that have been constructed near the bridge of Blas-Friars; they should have three steam engines and each one should drive ten mills. Only one of these machines was mounted, the other two should be mounted imminently.

[10] Reuleaux (1875), p. 13.

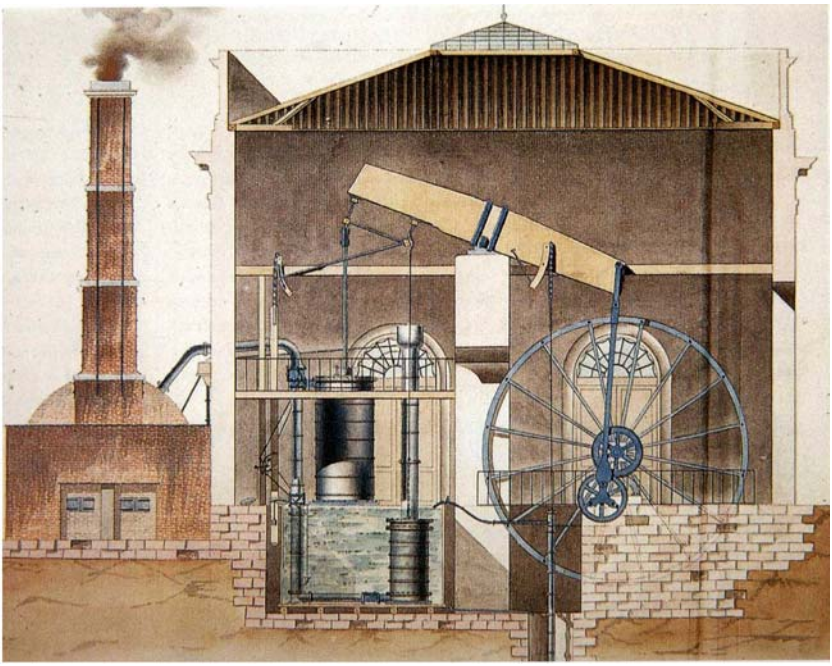

Fig. 3. Watt's Double Acting Steam Engine: Drawing by Agustín de Betancourt (1788). The drawing shows the novel aspects of the machine, some of which were seen and others guessed by him.

At first I was surprised when I saw that the chain joined to the rocker beam and from which the piston within the steam cylinder was suspended had been eliminated; it had been replaced by a parallelogram, of which I will give the description later on (. . .).

The day after which I saw this machine I started off for France; back in my house I dedicated myself with enthusiasm, remembering faithfully all the pieces that I had been able to see, trying to guess its use; for it I drew diverse diagrams and plots, and got to compose a double acting machine; from that very moment I undertook the construction of the model that has been a success beyond my hopes.

As this machine can be of a great usefulness in the mechanical arts and I have taken advantage of its economy of construction and combustible consumption, I have thought that the Academy would receive with pleasure the description that I am going to give. (Figure 3)

Betancourt presents the “Mémoire sur une machine à vapeur à double effet” the 15th of December of 1789 and signed as “Le chevalier de Betancourt Capitaine au service d’Espagne”. The registry of sessions of the 16th of December of 1789 of the Royal Academy of Sciences, states that “Mr. Betancourt has presented a Report on a double acting steam engine” and that commissioners Jean Charles Hut (1733–1799) and Gaspard Monge (1746–1818) have been designated to inform on this Report. In the session of the 10th of February of 1790 the commissioners’s report concludes in the following way:

> We thought that the Academy must welcome the fervour and the intelligence of Mr. Betancourt who has brought France the power of a discovery whose knowledge would not have arrived to him in natural form until much later and the report he presents, worthy of approval, must be printed in the collection of those of foreign wise people.

Simultaneously, in 1790, the industrialist Périer, Betancourt’s friend, who had asked for and obtained the privilege to install grain mills in the Parisian region, introduced the first double-acting steam engines installed in France in the mill located in l’Île des Cygnes, following Betancourt’s instructions.

These facts, related by the same Betancourt, have been cited as an example of industrial spying without enhancing the true prominence of Betancourt. Certainly his role in the diffusion of the double effect steam engine goes beyond the fact of transmitting the data obtained in England.

In the first place, it is a fact that Betancourt could not observe with enough detail all of the machine. In fact, he did not get to have knowledge of a very important element for the controlled operation of the machine, the Watt’s flyball governor.

Some historians have wanted to see in Betancourt a good mechanic and a good observer, mere transmitter of which he could see in Albion Mills. Nevertheless, it seems that these qualities are not enough for the development of the machine proposed by Betancourt.

In fact, in 1787, Albion Mills received the visit of three illustrious French visitors. One of them, the famous Coulomb, was an expert on the simple acting steam engine, as he had been a member of four commissions designated in 1783 by the Academy of Sciences to inspect Périer’s designs obtained from those of Watt. In his visit to Albion Mills it is a fact that he could see together with his colleagues the machine of double effect and that he even tried to make a sketch of it, when they were caught by an employee and, as a consequence, Boulton was against a later visit to the facilities of Soho. Despite

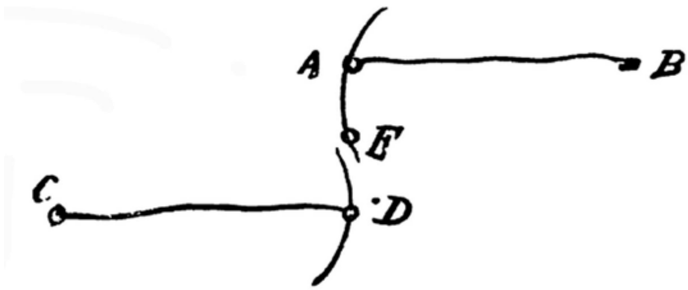

Fig. 4. Drawing by Watt of Watt's singular mechanism: This drawing accompanies Watt's explanation about the justification of the cause by which the singular mechanism of Watt generates an almost rectilinear trajectory.

having seen the machine in detail, until the point of trying to make a sketch, the information did not serve Coulomb to make any proposal of designs after his return to France.[11]

Gaspard F.C.M.R. de Prony (1755–1839), in his "Nouvelle Architecture Hydraulique" (Paris, 1796), makes reference to Betancourt's discovery and, in words that might have been transmitted directly to him by the same Betancourt, makes the following point:

> Artists must know that these observations are difficult to do, when only some few moments have been available to examine a machine masked by the building distributions, that isolate the different parts, even the outer ones, and prevent to have the matching, the set and the general effect.

In order to confirm that Betancourt, in addition to mechanical vision, had a high capacity of innovation when facing the problems of the design of mechanisms, we want to focus our view on a point that has been unnoticed to historians: without anybody revealing it to him, it was Betancourt himself who, when seeing the mechanism of connection of the piston with the rocker arm, deduced that the machine had a double effect. It is indeed at this moment when, in our opinion, an innovating contribution will take place.

Reuleaux, in the introduction to his "Theoretische Kinematik", makes the following comments about the development of the mechanism of rectilinear guidance made by Watt (Figure 4):

[11] Gouzévitch and Gouzévitch (2005), p. 21.

> Watt has shown to us in a letter some indications of the line of thought that led him directly to the alluded mechanism. 'The idea – he writes to his son in November of 1808 – was originated in the following way. Finding the double chains very inconvenient, or the racks and indented sectors for the transmission of the motion from the axis to the piston to the angular motion of the rocker arm, I worked to prove if I would be able to find some means to make the same by means of rotations around centers, and after some time it came up to me that being AB and CD two equal radios rotating around centers B and C, and connected among them by means of a rod AD, moving throughout arcs of equal length, the deviation of the straight line would be approximately equal and opposite, and the point E should describe approximately one straight line, and that if by convenience the radius CD was only half of AB, moving the closest point E to D it would occur the same, and from this construction it was derived the later denominated parallel motion'.
>
> Being interested in the content of this letter, a more meticulous examination of it reveals a deficiency that he might also have discovered. We found in it both the reasons and some of the final results of the exercise of Watt, but we do not obtain indications of any methodical sequence of ideas directed to this aim.

Reuleaux himself affirms that this letter was written by Watt twenty-four years after the invention, with a prolonged time for reflection.

In 1788 – four years after the presentation of the patent of Watt and twenty years before the letter above commented – in the Report on the Steam Engine Betancourt faces and works out the problem of dimensioning the four-bar mechanism. The first theoretical study of Watt's mechanism, that tried to determine the deviation with respect to the rectilinear trajectory, is the one carried out by Prony in the second volume of his "Architecture Hydraulique", published in 1796.[12] Nevertheless, we can affirm that it was Betancourt, in the Report on the Steam Engine presented in 1789, who made the first theoretical study of such mechanism.

In the first place he describes the problem that he tries to solve with the mechanism (Figure 5), in this detail of Betancourt's drawing, some of the parts to which he makes reference to explain the operation of Watt's singular mechanism can be observed: P′ and Q′ are the ends of the rocker arm; R′ and

[12] Prony (1796), seconde partie, pp. 123–142.

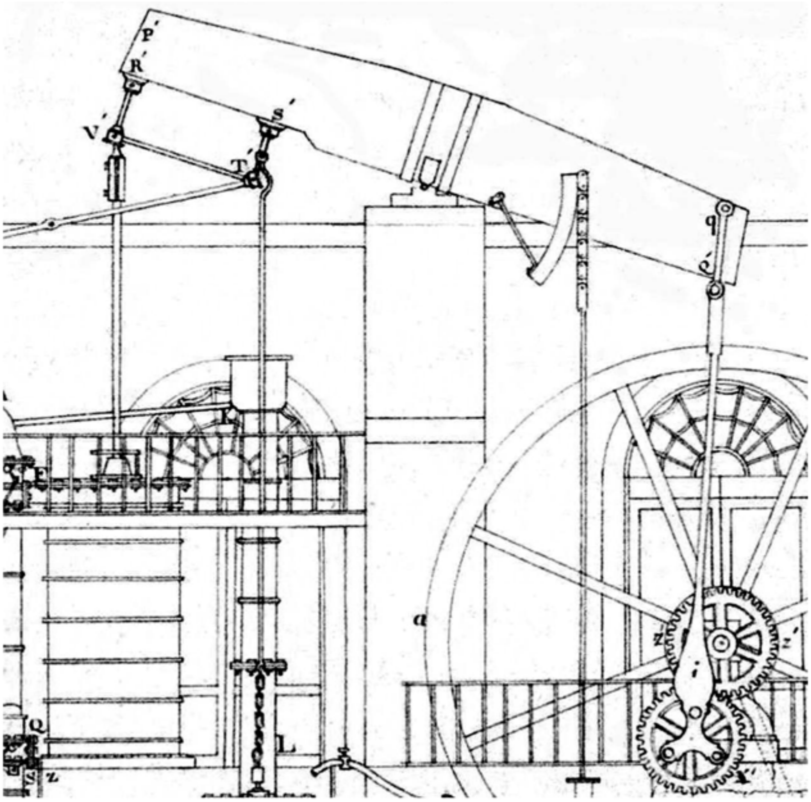

Fig. 5. Watt's double acting Steam engine: Betancourt (1789), illustration III.

S′ are the hinged connection of Watt's mechanism to the rocker arm; T′ is the hinged joint with the lower rocker arm; V′ is the hinged joint of Watt's mechanism to the axis of the piston, being the point that describes an almost rectilinear trajectory.

> We have seen that the piston WW that makes the rocker arm move, is pushed not only from top to bottom but also from bottom to top. In order to produce the first motion, it will be enough to suspend the piston from the rocker arm P′Q′ by means of a chain, as it is done in the common pumps, but is not the same for the motion from bottom to top, because the chain, folding itself, will not be able to communicate the motion to the rocker arm.

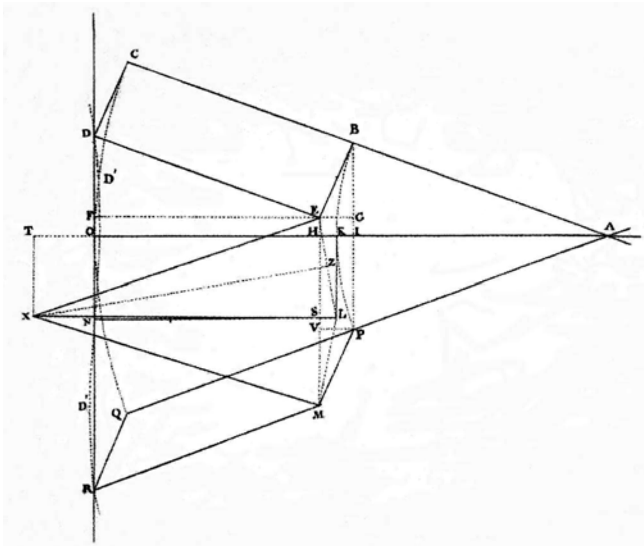

Fig. 6. Geometrical Scheme used by Betancourt to describe the operation of Watt's mechanism: Betancourt (1789), illustration IV.

> It is then necessary to find, in order to communicate to the rocker arm the double motion of the piston, means which are able to produce both effects, without moving sensibly the piston out of the vertical, since the end of the rocker arm travel a circle arc.
>
> This is what Watt has achieved by means of a parallelogram that I think it is of his invention and whose three vertices R′S′T′ have the property to travel circular arcs, whereas the fourth V′, together with the piston, describes a straight line approximately as we have seen it.

Next he set what he denotes as a general problem (Figure 6). In this figure to the upper rocker arm corresponds the segment AC, that rotates around the fixed point A; to the lower rocker arm corresponds the segment XE, that rotates around the fixed point X; to the pantograph would correspond the segments DE, EB, BC and CD, hinged in their four vertices; point D is the one that moves throughout the almost rectilinear trajectory and would be the joint point of Watt's mechanism to the axis of the piston. The drawing represents the mechanism in three positions: the upper rocker arm in the more elevated position (position AC); in the intermediate position (position AO); and in the lower position (position AQ):

> If it is wanted to deal with the problem of a parallelogram with all possible generality, trying to find all the possible solutions, the task could be presented to the geometricians enunciating the case in the following way:
>
> If in a parallelogram CB, DE the points CB draw up circle arcs of radio AC, AB and point E also draws up a circle arc of radio XE, what relation must exist between CB, BE, XE and XT so that the line drawn up by point D approximates most possible to straight line DO perpendicular to AO′

In contrast to the empirical approach of Watt, Betancourt sets out and solves the problem by means of geometrical methods, intuiting that the solution can only be approximated. That is something that will be enunciated by Hachette some years later.[13] At the sight of the complexity of the problem, Betancourt states what he denotes as the particular approach of the general problem:

> Since the only thing which interests to us in this matter is its application to the motion of the steam engine, we will limit ourselves to considering a particular case of the general statement that we have just presented.
>
> We have supposed that the sides of the parallelogram are given and are constant, that point D is on the line DR, perpendicular to AO, in three positions AC, AO and AQ and that angle CÂO is equal to OÂQ. We try to calculate the radius of the circle that goes through the three points ELM.

A clearer description of the procedure will be published later in the Essai. Therefore, Betancourt sets out and solves an example of what, many years later, will be known as the problem of path-generation Synthesis with three precision points.

In a clearer form, Lanz and Betancourt explain in the Essai this method of the so-called Evans' mechanism. A question, without answer at this time, is how this mechanism, attributed to Oliver Evans (1755–1819) and referenced in 1813, appears in the Essai in 1808 accompanied by a method for dimensioning it and without mentioning the sources from where it was taken.

[13] In "Histoire des machines à vapeur depuis leur origine jusqu'à nos jours" by Hachette (1830), he affirms without any proof that the curve described by the connection point of the axis of the piston and Watt's parallelogram is of sixth degree.

Following the description of the procedure to determine the dimensions of this mechanism that appears in the Essai, it can be notice that the method proposed is absolutely the same one that is used in the present texts of Synthesis of trajectory generation with three precision points in a four-bar mechanism.[14] In addition, it is added that this method can be used in Watt's parallelogram mechanism and in the one "solved" by Betancourt. The meaning in this text of the word "solved" is an unknown, but it is clearly about the four-bar singular mechanism patented by Watt. It is possible that, even if Betancourt had already known Watt's extended mechanism, he might have not known the singular mechanism and, therefore, it could have been rediscovered by Betancourt after Watt.

On the Circulation of Works

The Classification of Mechanisms

Although there is not much news on the influence of the Essai in later works, its successive printing in French in 1819 and 1840, in English in 1820 and German in 1829 gives an idea of the importance of the treatise.

Throughout the XIX century other contributions were made to the classifying criteria. In 1811, Borgnis, in his "Traité complet de mécanique", divides the machines in six types: receivers, connecting, modifiers, supports, regulators and operators.[15] As we can observe, it is a classifying criterion that gives more emphasis to the type of function done in the machine than to its kinematic characteristics. In 1830, Ampère, in his "Essai sur la philosophie des Sciences", classifies Monge's studies as a third order science and affirms that:

> This science must, therefore, define a machine, not as it is usually done, as a tool thanks to which the direction and intensity of a given

[14] In figure O17 in the Essai and in the text that accompanies it, the procedure for obtaining a synthesis of rectilinear guidance with three precision points is explained, in a way similar to the one used in a modern manual as, for example, Nieto (1978), pp. 102–103.

[15] According to Borgnis, the receivers are the organs of the machine destined to receive the immediate action of the motors; the connecting are those destined to transmit the movement; the modifiers are those that modify the speed of the diverse mobile bars; the supports are the centers of suspension, rotation or support of other organs; the regulators are those that correct the irregularities of the motion; and the operators are those that produce the final effect.

force can be altered, but as a tool by which the direction and speed of a given motion can be altered.

With it Ampère confers, on Monge's approach, a category of specific science for the kinematic study of machines. The Essai by Lanz and Betancourt is based on this approach.

Willis, in 1841, introduces in his book "Principles of Mechanisms", a totally new system of classification: instead of using the relation between the absolute input and output motions of the mechanism as classifying criterion, he uses the relative motions between the diverse elements, taking into account the change of direction and speed of the relative motion and whether this relation is constant or variable. He considers that Ampère's definition is in opposition to some of the examples included in the Essai, such as hydraulic wheels and wind-operated mills. Willis only considers mechanisms that are compounded exclusively of solid bodies. Reuleaux affirms that this criterion introduced by Willis was not successful even in England, so that, in general, it was preferred to continue using the Lanz and Betancourt criterion.

Laboulaye, in his "Cinématique" in 1849, rejects Willis's system, and divides the elements of the machines in three types: lever system, winch system and plane system, to which any mobile body belongs depending on whether it has one, two or three fixed points respectively. Nevertheless his approach, based on the motion of points, is not applicable to the motion of bodies.

Morin, in his book "Notions géométriques sur les mouvements", in 1851, and Weisbach, in his paper "Abänderung der Bewegung" (Alteration of the motion), in 1841, remained faithful to Lanz and Betancourt's system. Redtenbacher, Reuleaux's teacher, in "Die Bewegungsmechanismen", printed in 1857, describes and analyzes theoretically the collection of mechanisms in Karlsruhe, and he classifies them by their use, without using any kinematic criterion. Reuleaux himself will describe the approach as nihilistic.

In France, geometrical methods were developed to study the motion of rigid bodies. Example are in Poinsot's book "Théorie de la rotation des corps", which was followed by others, such as the "Eléments de géométrie applique à la transformation du movement" by Girault, in 1858; the "Cinématique" by Belanger, in 1864, and "Traité des mécanismes" by Haton, in the same year. Despite the different approach with respect to the Essai, Girault and Belanger follow the classifying criterion of the transformation of motion. Haton establishes nine categories: rollers, slides, eccentrics, gears, bars and cables, and he groups the last three under the denomination of accessory elements.

Through this development we can observe an insuperable separation between what we could denominate Theoretical Kinematics, of which several of the previously mentioned designers are examples, and Applied Kinematics, in which the approach of the Essai would not be surpassed until 1875. Reuleaux was wondering where the insufficiency of the methods developed until that moment so that in fact Monge's classification developed in Lanz and Betancourt's book was not surpassed. He responds by affirming that the insufficiency of such classification and the later contributions comes from the fact of using the transformation of motions as classifying criterion, without inquiring into the reason for such transformations. Reuleaux discovers that the fundamental reason for the transformation of motion lies on the constraints imposed by the kinematic pairs on the different types of joints between solids. This is the starting point for new classifying approaches.

It is possible to state that the Essai constitutes a first contribution to what would be later called Synthesis of Type. The approach started by this book, and continued by Reuleaux, reached its height in Artobolevsky (1976) that contains more than 5,000 mechanisms, classified by structural and application criteria.

The Rectilinear Guidance

Throughout the XIX century, enormous interest was raised on the part of engineers and mathematicians, to study the properties of the trajectories drawn up by the points of the coupler of a four-bar mechanism. The interest began to appear in France and later it transferred to England and Germany.

As we have already mentioned previously, in 1796, Prony printed the second volume of his "Architecture Hydraulique" and in it he developed the first study on the deviation of the trajectory of Watt's mechanism with respect to the straight line.

Hachette, in his "Histoire des machines à vapeur depuis leur origine jusqu'à nos jours" (1830), includes a proof of equivalence between Watt's mechanism and that developed by Oliver Evans in his Columbia machine before 1813. In 1836, Alexandre Joseph Hidulphe Vincent (1779–1868) published for the first time the equation of the curve of the point that generates trajectories that are almost rectilinear.

Simultaneously, the great Russian mathematician Pafnuti Chebyshev (1821–1894) tried to look for better solutions for the approximate drawing of rectilinear trajectories and showed his pessimism with respect to the possibility of finding a four-bar mechanism that could draw up precisely a straight line.

It seems that Vincent's work had influence on Charles Nicolas Peaucellier (1832–1913) who, in 1864, was the first to obtain a four-bar mechanism that drew up precisely a rectilinear trajectory.

Many have been the mechanisms developed after Watt's with the purpose of drawing up trajectories that are almost rectilinear. Some of them can be observed in Artobolevsky (1976).

A group of English mathematicians were affected by this interest in mechanisms and, particularly, in the trajectories drawn up by some of its creators, who were precisely the ones that made important contributions to theoretical kinematics: Arthur Cayley (1821–1895), Harry Hart (1848–?), Alfred Bay Kempe (1849–1922), Samuel Roberts (1827–1913) and James Joseph Sylvester (1814–1897).

Roberts and Cayley's works are connected more directly with developments in analytical geometry, mainly in the theory of algebraic curves. Roberts justifies his interest in mechanisms with the purpose of being able to draw up and study the properties of the; he states that the motion of a point of the coupler of a four-bar mechanism describes a curve of sixth degree and that there are three different four-bar mechanisms whose coupler generates the same trajectory. Sylvester and Kempe's works, more elementary, are connected clearly with the possibility of inventing new instruments. Their work reflects in a more evident way that mechanical engineering was, at that moment, one of the dominant technologies.

Beyond the mechanisms that draw up rectilinear trajectories in an exact or approximate form and the properties of the trajectories, a problem remained unsolved and it was guessed by Betancourt and it would be approached in a rigorous form one hundred years after the presentation of the Report on the steam engine: the obtaining of the dimensions of a mechanism that allows a certain trajectory to be generated. Burmester published his "Lehrbuch der Kinematik" (1888) where he sets out for the first time geometrical methods for the solution of the problem of path-generation synthesis.

Modern Interpretation of Main contributions to Mechanism Design

Classification of Mechanisms and Structural Synthesis

The collection and classification of mechanisms is still an important tool that helps design since it facilitates the search of mechanisms that fit the application you are trying to develop. The introduction of concepts such as kinematic pair, link and its different types, has allowed the handling of the classification of mechanisms under new approaches. Classification is not limited to motion transformation criteria, but also takes into account another important element: the structure of the mechanism, understanding as such the definition of the number of kinematic pairs, links, its types and the way in which they are interconnected. Another important classifying element appears associated to the structure, which is the number of degrees of freedom of the mechanism.

The best collection and classification of mechanisms can be found in Artobolevsky's work. Throughout its five volumes (Artobolevsky, 1975), some four thousand mechanisms have been gathered and classified by their structural and functional characteristics. The first and second volumes are devoted to the n-link mechanisms. The third volume is devoted to gear, cam and friction mechanisms. The fourth and fifth volumes are devoted to mechanisms with flexible links and to hydraulic, pneumatic and electrical mechanisms.

Beyond the collection and classification of mechanisms, the study of their kinematic structure has given rise to an important research field within the Machines and Mechanisms Science, which is structural kinematics. Different problems have been approached in kinematic chains and mechanisms: structural synthesis, the problem of isomorphism, structural analysis, the automatic development of mechanisms and the application of structural synthesis to creative design. For them, tools such as the Franke's notation, graph theory and group theory have been used.

There have been important contributions to this field in the 1960s and 1970s from authors such as Crossley, Freudenstein, Hain and Manolescu. In recent years outstanding work has been done in the field of isomorphism detection by authors such as Agrawal and Rao, and in the field of analysis aided by the computer about the structure of kinematic chains by Mruthyunjaya. Diverse works give a vision of the state-of-the-art and the latest contributions in this field (Mruthyunjaya, 2003; Kota, 1993). The research that is carried out in this field makes available to the machine designer computational tools that facilitate the generation of alternatives in an automatic way so that they fulfil the design criteria given.

Path Generation Synthesis

Path generation synthesis, as part of mechanism dimensional synthesis, has had an important development throughout the second half of twentieth century and continues in the twenty-first century.

The first methods of synthesis exploit the concept of precision points to obtain mechanisms whose trajectory goes exactly through a set of specified points. The problems that were encountered and solved had to do fundamentally with the development and solution of equations that include the condition of going through the precision points and with the problems associated with optimal spacing between the points in order to minimize error. Throughout the 1950s and 1960s, there have been important contributions from authors such as Freudenstein, Suh, Sandor, Roth, Gupta and others.

Due to limitations in the number of precision points, the importance of the application of optimization methods, whose objective is minimizing an objective function with or without constraints, has been greater and greater. In the case of the path generation synthesis, the objective function to minimize can be the error calculated as the difference between the desired trajectory and the generated trajectory. Diverse approaches have been used to solve the problem. A summary of contributions can be found (Angeles, 1993), where different families of methods appear: using least-square normality condition and Lagrange multipliers, general unconstrained optimization, constrained optimization using penalty functions, constrained optimization using general nonlinear programming techniques, mini-max optimization, methods based on probability and statistics. Recent developments include the use of genetic algorithms that try to avoid the problems associated with the selection of the initial solution and its consequences on convergence.

Another completely different approach is based on the use of the storage of coupler curves in a computer database for their comparison with the shape of the curves. To counter the problems generated by the slowness and the possible lack of convergence of optimization methods, other methods that use neural networks have appeared to try to take advantage of the previous knowledge available concerning the problem and its possible solutions.

Structural synthesis and path generation synthesis, to which Betancourt contributed so much to their origin, are still research fields of high interest as evidenced by the quality of MMS researchers that have devoted their efforts to these fields and by their practical importance to machine design.

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OENE BOTTEMA
(1901–1992)

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Abstract. Oene Bottema was a 20th century Dutch mathematician working in geometry. In the third quarter of the 20th century, the great majority of mathematicians had turned away from classical geometry. At the same time kinematics continued to play an important role in mechanical engineering. The engineers needed a man like Bottema who applied his encyclopedic knowledge of 19th century geometry with 20th century precision to many kinematical problems. He wrote almost 50 papers on kinematics. Most of them contain original contributions. He invented the method of instantaneous invariants in instantaneous kinematics. With Bernard Roth he wrote the influential survey *Theoretical Kinematics*, North Holland, Amsterdam, 1979.

Biographical Notes

Oene Bottema was born on December 25, 1901 in Groningen, the capital of the province of Groningen in the north of the Netherlands. He belonged to the 10th generation of a family of Bottemas living in Friesland, the province next to Groningen.

After primary school, Bottema attended the Hogere Burgerschool (H.B.S.) in Groningen. This type of school had been introduced in The Netherlands in 1863. Its five year courses were intended to prepare pupils for business and industry. The curriculum included mathematics and science in addition to Dutch and three modern foreign languages: English, French and German. The introduction of the H.B.S. in the Netherlands was of great significance. The school enabled many children with a middle class, or even lower class, background to get an excellent secondary education. Moreover in 1917 parliament decided that the H.B.S. diploma was good enough to give access to university studies in the sciences and medicine. Bottema was among the

M. Ceccarelli (ed.), Distinguished Figures in Mechanism and Machine Science, 61–78.

Fig. 1. Oene Bottema (1901–1992).

first to go directly from the H.B.S. to university. At the age of seventeen he enrolled at the university in his home town in the Department of Mathematics and Physics.

Bottema finished his university studies in 1924 and soon obtained a position as teacher at a secondary school, an H.B.S. He would work in secondary education until 1941, although from 1931 until 1935 and from 1937 until 1940 he taught classes at, respectively, the universities of Groningen and Leiden as an external lecturer (privaat-docent). He was a good manager and from 1933 to 1935 he was director of the State H.B.S. in Sappemeer, near Groningen. From 1935 until 1941 he was director of the State H.B.S. in Deventer.

In 1930 Bottema married Femmy Berendsen. She was two years younger. She gave him a daughter born in Groningen and a son born in Deventer.

As we will see below, while working as a teacher and director at secondary schools Bottema got his doctor's degree in 1927. He published an average of more than three papers per year in this period and he wrote a book. At the age of forty the name Bottema had become very well known among Dutch mathematicians. His scientific qualities were generally recognized and in 1941 he became full professor for pure and applied mathematics at the Technological University in Delft. From 1951 to 1959 he was rector magnificus (president or vice chancellor) of the university. He was a born manager, honest and hard working. As a rector he was admired and feared. He was, moreover, a gifted orator. The speeches that he gave as rector were works of art. It is also amazing that while rector he still managed to write 45 papers.

After 1959 Bottema devoted himself to teaching and scholarly work in mathematics and kinematics.

In 1961 Bottema met Ferdinand Freudenstein and through Freudenstein he met Bernard Roth. With his encyclopedic geometrical knowledge Bottema was an ideal partner for the Americans. The cooperation undoubtedly stimulated Bottema to do some excellent work in kinematics which he otherwise would not have done.

He received many distinctions. In 1954 he became Officier de l'Académie Française. The Dutch government made him, in 1955, Knight in the Order of the Nederlandsche Leeuw and, in 1959, he became Commander to the Order of Orange Nassau. In 1958 he got the degree of Doctor of Laws, honoris causa from the University of Leeds. He was honorary member of the Dutch Mathematical Society and of IFToMM (International Federation for the Theory of Machines and Mechanisms). At his retirement in 1971 he received the Gold Medal of the city of Delft.

After his retirement he continued to work until a few years before his death (of old age) in Delft on November 30, 1992. His wife Femmy had died in 1981. Femmy died of multiple sclerosis after over thirty years of gradual physical deterioration during which she became completely dependent on a wheel chair. In all these years Bottema took care of her.

Bottema was an erudite man, the opposite of a man only occupied with mathematics. He had a vast knowledge of history and literature, which is obvious from his public lectures and articles in literary journals. On his seventieth birthday his friends gave him a book, which he had in fact written himself. It contains a selection of his best public addresses and literary articles. The title of the book *Steen en Schelp* (Stone and Shell) was taken from an address with the same title. Bottema admired Wordsworth, who in Book V of *The Prelude* uses the stone and the shell to represent respectively geometrical truth and poetry. For Bottema the unshakable argument of geometry and the inspiration of poetry were the two indispensable guides in life. He disdained belabored and messy solutions in his mathematics and in all other aspects of his life. As a mathematician, an administrator and a human being, Bottema always attempted to create a good balance between content and form. That is how we remember him; mathematics, poetry and, of course, the presence of his inseparable pipe.

Bottema's Main Works

Bottema published more than 400 papers and 10 books (including his doctoral dissertation). He published nearly 50 papers on kinematics (see the references). Many of the papers can be considered as preparatory work for (Bottema and Roth, 1979).

Review of Main Work Related to Mechanism Design

The Background

In order to understand Bottema's work, it is necessary to briefly consider the developments in geometry in the 19th century. In the eighteenth century in mathematics developments in analysis and its applications in mechanics had dominated. This changed radically in the nineteenth century. The nineteenth century was a golden age for geometry. Renaissance architects already knew the technique of projecting a building on two perpendicular planes, but in the late 1760s Gaspard Monge started to use and advocate such methods at the military school at Mézières. It was classified knowledge, so only in the 1790s could Monge go public with the details. He gave courses on descriptive geometry at the Ecole Polytechnique and in 1799 J.N.P. Hachette published Monge's lectures as the book *Géométrie descriptive*. The largest part of this book is devoted to surfaces and skew curves. Monge considered in particular surfaces that can be kinematically generated by means of moving curves. His research in this respect can be seen as a continuation of work done by Gilles Personne de Roberval and Isaac Newton. Monge created a new school of geometers in France. Monge also established the theory of machines and mechanisms as a separate subject at the Ecole Polytechnique. This was the beginning of spectacular developments in geometry and in kinematics that would last until the middle of the 20th century. Monge was one of the first mathematicians participating in these developments who combined an interest in both geometry and mechanisms.

As for geometry, after Monge, Jean Victor Poncelet rediscovered and improved projective geometry. In the further development there was a split.

Some, like C.G.C. von Staudt concentrated on synthetic methods in geometry. Von Staudt was joined by, for example, Steiner, Chasles, Cayley and Cremona. Others like August Ferdinand Möbius and Julius Plücker concentrated on algebraic methods. They were followed by Clebsch, Gordan, Brill and Klein. The 19th century was also the period in which the first non-Euclidean geometries were introduced by Gauss, Bolyai and Lobachevski. Bernhard Riemann introduced the idea of geometries of arbitrarily high dimensions. All these developments made geometry look like a discipline consisting of many very different, not always cohering, subjects. Very famous in this respect is Felix Klein's *Erlanger Programm* of 1872 in which he reunified the subject by showing that all these different geometries could be defined by means of a group of transformations. In geometry, Klein taught us, one always studies properties that are *invariants* under a group of transformations.

Although in the 19th century geometry was one of the most fertile research areas in mathematics, often the methods that were used lacked rigor. At the end of the 19th century, in 1899, David Hilbert published *Grundlagen der Geometrie* in which the modern axiomatic approach to geometry was introduced. At the end of the 18th century geometry was restricted to Euclidean space, which could be studied synthetically or analytically. In the first half of the 20th century the word geometry had begun to refer to the properties of many different spaces and their relations. Also methodologically the subject had changed. Many new methods had been designed. The use of matrices to describe displacements, quaternions, dual numbers, isotropic coordinates, were all introduced in the 19th century. Moreover, when Bottema was introduced to this new world, the foundational problems that had vexed 19th century geometry had been solved, and it had become possible to do geometry in a very rigorous way.

Bottema was working in this essentially 19th century tradition in which many mathematicians were working in geometry; many of them combined this with a great interest in kinematics. As for kinematics, he was aware of the fact that he stood on the shoulders of the famous 19th century mathematicians Chasles, Mannheim, Darboux, Burmester and many others. And he was always very satisfied when he succeeded in reaching a little bit further with respect to the problems that these great men had studied as well.

Bottema's Geometrical Work

The University of Groningen was founded in 1614. From 1695 until 1705 Johann Bernoulli (1667–1748) had been professor of mathematics at the university in Groningen. It should be noted that Bernoulli discovered the instantaneous centre of rotation in planar instantaneous kinematics. After the departure of Bernoulli the level of mathematics education in Groningen had fallen. Fortunately things had changed in the second half of the 19th century. Pieter Hendrik Schoute (1846–1913), a specialist in multi-dimensional geometry, had also been professor of mathematics in Groningen and he had raised the level of geometry teaching considerably. The famous Dutch astronomer Jacobus Cornelius Kapteyn (1851–1922) worked in Groningen as well. When Bottema enrolled at the University of Groningen, mathematics and science were taught at an international level. In particular the classes of J.A. Barrau (1873–1953), professor of geometry, had a great influence on Bottema and so did the lessons in theoretical physics of Frits Zernike (1888–1966), winner of a Nobel Prize for physics in 1953.

Bottema started publishing even before he finished his university studies in 1924, and he went on doing so until a few years before his death. In 1927 he defended his doctoral thesis at the University of Leiden. His supervisor was W. van der Woude. The title of his thesis was *De figuur van vier kruisende rechte lijnen* (The figure consisting of four mutually skew lines). In the thesis Bottema deals with the projective properties of such figures. Grassmann had discovered that the projective properties of such a figure are characterized by the so-called Grassmann harmonic ratio, a projective invariant for four mutually skew lines analogous to the harmonic ratio for four points on a line. In 1878 Voss had discovered that although a calculation by means of degrees of freedom suggests otherwise, in general there does not exist a third degree curve of double curvature that touches four given straight lines. And if there is such a curve there exist infinitely many of them. In this case the four lines form a Voss-quadruple. In the thesis Bottema studies such Voss-quadruples and other special quadruples of skew lines.

In his thesis Bottema's elegant style in mathematics is already present. Very elegant is also a paper that Bottema published in 1928 on the introduction of coordinates in projective geometry. The question whether the analytical method and the synthetic method in geometry are, with respect to their results, equivalent had been answered before. Bottema's solution, however, is lovely. Addition and multiplication are defined with respect to points on a conic section. Pascal's theorem and the well-known properties of projectivities on a conic section imply that the resulting system is a field.

In 1938 Bottema published his first book, *De elementaire meetkunde van het platte vlak* (The elementary geometry of the plane). His goal was twofold. He had noticed that most books on plane geometry were, either, not completely rigorous, although they proceeded quickly to non-foundational problems, or were restricted to foundational questions. Bottema wanted both in one book. He wanted, on the one hand, to give a completely rigorous treatment of plane geometry, and, on the other hand, to deal with interesting non-foundational questions as well.

De elementaire meetkunde van het platte vlak was well written and well received. It is remarkable that from the book it is Bottema's theorem on pedal points which is best remembered (Dergiades et al., 2003; Sashalmi et al., 2004). It says that if we consider a triangle ABC and on its three sides AB, BC, CA, respectively, three points P_c, P_a and P_b, the relation

$$AP_b^2 + CP_a^2 + BP_c^2 = AP_c^2 + BP_a^2 + CP_b^2$$

is a necessary and sufficient condition for P_a, P_b and P_c to be pedal points: projections on the three sides of one and the same point P inside the triangle.

Bottema's Kinematical Work

Bottema developed research interest in kinematics during World War II. In 1944 he published three papers on kinematical subjects. Two of them concern the Darboux motion in elliptic space. In 1881 Darboux completely solved the following problem: Determine all possible motions of a moving Euclidean space E with respect to a fixed Euclidean space Σ such that the path of every point of E is a plane curve in Σ. There are trivial solutions, for example when the planes of the paths are all parallel. Darboux has shown that the only existing non-trivial solutions are such that all paths are ellipses. The easiest way to see that such motions exist is the following. In plane elliptic motion the points on the moving polhode, a circle, oscillate on straight line segments, diameters of the fixed polhode, also a circle. When we combine such elliptic motion with a harmonic oscillation with a corresponding period in a direction perpendicular to the plane, those points obviously describe planar oval curves.

In 1944 Bottema solved the analogous problem for 3-dimensional elliptic kinematics: in elliptic space there exist motions analogous to the Darboux motions in ordinary kinematics.

The third 1944 paper on kinematics is on the coupler curve. Darboux had already shown that the different positions of a four bar mechanism can be mapped on a third degree curve. Bennett, Hippisley, Morley and Weiss had shown that pairs of isogonal points on a coupler curve could be Laguerre mapped on Hessian pairs of points on a third degree curve. It is then tempting to suppose that the set of coupler curves of a given four bar mechanism corresponds to the set of coupler curves that can be generated from a certain third degree curve by means of such a Laguerre mapping. Bottema showed by means of two examples that this supposition is wrong.

The two papers on Darboux motion in elliptic space and the paper on the coupler curve nicely illustrated what Bottema was good at. In the middle of the 20th century many mathematicians were losing interest in classical 19th century geometry. Leading mathematicians like David Hilbert, Emmy Noether, Barteld van der Waerden and the members of the French Bourbaki school had developed a new view of mathematics: mathematics was no longer merely the science of magnitude and number, but it had become the science of all mathematical structures. Although in mechanical engineering the interest in kinematics was growing, many mathematicians were caught up in those new developments and turned away from classical geometry and kinematics to the investigation of the multitude of new abstract structures. This new structuralist research programme in mathematics was extremely fertile and it has led to spectacular developments. Bottema however, continued to work on classical subjects. Most of his papers find their starting point in existing results, which are then corrected, improved or generalized by Bottema. They do not contain huge jumps forward, but his extensive knowledge, his critical sense, his feeling for elegance and his ability to choose exactly the right method for the problem involved, enabled him again and again to add new results to the existing ones. There is, however, one exception. With his notion of instantaneous invariants he succeeded in defining a new research program. The idea was born in Bottema (1949) and Bottema seems to have shelved the idea in the period in which he was rector magnificus of the Technological University. Only in (1961a) did he return to this line of research. In 1944 the outstanding German kinematician Hermann Alt (1889–1954) from Dresden had written a paper in which he had criticized a long paper from 1938 by Kurt Rauh et al. on Cardan positions for the plane motion of a rigid body. Such positions are positions of a moving plane which have a third order

We have

$$\left.\begin{array}{ll} X' = -x\sin\varphi - y\cos\varphi + a', & X'' = -x\cos\varphi + y\sin\varphi + a'' \\ Y' = x\cos\varphi - y\sin\varphi + b', & Y'' = -x\sin\varphi - y\cos\varphi + b'' \\ \multicolumn{2}{c}{X''' = x\sin\varphi + y\cos\varphi + a'''} \\ \multicolumn{2}{c}{Y''' = -x\cos\varphi + y\sin\varphi + b'''} \end{array}\right\} . \quad (2)$$

For the instantaneous centre of rotation P, one has $X' = Y' = 0$, thus

$$x = a'\sin\varphi - b'\cos\varphi, \quad y = a'\cos\varphi + b'\sin\varphi \quad . \; . \; . \quad (3)$$

and

$$X = -b' + a, \quad Y = a' + b. \quad . \; . \; . \; . \; . \; . \; . \quad (4)$$

These equations determine p_m and p_f respectively. For $t = 0$ the coordinates of P are $X = x = -b'_0$, $Y = y = a'_0$. Thus if we choose the coinciding origins for $t = 0$ in the centre P, we have $a'_0 = b'_0 = 0$. The direction of the tangent of p_f in P is given by the ratio $-b'' + a'$, $a'' + b'$, thus for $t = 0$ by $-b''_0$, a''_0 ; this is also the direction of the tangent of p_m in P for $t = 0$. Taking this common tangent p for the X-axis we have $a''_0 = 0$. The possibilities arising from the arbitrary choise of the system of coordinates have now been exhausted and we arrive at the following conclusion: the equations for the motion (1) can be put in a canonical form for which:

$$a_0 = b_0 = a'_0 = b'_0 = a''_0 = 0. \quad . \; . \; . \; . \; . \; . \; . \quad (5)$$

Then we have for $t = 0$ in view of (2):

$$\left.\begin{array}{llll} X = x, & X' = -y, & X'' = -x, & X''' = y + a'''_0 \\ Y = y, & Y' = x, & Y'' = -y + b''_0, & Y''' = -x + b'''_0 \end{array}\right\} . \quad (6)$$

Fig. 2. The birth of the instantaneous invariants in Bottema (1949).

contact with an elliptic motion. Although he appreciated Alt's work, Bottema felt that the synthetic method that Alt had used lacked precision.

Bottema solved the controversy by introducing the instantaneous invariants although he did not yet use the name. The basic idea of the method of instantaneous invariants is simple. The position of a moving frame of reference with respect to a fixed frame of reference is determined by a function which in a particular position can be developed in a Taylor series. The parameters that describe the motion (including the parameter that represents time) and the two frames of reference are chosen in such a way that as many as possible of the coefficients of the Taylor series vanish. The other coefficients are invariants under the group of Euclidean transformations. The instantaneous properties of the motion are determined by these invariants. The example of plane instantaneous Euclidean kinematics will illustrate this.

If x, y; X, Y are the Cartesian coordinates of a point in the moving plane and the fixed plane respectively, we have

$$X = x\cos\varphi - y\sin\varphi + a; \quad Y = x\sin\varphi + y\cos\varphi + b, \tag{1}$$

a and b being functions of φ. We will denote the values of the n-th derivatives with respect to φ of the variables a and b for the position $\varphi = 0$ with the suffix n. This means that the zero-order properties (they concern merely position) in the position $\varphi = 0$ are determined by a_0 and b_0. The first-order properties (they concern tangents) in the position $\varphi = 0$ are determined by a_1 and b_1. And the second-order properties (they concern curvature) in the position $\varphi = 0$ are determined by a_2 and b_2.

When at $\varphi = 0$ the two coordinate systems coincide we have in that position $a_0 = b_0 = 0$ and thus

$$X = x \quad \text{and} \quad Y = y. \tag{2}$$

When the origin of the two systems coincides with the instantaneous centre of rotation at the moment $\varphi = 0$, we have $a_1 = b_1 = 0$ and thus

$$X' = -y \quad \text{and} \quad Y' = x. \tag{3}$$

The primes denote the derivative with respect to φ in the position $\varphi = 0$. We let the velocity vector of the position of the pole in the fixed system coincide with the coinciding x-axes at this moment, so $a_2 = 0$ as well. We then get for the second-order properties

$$X'' = -x \quad \text{and} \quad Y'' = -y + b_2. \tag{4}$$

Nota bene: the result is that in the position under consideration we can finally choose the direction of the two coinciding x-axes at the moment $\varphi = 0$ opposite to the direction in which the pole is moving on the fixed polhode. This means that b_2 is positive. The derivatives a_n and b_n with $n > 1$ are the instantaneous invariants. We will illustrate the geometrical significance of b_2. The points in the moving plane that are in an inflexion point of their trajectory satisfy the relation X'': $Y'' = X' : Y'$, which gives the equation of the inflexion circle by means of (3) and (4).

$$x_2 + y_2 - b_2 y = 0. \tag{5}$$

Clearly b_2 is equal to the diameter of the inflexion circle. By means of Equations (4) and (6) we can calculate the curvature of the two polhodes at the pole.[1] For the curvatures k_f and k_m of respectively the fixed and moving polhodes at the pole we find

$$k_f = -(a_3 + b_2)/b_2^2 \quad \text{and} \quad k_b = -(a_3 + 2_b 2)/b_2^2 \tag{6}$$

and this immediately yields a version of the Euler–Savary relation

$$k_f - k_b = 1/b_2. \tag{7}$$

In Bottema (1949) Bottema applied this method for the first time. He proved that for elliptic motion we have $a_3 = b_3 = 0$. Then (6) and (7) yield that $k_f = 2 \cdot k_b$. This is obviously a necessary condition, but it is not a sufficient one for a Cardan position. In this way Bottema could precisely point out where Alt had been right and where Rauh had been right on Cardan positions.

The fact that Bottema met Ferdinand Freudenstein in 1961 and that he later got in touch with Bernard Roth had a great influence on Bottema's research. At the time the graphical methods that had been very popular in mechanism design until after World War II were being replaced by analytical methods leading to systems of equations that could be solved by a computer. The two Americans were very much part of that development and Bottema with his extensive knowledge of geometry was an ideal partner. The excellent publication (Bottema, 1964) on the analytical determination of the four Burmester points was clearly written with the intention to yield a method suitable for numerical computation.

The method of instantaneous invariants was repeatedly used by Bottema. Bottema (1967a) and Bottema (1971a) are excellent papers. In Bottema (1976a) he showed how the use of the instantaneous invariants makes it possible to generalize the well-known results concerning the existence in instantaneous plane kinematics of the inflexion circle, Ball's point, the circling-point curve and Burmester's points, by asking the question of which points lie on a parabola for five or six positions or on a conic section for six or seven positions. One of the results is that at each moment there are six points of the moving plane which pass through a stationary parabolic or sextactic point of

[1] We apply the formula: curvature $k = (X'Y'' - X''Y')/(X'^2 + Y'^2)^{3/2}$.

their path, namely a point for which six consecutive positions are on a parabola. Similarly there are at each moment eleven points in the moving plane which pass through a septactic point of their path, namely a point for which seven consecutive positions are on a conic. In (1971a) Bottema studied the motion of a rigid body with two degrees of freedom. Obviously the trajectories are surfaces. The famous German geometer and kinematician, Wilhelm Blaschke, had expressed pessimism with respect to the possibility to study the second order properties of these surfaces, for example, the properties of the curvature. Yet in (1971a) Bottema succeeded in deriving some results for the curvature.

The method of instantaneous invariants was also successfully used by others (Kamphuis, 1969) and in particular by Bottema's pupil Geert Remmert Veldkamp (1963, 1976, 1983). Veldkamp's thesis (1963) was supervised by Bottema. It contains a very thorough treatment of planar instantaneous kinematics by means of instantaneous invariants.[2]

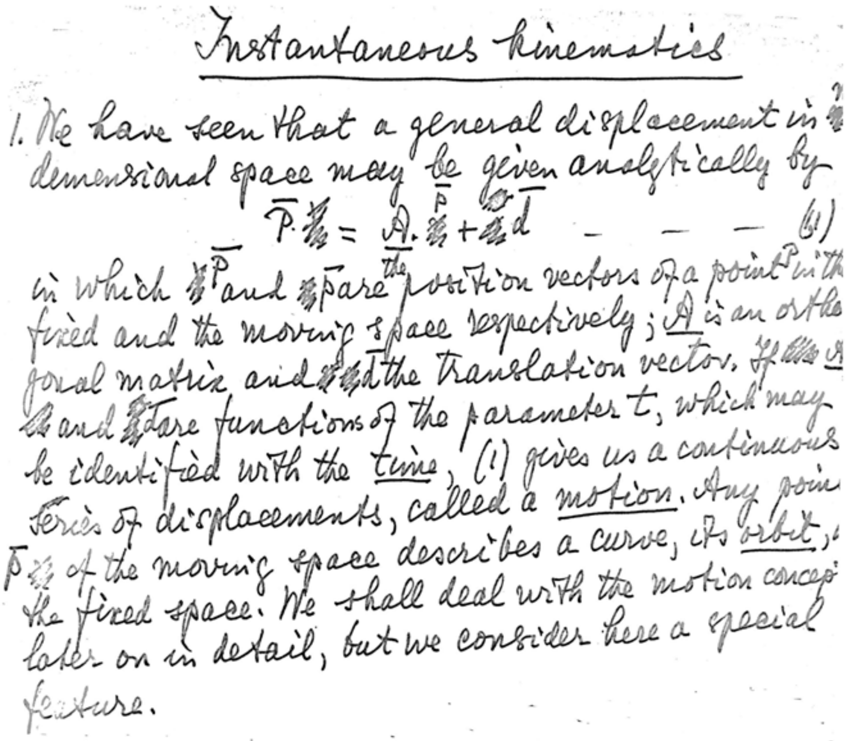
The method is also used in what must be considered as Bottema's opus magnum in kinematics, the textbook *Theoretical Kinematics* which he wrote together with Bernard Roth (Bottema and Roth, 1979). To me the book seems to be an example of a perfect cooperation between a mathematician and an engineer. Without Roth, Bottema would never have written a book on kinematics and without Bottema, Roth would have written a different book.

Theoretical kinematics deals with the general kinematical properties of motion; in theoretical kinematics mechanisms only occur as devices that generate specific motions of which the properties either illustrate the theory or are remarkable from the point of view of the general theory. Applied kinematics specifically deals with mechanisms and their kinematical properties. In (1979) Bottema and Roth deal with theoretical kinematics. Most of the book is devoted to Euclidean kinematics. The following list of the first eight chapters gives a good idea of the structure of the book:

Ch. I: Euclidean displacements
Ch. II: Instantaneous kinematics
Ch. III: Two positions theory
Ch. IV: Three positions theory
Ch. V: Four and more positions
Ch. VI: Continuous kinematics

[2] G.R. Veldkamp, professor at the Technological University of Eindhoven, was an excellent kinematician. His textbook (Veldkamp, 1970) is lovely, but unfortunately in Dutch.

Ch. VII: Spherical kinematics
Ch. VIII: Plane kinematics


Instantaneous kinematics

1. We have seen that a general displacement in n-dimensional space may be given analytically by

$\bar{P} = \underline{A}.\bar{p} + \bar{d}$ — — — (1)

in which $\bar{P}$ and $\bar{p}$ are the position vectors of a point in the fixed and the moving space respectively; $\underline{A}$ is an orthogonal matrix and $\bar{d}$ the translation vector. If $\underline{A}$ and $\bar{d}$ are functions of the parameter t, which may be identified with the time, (1) gives us a continuous series of displacements, called a motion. Any point $\bar{p}$ of the moving space describes a curve, its orbit, in the fixed space. We shall deal with the motion concept later on in detail, but we consider here a special feature.

Fig. 3. Bottema's handwriting. A concept of the beginning of chapter 2 of Bottema and Roth (1979).

The fundamental notion is the notion of Euclidean displacement. The treatment is analytical, based on vector algebra, matrix algebra and calculus. The basic properties of Euclidean displacements for n-dimensional space are derived in the first chapter. The second chapter introduces the investigation of a time-dependent displacement of n-dimensional space at a particular instant. The next three chapters are devoted to discrete 3-dimensional kinematics, respectively to two, three and four position theory. The next three chapters deal with 3-dimensional space kinematics and two of its very important special cases: 3-dimensional space kinematics, spherical kinematics and plane kinematics. The authors start with general considerations and then move slowly to special cases. The last five chapters of the book deal with special subjects:

Ch. IX: Special motions
Ch. X: n-Parameter motions
Ch. XI: A mapping of plane kinematics
Ch. XII: Kinematics in other geometries
Ch. XIII: Special mathematical methods in kinematics.

Chapter XI is remarkable. In 1911 Grünwald had introduced the idea to map the set of displacements on the points of a three-dimensional space. The motion of a one-degree of freedom mechanism then corresponds to a curve in space and properties of this curve to properties of the mechanism. Blaschke and Müller had used quaternions to bring this about and they had used the method to derive theorems in plane kinematics. Bottema's friend, H.J.E. Beth, had written a book about it using geometrical reasoning (Beth, 1949). Although the mapping was mathematically interesting in 1979 the subject may have seemed very far away from applications. However, in the mean time the kinematic mapping turned out to be a useful and elegant approach to the kinematic analysis of parallel robot platforms (cf. Husty, 2003).

Fig. 4. Oene Bottema after his retirement. (Photo courtesy of Jurgen M. Bottema.)

On the Reception of His Work

Theoretical Kinematics was very well received. In *Mechanism and Machine Theory*, A.S. Hall Jr. wrote: "This is a very important book because it is unique. There is nothing comparable available." Hall was right, a comparable book did not exist. For several decades the book provided the much needed background knowledge for much work in applied kinematics. In the *Zentralblatt für Mathematik*, D. Mangeron called the book "a masterly addition to theoretical kinematics". And indeed the authors expanded the theory and filled various gaps.

The only critical remark can be found in the review written by R. Connelly for the *Mathematical Reviews*. He wrote: "The results are never put in the form of theorems and proofs. Important statements appear only in italics as part of the general discussion." Yet this is hardly a serious criticism. The book indeed has its own style which deviated from what had in 1979 become the standard way to present mathematical results. This rigorous formal style based on the language of modern logic and set theory is for non-mathematicians often very inaccessible. Of course, the authors wrote for mathematicians and many others, but primarily they wrote for mechanical engineers. The book was a great success. In 1990 a reprint of the book appeared in the Dover series.

Acknowledgements

Most of the biographical details in this paper are based on Veldkamp (1986). I am also grateful to Bernard Roth for reading a previous version.

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WILLIAM KINGDON CLIFFORD (1845–1879)

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Abstract. William Kingdon Clifford was an English mathematician and philosopher who worked extensively in many branches of pure mathematics and classical mechanics. Although he died young, he left a deep and long-lasting legacy, particularly in geometry. One of the main achievements that he is remembered for is his pioneering work on integrating Hamilton's *Elements of Quaternions* with Grassmann's *Theory of Extension* into a more general coherent corpus, now referred to eponymously as Clifford algebras. These geometric algebras are utilised in engineering mechanics (especially in robotics) as well as in mathematical physics (especially in quantum mechanics) for representing spatial relationships, motions, and dynamics within systems of particles and rigid bodies. Clifford's study of geometric algebras in both Euclidean and non-Euclidean spaces led to his invention of the *biquaternion*, now used as an efficient representation for *twists* and *wrenches* in the same context as that of Ball's *Theory of Screws*.

Biographical Notes

William Kingdon Clifford was a 19th Century English mathematician and scientific philosopher who, though he lived a short life, produced major contributions in many areas of mathematics, mechanics, physics and philosophy. This he achieved during a mere fifteen-year professional career. He was the archetypal polymath, since as well as displaying remarkable mathematical skills, he was also an accomplished literature and classics scholar. Clifford was fluent in reading French, German and Spanish, as he considered these to be important for his mathematical work. He learned Greek, Arabic and Sanskrit for the challenge they presented, Egyptian hieroglyphics as an intellectual exercise, and Morse code and shorthand, because he wished to understand as many forms for communicating ideas as possible. During his lifetime Clifford was energetic and influential in championing the scientific method in

M. Ceccarelli (ed.), Distinguished Figures in Mechanism and Machine Science, 79–116.

Fig. 1. A portrait of William Kingdon Clifford (1845–1879). (Source: School of Mathematics and Statistics, University of St Andrews, Scotland) (URL: http://www-history.mcs.st-and.ac.uk/history/PictDisplay/Clifford.html)

social and philosophical contexts, and was a leading advocate for Darwinism. He and his wife Lucy socialised regularly with many famous scientists and literary figures of the period. He even had several non-academic strings to his bow, notably gymnastics and kite flying, at which he impressed his contemporaries on numerous occasions. He was slight of build but his renowned physical strength and athletic skills no doubt came to the fore when, on a scientific expedition to Sicily (for the 22 December 1870 solar eclipse), he was shipwrecked near Catania and survived. Despite this experience he fell in love with the Mediterranean area. Sadly his health was relatively poor throughout his life and he died of pulmonary tuberculosis (then referred to as consumption) at the early age of 33 (Chisholm, 2002).

William Kingdon Clifford was born on 4th May 1845 at Exeter in the county of Devon in the south-west of England. His father (William Clifford) was a book and print seller (mainly of devotional material), an Alderman and a Justice of the Peace. His mother Fanny Clifford (née Kingdon) was the daughter of Mary-Anne Kingdon (née Bodley) who was related to Sir Thomas Bodley (1545–1613). The latter had been a lecturer in natural philosophy at Magdalen College, Oxford University during the reign of Queen Elizabeth I, and was one of the main founders of the Bodleian Library in Oxford. As a child William Kingdon lived at 9 Park Place in Exeter, the house where his mother had been born, just a short walk from 23 High Street, Exeter, where the family later moved. Exeter Civic Society has since placed a commemorative plaque on the wall of 9 Park Place, for ease of identification. Clifford suffered early tragedy in his short life with the death of his mother in 1854, aged 35, when he was only nine. His father re-married, had four more children, and eventually died in 1878, aged 58, in Hyères, France.

On 7 April 1875 William Kingdon Clifford, aged 29, married Sophia Lucy Jane Lane, aged 28, of Camden Town, London. Lucy (her preferred appellation) claimed, romantically, to have been born in Barbados, but it seems that her only association with the island was through her grandfather John Brandford Lane, who had been a landowner there. It is unlikely that Lucy herself was ever there, and indeed there was some mystery about her background, not least because she continually lied about her age, reducing it eventually by ten years, apparently to conceal details of her past (Chisholm, 2002).

Ostensibly, William and Lucy had a happy marriage and produced two daughters. However, he was prone to overwork, lecturing and performing administrative duties during the daytime, and doing research and writing his many papers and articles at night. This probably led to a deterioration in his health, which had never been robust, and in the Spring of 1876 he accepted his poor state of health and agreed to take a leave of absence from his duties. The family then spent six months in the Mediterranean region (Algeria and Spain) while he convalesced, before returning to his academic post at University College, London in late 1876. Within eighteen months his health failed again and he travelled to the Mediterranean once more, this time returning in a feeble state in August 1878. By February 1879, with the rigours of the English winter in full force, desperate measures were required, and despite the dangers of travel in such a poor state of health William sailed with the family to the Portuguese island of Madeira in the North Atlantic Ocean to attempt to recuperate. Unfortunately he never recovered and after just a month of debilitating illness he died on 3 March 1879 at Madeira of pulmonary tuberculosis. His body was brought back to England by his wife and was buried in Highgate Cemetery in London. The following epitaph (taken from Epictetus) on his tombstone was chosen by Clifford himself on his deathbed:

I was not, and was conceived.
I loved and did a little work.
I am not, and grieve not.

Sadly the marriage had been cut short after only four years with the untimely death of William aged 33. During their four-year marriage, and subsequently as his widow, Lucy had become a successful novelist, playwright and journalist. Throughout their time together they moved in sophisticated social circles – scientific as well as literary. After William's death Lucy became a close friend of Henry James and regularly mixed socially with many other prominent figures including Virginia Woolf, Rudyard Kipling, George Eliot, Thomas Huxley and Thomas Hardy. Lucy outlived William by fifty years and died on 21 April 1929, aged 82. She was buried beside her husband in Highgate Cemetery. The following epitaph for Lucy was added to Clifford's tombstone:

Oh, two such silver currents when they join
Do glorify the banks that bound them in.

Clifford's formal education had begun when he gained a place at the Exeter Grammar School. However, he only spent a few months there before he transferred in 1856 to the Mansion House School, also in Exeter. This school was subsequently renamed Mr. Templeton's Academy, and was eventually demolished by Exeter City Council after having been bombed in 1942 during World War II. In 1858 and 1859, whilst at Mr. Templeton's Academy, Clifford took both the Oxford and the Cambridge University Local Examinations in an impressive range of subjects, gaining many distinctions. Continuing his excellent academic record, Clifford won, at age 15, a Mathematical and Classical Scholarship to join the Department of General Literature and Science at King's College, London, and so he left Mr Templeton's Academy in 1860. At King's College more achievements followed when he won the Junior Mathematical Scholarship, the Junior Classical Scholarship and the Divinity Prize, all in his first year. He repeated the first two of these achievements in both his second and third years at King's College, and additionally he won the Inglis Scholarship for English Language, together with an extra prize for the English Essay. Clifford left King's College in October 1863, at age 18, after securing a Foundation Scholarship to Trinity College, Cambridge, to study Mathematics and Natural Philosophy. At Cambridge he continued to shine academically, winning prizes for mathematics and for a speech he presented on Sir Walter Raleigh. He was Second Wrangler in his final examinations in 1867 and gained the Second Smith's Prize. Clifford was awarded his BA degree in Mathematics and Natural Philosophy in 1867. He completed his formal education on receiving an MA from Trinity College in 1870.

On 18 June 1866, prior to obtaining his BA, Clifford had become a member of the London Mathematical Society, which held its meetings at University College. He had served on its Council, attending all sessions in the periods 1868/69 to 1876/77. Within a year of being awarded his BA, Clifford was elected in 1868 to a Fellowship at Trinity College. He remained at Trinity College until 1871 when he left to take up an appointment as Professor of Mathematics and Mechanics at University College, London. It appears that he had 'lost his (Anglican Christian) faith' and realistically could no longer remain at Trinity College. Unlike Trinity College, University College had been founded in 1827 as a strictly secular institution and the Professors were not required to swear allegiance to a religious oath. This liberal-mindedness appealed to Clifford's freethinking viewpoint at the time, although he had been a staunch Anglican in his youth. In June 1874 Clifford was elected as a Fellow of the Royal Society, and soon afterwards he was also elected as a member of the Metaphysical Society. The latter was chiefly concerned with discussing arguments for or against the rationality of religious belief, in the prevailing intellectual climate where Darwinian evolutionary theory was at the forefront of debate. At this time he also delivered popular science lectures as well as investigating psychical research and he was instrumental in debunking spirit mediums and general claims for so-called paranormal activity.

Clearly, Clifford had wide-ranging interests, producing a considerable output of work, considering his brief life span. However, much of his academic writings were published posthumously. His academic publications fall mainly into three categories – Popular Science, Philosophy and Mathematics.

List of Main Works

A good representative example of Clifford's *Popular Science Lectures* is "Seeing and Thinking". His main *Philosophical Works* include the important "The Ethics of Belief" (Clifford, 1877), "Lectures and Essays", and "The Common Sense of the Exact Sciences". However, his *Mathematical Works*, such as "Elements of Dynamic Vol. 1", "Elements of Dynamic Vol. 2", and "Mathematical Papers" (Clifford, 1882), are especially interesting in the present context. In particular, the "Mathematical Papers" (edited by R. Tucker) published originally in 1882, and reprinted in 1968 (by Chelsea Publishing Company, New York), is the most relevant reference here. These mathematical papers were organised by their editor into two main groupings, namely: *Papers on Analysis*, and *Papers on Geometry*. The former analysis papers were grouped into papers on *Mathematical Logic*, *Theory of Equations and of Elimination*, *Abelian Integrals and Theta Functions*, *Invariants and Covariants*, and *Miscellaneous*. Although at least four papers within this

Analysis grouping are relevant in Mechanism and Machine Science, they are not of central importance. The papers in Tucker's *Geometry* grouping are the more relevant ones. Tucker organised Clifford's geometry papers into papers on *Projective and Synthetic Geometry*, *Applications of the Higher Algebra to Geometry*, *Geometrical Theory of the Transformation of Elliptic Functions*, *Kinematics*, and *Generalised Conceptions of Space*. At least sixteen papers from this *Geometry* grouping are directly related to the Mechanism and Machine Science field, but the following six are of fundamental importance:

Preliminary Sketch of Biquaternions (Clifford, 1873)
Notes on Biquaternions
Further Note on Biquaternions
On the Theory of Screws in a Space of Constant Positive Curvature
Applications of Grassmann's Extensive Algebra (Clifford, 1876a)
On the Classification of Geometric Algebras (Clifford, 1876b)

Here, only the first of these papers (on biquaternions) will be reviewed.

Review of Main Works on Mechanism and Machine Science

Preamble

Partly because of his short life, much of Clifford's academic work was published posthumously. However, his widespread network of scientific contacts, and his reputation as an outstanding teacher, together with his clear notes and instructive problems ensured that he gained the acknowledgement that he deserved during his lifetime. In the context of the history of mechanism and machine science, his papers on geometry (Clifford, 1882) are most relevant, particularly those on kinematics, and on generalised conceptions of space.

A general rigid-body spatial displacement with no fixed point can be achieved as a *twist* about a screw axis. This is a combination of a rotation about and a translation along a specific straight line (the axis) in 3D space (Ball, 1900). A similar situation arises when the rigid body undergoes continuous spatial motion, in which case, at any instant of time, it is performing a twist-velocity about a screw axis. Analogously, the most general system of forces acting on a rigid-body may be replaced with an equivalent *wrench* about a screw axis. This is a combination of a single force acting along a specific straight line (the axis) in 3D space, together with a couple, first introduced by Poinsot (1806), acting in any plane orthogonal to the line. These scenarios may be represented algebraically in many different ways (Rooney, 1978a), but Clifford's *biquaternion* (Clifford, 1873) offers one of the most elegant and efficient representations for kinematics.

All three of Clifford's papers on biquaternions discuss and develop the concept, although the first paper, *Preliminary Sketch of Biquaternions*, is the main one that introduces the biquaternion – it is considered to be a classic and is the main one to be reviewed here. The second paper, *Notes on Biquaternions*, was found amongst Clifford's manuscripts and was probably intended as a supplement to the first paper. It is short and develops some of the detailed aspects of biquaternion algebra. The third paper, *Further Note on Biquaternions*, is more extensive and it discusses and clarifies why a biquaternion may be interpreted in essentially two different ways, either as a generalised type of number, or as an operator.

Preliminary Sketch of Biquaternions

The idea of a *biquaternion*, as presented in Clifford's three papers, *Preliminary Sketch of Biquaternions*, *Notes on Biquaternions* and *Further Note on Biquaternions* (Clifford, 1882), originated with Clifford, although the term "biquaternion" had been used earlier by Hamilton (1844, 1899, 1901). to denote a quaternion consisting of four complex number components, rather than the usual four real number components. Clifford acknowledges Hamilton's priority here but he considers that because "all scalars may be complex" Hamilton's use of the term is unnecessary. Clifford adopts the word for a different purpose, namely to denote a combination of two quaternions, algebraically combined via a new symbol, ω, defined to have the property $\omega^2 = 0$, so that a biquaternion has the form $q + \omega r$, where q and r are both quaternions in the usual (Hamiltonian) sense.

The symbol ω (and its modern version, ε) has been the focus of much misunderstanding since it is a quantity whose square is zero and yet is not itself zero, nor is it 'small'. It should be viewed as an operator or as an abstract algebraic entity, and not as a real number. Clifford confuses matters further by using the symbol in several different contexts. In Part IV (on elliptic space) of the *Preliminary Sketch of Biquaternions* paper ω has a different meaning and an apparently different multiplication rule $\omega^2 = 1$, and in the papers *Applications of Grassmann's Extensive Algebra* and *On the Classification of Geometric Algebras* there is yet another related use of the symbol ω, and this time its defining property is $\omega^2 = \pm 1$. In the early part of the *Preliminary*

Sketch of Biquaternions paper Clifford even uses ω to denote angular velocity, so there is much scope for confusion.

The classic first paper on biquaternions, *Preliminary Sketch of Biquaternions*, is organised into five sections. *Section I* introduces and discusses the occurrence of various different types of physical quantity in mechanical systems, and how they may be represented algebraically. *Section II* proceeds to construct a novel algebra, for manipulating various physical quantities, based on an extension and generalisation of Hamilton's algebra of quaternions, and this is where the term *biquaternion* is introduced. *Section III* briefly investigates non-Euclidean geometries (and specifically elliptic geometry with constant positive curvature) for the purpose of interpreting some of the projective features and properties of biquaternions. *Section IV* examines several particular physical quantities and shows that in some sense their 'ratio' is a biquaternion. Finally *Section V* looks at five specific geometrical scenarios involving biquaternions. The short second paper of the trio, *Notes on Biquaternions*, appears to continue this latter Section V with a further two geometrical scenarios.

Clifford's motivation in creating his biquaternion derives essentially from mechanics, and in Section I of *Preliminary Sketch of Biquaternions* he draws attention to the inadequacies of algebraic constructs such as scalars and vectors for representing some important mechanical quantities and behaviours. Many physical quantities, such as energy, are adequately represented by a single magnitude or *scalar*. But he states that other quantities, such as the translation of a rigid body, where the translation is not associated with any particular position, require a magnitude and a direction for their specification. Another example is that of a couple acting on a rigid body, where again a magnitude and direction are required but the position of the couple is not significant. The magnitude and direction of either a translation or of a couple may be represented faithfully by a *vector*, as Hamilton had shown.

However, Clifford emphasises that there are several mechanical quantities whose *positions* are significant, as well as their magnitudes and directions. Examples include a rotational velocity of a rigid body about a definite axis, and a force acting on a rigid body along a definite line of action. These cannot be represented adequately just by a vector, and Clifford introduces the term *rotor* (probably a contraction of 'rotation vector') for these quantities, that have a magnitude, a direction and a position constrained to lie along a straight line or axis.

In order to combine or to compare scalar, vector and rotor quantities, some form of consistent algebra is desirable that faithfully represents the required processes of combination/comparison. Scalar quantities may be dealt with using the familiar real number algebra. Its standard operations of addition, subtraction, multiplication and division usually yield intuitively plausible results for the magnitudes of scalars.

For vectors in a 2D space the complex numbers (considered as 2D vectors), offer something fundamentally new in that they can be used to represent and compare directions as well as magnitudes, by forming the ratio of two complex numbers and hence by using the complex number division operation. Moreover, complex numbers may be used as operators for rotating and scaling geometrical objects in a 2D space.

In the case of vector quantities in 3D space, consistent addition and subtraction operations may be defined using "vector polygons" to combine vectors (denoted by straight line segments) in a way that takes account of their directions. The standard approach is based on empirical knowledge of how two translations (or two couples) behave in combination. So, two vectors may be added (subtracted) to give a meaningful sum (difference), which is itself another vector. But if an attempt is made to *compare* two vectors, in the way that two scalars might be compared, by forming their ratio using algebraic division, there is a problem.

In 3D space Hamilton (1844, 1899, 1901) had shown that it is difficult to define any form of division operation to obtain the ratio of two vectors because such a ratio could not itself be a vector. He had demonstrated conclusively that forming the 'ratio' of two 3D vectors requires the specification of four independent scalar quantities, and so the outcome must be a 4D object in general. He had also shown that two different vector 'ratios' are obtained from 'left-division' and 'right-division' (left- and right-multiplication by an inverse), and so the operation is non-commutative. Hamilton had solved the problem by inventing quaternions and their consistent non-commutative (4D) algebra. A 3D vector algebra cannot be closed under multiplication and 'division', despite the fact that it is closed under addition and subtraction. Instead the 3D vectors must be embedded in a 4D space and treated as special cases of 4D vectors with one zero component. Hamilton's algebra was based on a *quaternion product* that could be partitioned into a *scalar* part and a *vector* part. These parts were subsequently treated separately by Gibbs as a *'dot'* *product* and a *'cross' product* to form the basis of his later vector algebra (Gibbs, 1901).

Clifford extends Hamilton's approach and investigates the situation with rotors, which are even more problematical than 3D vectors. In this case empirical knowledge and experience demonstrate that the combination of two rotational velocities with different (skew) axes does not produce a rotational velocity. Instead it produces a general rigid-body motion. In a similar way the combination of two forces with different (skew) lines of action does not produce another force with a definite line of action. Instead it produces a general system of forces (Ball, 1900). So, an algebra for rotors in 3D space cannot be expected to be closed under addition or subtraction, and by analogy with the situation with 3D vectors neither can it be expected to be closed under multiplication or division. Clifford tackles the problem by proceeding to develop a consistent approach that can deal comprehensively with scalars, vectors and rotors, together with their combinations under suitably defined operations of addition, subtraction, multiplication and division. His aim is to provide an algebra for an extended range of physical quantities in mechanics.

As a first step Clifford refers to Ball's work on screw theory (Ball, 1900) which he acknowledges as a complete exposition of general velocities of rigid bodies and of general systems of forces on rigid bodies. Ball had shown that the most general velocity of a rigid body is equivalent to a rotation velocity, about a definite axis, combined with a translation velocity along this axis, thus forming a helical motion, which he referred to as a *twist* velocity about a screw. The screw consists of a screw axis (the same line as the rotation axis) together with a *pitch* (a linear magnitude) given by the ratio of the magnitude of the translational velocity to the magnitude of the rotational velocity. The twist velocity is hence a screw with an associated (angular speed) magnitude.

Analogously Ball had also shown that the most general system of forces on a rigid body is equivalent to a single force with a definite line of action, combined with a couple in a plane orthogonal to this axis, thus forming a helical force system that he referred to as a *wrench* about a screw. In this case the screw consists of a screw axis (the same line as the line of action of the single force) together with a *pitch* (a linear magnitude) given by the ratio of the magnitude of the couple to the magnitude of the single force. The wrench is hence a screw with an associated (force) magnitude.

Clifford completes Section I of the paper by introducing the term *motor* (probably a contraction of 'motion vector') to denote this concept of a (force

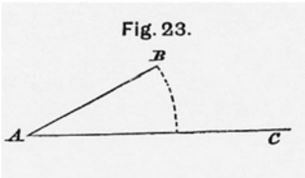

Fig. 2. The ratio of two vectors. (Source: W.K. Clifford, 1882, Collected Papers, facing p. 228, Chelsea Publishing Company, New York)

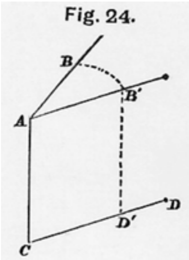

Fig. 3. The ratio of two rotors. (Source: W.K. Clifford, 1882, Collected Papers, facing p. 228, Chelsea Publishing Company, New York)

or angular speed) magnitude associated with a screw. He thus designates the sum of two or more rotors (representing forces or rotation-velocities) as a new object, namely a motor, and then establishes that although the addition of rotors is not closed, the addition of motors is closed. By considering any vector and any rotor to be degenerate forms of motor, and noting that the sum of two motors is always a motor, Clifford effectively achieves an algebra of vectors, rotors and motors that is closed under addition and subtraction.

In Section II of *Preliminary Sketch of Biquaternions* Clifford proceeds to develop further his algebra of motors by examining whether or not he can define their multiplication and 'division'. He begins by noting that Hamilton's quaternion may be interpreted either as the ratio of two 3D vectors, or as the operation which transforms one of the vectors into the other. He illustrates this with a figure (Figure 2) showing two line segments, labelled AB and AC, to represent the two vectors. These have different lengths (magnitudes) and directions, and although the vectors have arbitrary positions, the line segments are positioned conveniently so that they both emanate from the same point, A. He explains that AB may be converted into AC by rotating it around a rotation axis through A that is perpendicular to the plane BAC, until AB has the same direction as AC, and then stretching or shrinking its length until it coincides with AC. The process of combining the rotation with the magnification may be thought of as taking the ratio of AC to AB, or alternatively as operating on AB to produce AC. Hamilton had previously shown this process to be representable as a quaternion q. It may be written either in the form of a ratio $AC/AB = q$ or in the form of an operation $q \cdot AB = AC$. If the magnification is ignored, the rotation by itself essentially represents the ratio of two directions, namely those of AC and AB, or, equivalently, the process of transforming one direction into the other.

Clifford states that this particular quaternion q will operate on any other vector AD in the plane BAC in the same way, so that another such vector AD will be rotated about the same axis perpendicular to the plane BAC through the same angle, and be magnified in length by the same factor to become AE in the plane BAC, where angle DAE equals angle BAC. However, he also states that this quaternion q operating on any vector, say AF, not lying in the plane BAC does not rotate and magnify AF in this way. In fact he gives no meaning to this operation. So, a quaternion formed from the ratio of the two vectors AB and AC can operate only on vectors in the plane BAC.

By analogy with Hamilton's quaternion, used for the ratio of two 3D vectors, Clifford considers forming the ratio of two rotors. He describes how two rotors (with different (skew) axes) may be converted one into the other. Again he uses a diagram (Figure 3) to illustrate the procedure. The two rotors are represented as two line segments lying along (skew) axes, and labelled AB and CD. These have different lengths (magnitudes), directions and positions, but they are partially constrained in position to always lie somewhere along their respective axes. He states that there is a unique straight line that meets both rotor axes at right angles, and he positions the line segments so that the points A and C lie on this unique line. The length of the line segment AC then represents the shortest distance between the two rotor axes. Clifford outlines how the rotor AB may be converted into the rotor CD, in three steps. Firstly, rotate AB about the axis AC into a position AB', which is parallel to CD. Secondly, translate AB' along AC, keeping it parallel to itself, into the position CD'. Thirdly, stretch or shrink the length of CD' until it coincides with CD. The combination of the first two operations is clearly seen to be a twist about the screw with axis AC with pitch given by length AC/angle BAB'. The third operation is simply a magnification (a scale factor). So, Clifford demonstrates that the ratio of the two given rotors AB and CD is a twist about a screw combined with a (real number) scale factor. He writes this ratio in the form $CD/AB = t$ or alternatively as an operation in the form $t \cdot AB = CD$, and he refers to t as a *tensor-twist* (the word "tensor" in the sense that he uses it here is not related to the modern use of the word). If the scale factor is ignored, the twist about the screw by itself essentially represents the ratio of two (skew) axes, namely those of CD and AB, or, equivalently, the process of transforming one axis into the other.

Clifford states that this particular tensor-twist t (the ratio of the two rotors AB and CD) will operate on any other rotor EF whose axis meets the axis of t (that is the axis of AC) at right angles. It will rotate EF about the axis of AC through an angle BAB', translate it along this axis through a distance equal to the length of AC, and stretch or shrink its length in the ratio of the lengths of CD to AB. However, Clifford also states that t operating on any rotor, say GH, that does not meet the axis of AC, or that does not meet it at right angles, will not rotate, translate and magnify GH in this way. In this case he gives no meaning to the operation. So, a ratio of two rotors AB and CD can operate only on other rotors whose axes intersect their screw axis orthogonally.

At this stage the ratio of two vectors has been considered (following Hamilton) and the ratio of two rotors has been derived. Clifford now investigates the ratio of two motors. He first looks at a special case, namely that where the two motors have the same pitch. He shows that in this case the ratio of these two motors is again a tensor-twist. His proof relies on expressing each of the motors as the sum (actually he uses a linear combination) of two rotors (he had stated earlier that the sum of two rotors is a motor). Clifford considers the first motor to be a linear combination of two rotors α and β, so the first motor is $m\alpha + n\beta$, where m and n are real scale factors (scalars). Then he considers a tensor-twist t whose axis intersects both of the axes of α and β at right angles (hence the axis of t lies along the common perpendicular of the axes of α and β). The effect of t on the rotor α is to produce a new rotor $\gamma = t\alpha$, and similarly t acting on the rotor β produces another new rotor $\delta = t\beta$. He now forms a second motor, this time from a linear combination of the two new rotors γ and δ, by using the same scale factors m and n as he used in constructing the first motor, giving the second motor as $m\gamma + n\delta$. This ensures that the second motor has the same pitch as the first motor. Finally, he assumes that the distributive law is valid for rotors and constructs the following sequence: $t(m\alpha + n\beta) = m(t\alpha) + n(t\beta) = m\gamma + n\delta$. Hence he shows that:

$$t = \frac{m\gamma + n\delta}{m\alpha + n\beta}$$

is the ratio of the two motors having the same pitch.

This establishes that the ratio of two motors with the same pitch is again a tensor-twist. Unfortunately, if the motors do not have the same pitch, their ratio is not a tensor-twist, and so Clifford then sets out to derive the general case. The procedure is quite lengthy and involves the introduction of a new operator ω with a somewhat counter-intuitive property, namely $\omega^2 = 0$. In modern times this symbol has been changed to ε, partly to avoid confusion with the commonly used symbol for rotational speed, and partly to suggest pragmatically that it is akin to a small quantity whose square may be neglected in algebraic calculations and expansions. It is now referred to as a *dual number*, or more specifically as the *dual unit*.

Clifford considers that the ratio of two general motors will be established if a geometrical operation can be found that converts one motor, say A, into another motor, say B. He begins his analysis of the general case by observing that every motor can be decomposed into the sum of a rotor part and a vector part, and that the pitch of the motor is given by the ratio of the magnitudes of the vector and rotor parts. This is justified empirically by remembering that a wrench (an example of a motor) consists of the sum of a force with its line of action (a rotor), and a couple in a plane orthogonal to the line of action (a vector). Another example is a twist velocity (a motor) consisting of the sum of a rotational velocity about an axis (a rotor), and a translational velocity along the axis (a vector). Clifford states that, because of this generally available decomposition of any motor into a rotor plus a vector, it is possible to change arbitrarily the pitch of the motor without changing the rotor part, by combining the motor with some other suitable vector. So, to convert a given general motor A into another given general motor B, he proceeds by introducing an auxiliary motor B' that has the same rotor part as B but that has the same pitch as A. He has already shown that the ratio of two motors with the same pitch is a tensor-twist, so he immediately knows the ratio $B'/A = t$. He expresses B' in terms of B by adding an appropriate vector, $-\beta$, so that: $B = B' + \beta$ where β is a vector parallel to the axis of B.

Clifford can then write the ratio of B to A as:

$$\frac{B}{A} = \frac{B'}{A} + \frac{\beta}{A} = t + \frac{\beta}{A}.$$

This is the sum of a tensor-twist t with a new object β/A. The latter is the ratio of a vector in some direction, to a motor with an axis generally in a different direction, and as yet its nature is unknown. He proceeds to investigate the nature of this new ratio by introducing a symbol ω to represent an operator that converts any motor into a vector parallel to the axis of the motor and of magnitude equal to the magnitude of the rotor part of the motor. Thus, for example, ω converts rotation about any axis into translation parallel to that axis. Similarly, ω converts a force along its line of action into a couple in a plane orthogonal to that line of action. By definition ω operates on a motor, and the effect of ω operating on a vector such as a translation or a couple is to reduce these to zero. So, ω operating on a motor, produces a (free) vector from its rotor part and simultaneously eliminates the vector part of the motor. Thus, operating with ω twice in succession on any motor A always reduces the motor to zero, that is $\omega^2 A = 0$, or expressed simply, in more modern form, $\omega^2 = 0$.

Clifford states the above operation algebraically as $\omega A = \alpha$, where A is a general motor and α is the (free) vector with the same direction and magnitude as the rotor part of A. He recalls that the ratio of two vectors is a quaternion and hence $\beta/\alpha = q$ is a quaternion, so $\beta = q\alpha$. This allows him to write the following sequence: $\beta = q\alpha = q\omega A$, so that: $\beta/A = q\omega$, and therefore: $B/A = t + q\omega$.

The latter expresses the ratio of two general motors A and B as the sum of two parts, namely a tensor-twist t and a quaternion q multiplied by ω. At this stage Clifford has derived a clear interpretation for the ratio of two motors but he is not content with this form. He proceeds to interpret the ratio B/A differently and eventually expresses it in an alternative interesting form.

His alternative interpretation requires some further analysis, but it leads to a more sophisticated result involving the new concept of a *biquaternion*. He starts by considering an arbitrary point, O, in space as an origin. From empirical knowledge of forces, couples, rotational and translational velocities, he is able to state that, in general, any motor may be specified uniquely as the sum of a rotor with axis through the origin, O, and a (free) vector, with a different direction from that of the rotor. He proceeds to observe that rotors whose axes always pass through the same fixed point behave in exactly the same way as (free) vectors. The ratio of any two of these rotors is of course a tensor-twist, because both have the same (zero) pitch. But the pitch of this tensor-twist is zero because the rotor axes intersect (in modern terms there is no translation along a common perpendicular line), and so the ratio of the two rotors through the same fixed point is essentially a quaternion with axis constrained to pass through the fixed point.

At this stage Clifford's notation becomes slightly confusing. Now he chooses to use a cursive Greek letter to represent a rotor whose axis passes through the origin, and the same cursive Greek letter prefixed by the symbol ω to represent a vector with the same magnitude and direction as the corresponding rotor. So, the rotor α whose axis passes through the origin, and the vector $\omega\alpha$ are parallel in direction and they have the same magnitude. This does make sense because, as stated earlier, the effect of ω operating on any motor, including the zero-pitch motor α (a rotor) with axis through the origin, is to convert it into a vector in just this way. Clifford writes (à la Hamilton) the ratio of two such vectors $\omega\alpha$ and $\omega\beta$ as a quaternion $p = \omega\beta/\omega\alpha$ and by 'cancelling' the ω this becomes $p = \beta/\alpha$ where α and β are rotors with axes through the origin. [Clifford uses the letter q, rather than p, as a general symbol for a quaternion, but the letter p has been substituted here instead to distinguish it as a different quaternion from the one to be introduced below.] So, in this way he has shown that the quaternion p represents either the ratio of two vectors $\omega\alpha$ and $\omega\beta$, or, equivalently, the ratio of two respectively parallel rotors α and β with axes passing through the origin.

Clifford is now able to state the general expression for a motor as $\alpha + \omega\beta$. This agrees with empirical evidence since it is the sum of a rotor α, with axis through the origin, and a (free) vector $\omega\beta$, with a direction that differs, in general, from the direction of α. The ratio of two such general motors, $\alpha + \omega\beta$ and $\gamma + \omega\delta$, is the algebraic expression:

$$\frac{\gamma + \omega\delta}{\alpha + w\beta}.$$

To evaluate this, Clifford continues by recognising that the ratio of the two rotors α and γ, with axes through the origin, is some quaternion, $\gamma/\alpha = q$. From this he has that $q\alpha = \gamma$, and so $q(\alpha + \omega\beta) = q\alpha + q\omega\beta = \gamma + \omega q\beta$. But now he has to determine the geometrical nature of the algebraic product $q\beta$ in this expression. Operating on α with q clearly rotates it into γ, but since β does not in general lie in the same plane as α and γ, the geometrical effect of operating on β with q is not yet known, although algebraically it is just another quaternion.

Clifford tackles this problem of geometrical interpretation by introducing yet another quaternion r and using the algebra of quaternions to derive, in the first instance, some formal algebraic expressions. Since any algebraic combination of quaternions, vectors (equivalent to quaternions with zero first component) and rotors through a fixed point (equivalent to vectors) is a quaternion, he defines r as the quaternion,

$$r = \frac{\delta - q\beta}{\alpha},$$

from which he has: $r\alpha = \delta - q\beta$. He then operates on this with ω and obtains $\omega r\alpha = \omega\delta - \omega q\beta$. Finally, he adds this equation to the earlier one

$q(\alpha + \omega\beta) = \gamma + \omega q\beta$, to derive the following expression:

$$(q + \omega r)(\alpha + \omega\beta) = \gamma + \omega\delta,$$

in which the defining property $\omega^2 = 0$ is used. Re-writing the final expression in the form:

$$\frac{\gamma + w\delta}{\alpha + \omega\beta} = q + \omega r$$

shows that the ratio of two general motors is the sum of two terms. The first is a quaternion and the second is a quaternion operated on by ω. Clifford refers to this new quantity, representing the ratio of two general motors, as a *biquaternion*. Unfortunately, he then states that this biquaternion has no immediate interpretation as an operator in the way that a quaternion operates on a vector to give another vector (if the first vector is orthogonal to the axis of the quaternion). This conclusion is somewhat unsatisfactory but in the remaining Sections III–V of the paper *Preliminary Sketch of Biquaternions* he addresses the shortcoming by setting the biquaternion concept in the wider context of projective geometry. He ends the section with the following Table 1, summarising his perception of the situation so far.

Table 1. Summary of geometrical forms and their representations. (Source: W.K. Clifford, 1882, Collected Papers, p. 188, Chelsea Publishing Company, New York)

Geometrical Form	Quantity	Example	Ratio
Sense on st. line	Vector on st. line	Addition or Subtraction	Signed Ratio
Direction in plane	Vector in plane	Complex quantity	Complex Ratio
Direction in space	Vector in space	Translation, Couple	Quaternion
Axis	Rotor	Rotation-Velocity, Force	Twist
Screw	Motor	Twist-Velocity, System of Forces	Biquaternion

In Section III of *Preliminary Sketch of Biquaternions*, Clifford amplifies the concept of the biquaternion in the context of non-Euclidean spaces, particularly the elliptic geometry of constant positive curvature. This is a generalisation into 3D (curved space) of the 2D geometry of the (curved) surface of a sphere. Using the formalism of projective geometry he outlines the following facts, relating to this elliptic (constant positive curvature) non-Euclidean space:

- Every point has a unique set of three coordinates, and conversely every set of three coordinate values defines a unique point;
- There is a quadric surface, referred to as the *Absolute*, for which all its points and tangent planes are imaginary;
- Two points are referred to as *conjugate* points, with respect to the absolute, if their 'distance' (an angle) apart is a quadrant, and two lines or two planes are conjugate if they are at right angles to each other;
- In general, two lines can be drawn so that each meets two given lines at right angles, and the former are referred to as *polars* of each other;
- A twist-velocity of a rigid body has two axes associated with it because translation along one axis is equivalent to rotation about its polar axis and vice versa;
- A twist-velocity of a rigid body has a unique representation as a combination of two rotation-velocities about two polar axes;
- The motion of a rigid body may be expressed in two ways, either as a twist-velocity about a screw axis with a certain pitch, or as a twist-velocity about the polar screw axis with the reciprocal of the first pitch;
- In general, a motor may be expressed uniquely as the sum of two polar rotors;
- A special type of motor arises when the magnitude of the two polar rotors are equal, because the axes of the motor are then indeterminate, so that the motor behaves as a (free) vector;
- There are *right vectors* and *left vectors* in elliptic space, depending on the handedness of the twist of the motor from which they are derived, whose axes are indeterminate;
- In elliptic space if a rigid body rotates about an axis through a certain 'distance' and simultaneously translates along it through an equal 'distance', then all points of the body travel along 'parallel straight lines' and the motion is effectively a rotation about any one of these lines together with an 'equal' translation along it.

From these facts Clifford derives the following proposition at the end of Section III:

Every motor is the sum of a right and a left vector.

This he expresses in the form

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A'),$$

where A is the motor and A' is its polar motor, and where $(A + A')$ and $(A - A')$ are both motors of pitch unity, but one is right-handed and the other is left-handed.

In Section IV of *Preliminary Sketch of Biquaternions*, Clifford continues with his treatment of motors in the context of elliptic geometry and essentially sets up a coordinate system for rotors passing through the origin. He bases this on the three mutually perpendicular unit rotors i, j, and k whose axes are concurrent at the origin. Any rotor through the origin then has the form $ix + jy + kz$, where x, y, and z are scalar quantities (ratios of magnitudes). He gives another interpretation to i, j, and k as operators. Thus for instance i operates on any rotor that intersects the axis of i at right angles and rotates it about the axis of i through a right angle. Similar comments apply to j and k, and their axes. Clifford refers to these operations as *rectangular rotations*. Performing repeated rectangular rotations leads to the familiar quaternion equations $i^2 = j^2 = k^2 = ijk = -1$ and hence Clifford interprets the unit quaternions i, j and k as rectangular rotations about the coordinate axes. He states that for operations on rotors which are orthogonal to, but do not necessarily intersect, the axes of i, j, and k, the quaternion equations are still valid.

The rotor $ix+jy+kz$ is interpreted as a rectangular rotation about the axis of the rotor, combined with a scale factor $(x^2 + y^2 + z^2)^{1/2}$. It operates only on those rotors whose axes intersect its axis at right angles. The remainder of Section IV explores various consequences of these interpretations and concludes with another proof that the ratio of two motors is a biquaternion, as defined in Section II.

The final Section V of *Preliminary Sketch of Biquaternions*, is short and deals with some applications of the rotor concept in elliptic geometry looking at special cases of geometrical interest. There are five sub-sections as follows: Position-Rotor of a point; Equation of a Straight Line; Rotor along Straight Line whose Equation is given; Rotor *ab* joining Points whose Position-Rotors are α, β; Rotor parallel to β through Point whose Position-Rotor is α. These are not reviewed here since they are not of central interest to the field of Mechanism and Machine Science.

Modern Interpretation of Contributions to Mechanism and Machine Science

Preamble

Since the time of Clifford's seminal papers on biquaternions considerable progress has been made in this topic. The theoretical aspects have been significantly advanced by mathematicians developing new types of 'number', such as dual numbers and double numbers (Dickson, 1923, 1930; Yaglom, 1968), and new fields in abstract algebra such as the eponymous Clifford algebras (Grassmann, 1844; Clifford, 1876a, 1876b; Altmann, 1986; Hestenes and Sobczyk, 1987; Conway and Smith, 2003; Rooney and Tanev, 2003; Rooney 2007). In the realm of applications, major progress has been made in mechanics (particularly in kinematics) using various dual quantities and or motors (Denavit, 1958; Keler, 1958; Yang, 1963, 1969; Yang and Freudenstein, 1964; Dimentberg, 1965; Yuan, 1970, 1971; Rooney, 1974, 1975b), and many other leading researchers in mechanics refer to quaternions and biquaternions in dealing with screw theory, notably (Hunt, 1978; Davidson and Hunt, 2004). In physics also (and particularly in quantum mechanics) various types of Clifford algebras are in use (Hestenes, 1986; Penrose, 2004). Furthermore, other related application areas have used or could profitably use the quaternion concept (Rogers and Adams, 1976; Kuipers, 1999) and could benefit from a generalisation to the biquaternion. However, it must be said that Clifford's inventions have not had universal acceptance, and, as noted by Baker and Wohlhart (1996), one early researcher in particular (von Mises, 1924a, 1924b) deliberately set out to establish an approach to the analysis of motors that did not require Clifford's operator ω.

Clifford's important achievements are numerous and wide-ranging, but in the present context the more significant ones include: the invention of the operator ω; the clarification of the relationship between (flat) spatial geometry and (curved) spherical geometry; the derivation of the biquaternion concept; and the unification of geometric algebras into a scheme now referred to as Clifford algebras. But before considering these in more detail from a modern viewpoint it is worth drawing attention to some problematical aspects.

Despite the elegance of Clifford's work on biquaternions, there are several subtleties that must be considered in the context of mechanics. The immediate problem, apparent from the outset is that a biquaternion, defined originally by Clifford as a ratio of two motors, does not appear to be interpretable as an operator that generally transforms motors, one into another. Actually it can be interpreted in a restricted sense in this way provided that it operates only on motors whose axes intersect the axis of the biquaternion orthogonally. This is analogous to the situation with Hamilton's quaternion and rotations. However, these difficulties are resolved by forming a triple product operation, involving three terms, rather than the one used initially by Clifford, involving just two terms, and by *not* interpreting the quaternion units i, j, and k as rotations through a right angle about the x, y, and z axes, respectively, as is commonly done (Porteous, 1969; Rooney, 1978a; Altmann, 1989). This new three-term biquaternion operation allows any motor to be screw displaced into another position and orientation and not just those motors whose axes are orthogonal to and intersect the biquaternion axis. By way of comparison the equivalent operation for quaternions rotates any vector and not just those orthogonal to the axis of the quaternion (Brand, 1947, Rooney, 1977; Hestenes, 1986).

A second problem arising from Clifford's work on biquaternions relates to the use in dynamics of his operator ω, with the property $\omega^2 = 0$. Since its introduction it has taken on a wider life of its own and is now studied (independently of its roots in mechanics) as an abstract algebraic entity (Dickson, 1923, 1930). Currently, it is referred to as a *dual number*, and is designated by the symbol ε, where $\varepsilon^2 = 0$ (Yaglom, 1968). It was introduced, , essentially in the contexts of geometry, statics and kinematics and has been employed very successfully there. In the realm of mechanics in general it has spawned a range of dual-number and other dual-quantity techniques applicable in the analysis and synthesis of mechanisms, machines and robots (Denavit, 1958; Keler, 1958; Yang, 1963, 1969; Yang and Freudenstein, 1964; Dimentberg, 1965; Yuan, 1970, 1971; Rooney, 1974, 1975b). However, although these techniques generally work well in geometry, statics and kinematics, where spatial relationships, rotational velocities, forces and torques are the focus, they are often of more limited use in dynamics, where accelerations, and inertias are additionally involved. Here again there is some difficulty of interpretation but perhaps more importantly the algebraic structure of the dual number and other dual quantities do not appear properly to represent the nature of the underlying dynamical structures (von Mises, 1924a,b; Kislitzin, 1938; Shoham and Brodsky, 1993; Baker and Wohlhart, 1996).

Dual Numbers, Dual Angles, Dual Vectors and Unit Dual Quaternions

When considering the geometry or motion of objects in 3D space the most common transformations in use are those that operate on points. These are referred to as *point transformations* and the familiar 4×4 real matrix, operating on the homogeneous coordinates of any point, falls into this class (Maxwell, 1951; Rooney, 1977). However, the modern use of Clifford's operator ω and his biquaternion, together with quantities derived from them, essentially rests on a consideration of 3D space as a collection of (straight) lines as well as points, because lines occur (as rotation and screw axes) in any discussion of motion and the forces that cause motion. A line has four degrees of freedom of position and orientation and requires four independent coordinates for its specification (Semple and Roth, 1949), whereas a point needs only three coordinates. The transformations required for lines are naturally referred to as *line transformations*. One type of representation of lines, and also of transformations of lines, involves dual numbers (the modern version of Clifford's operator ω). Lines may be represented using dual vectors, whereas transformations are represented using dual quaternions (the modern version of Clifford's biquaternions).

It has proved convenient to use six so-called Plücker coordinates in the mathematical description of a line (Plücker, 1865; Brand, 1947). These are analogous to the four homogeneous coordinates used to represent a point (Maxwell, 1951).

The six Plücker coordinates arise as the components of two vectors (Figure 4). The first vector, $\mathbf{L}$, with three components, L, M and N, defines the direction of the given line. The second vector, $\mathbf{L}_0$, with components, L_0, M_0 and N_0, is the moment of the line about the origin. So, $\mathbf{r} \times \mathbf{L} = \mathbf{L}_0$, where $\mathbf{r}$ is the position vector of any point on the line. Now, it is clear from contemporary standard vector algebra (Gibbs, 1901; Brand, 1947) that $\mathbf{L} \cdot \mathbf{L}_0 = \mathbf{L}_0 \cdot \mathbf{L} = (\mathbf{r} \times \mathbf{L}) \cdot \mathbf{L} = 0$, and so the two vectors $\mathbf{L}$ and $\mathbf{L}_0$ are always orthogonal. The six Plücker coordinates satisfy the relationship $\mathbf{L} \cdot \mathbf{L}_0 = LL_0 + MM_0 + NN_0 = 0$. Additionally, the vectors, $\mu\mathbf{L} = (\mu L, \mu M, \mu N)$ and $\mu\mathbf{L}_0 = (\mu L_0, \mu M_0, \mu N_0)$ for arbitrary non-zero μ, give the same line as before. It is usual to choose $\mathbf{L}$ as a unit vector and hence to choose L, M and N such that $L^2+M^2+N^2 = 1$, so that they represent the direction cosines of the line. There are hence two conditions imposed on the six Plücker coordinates and only four independent coordinates remain, as expected for a line in 3D space.

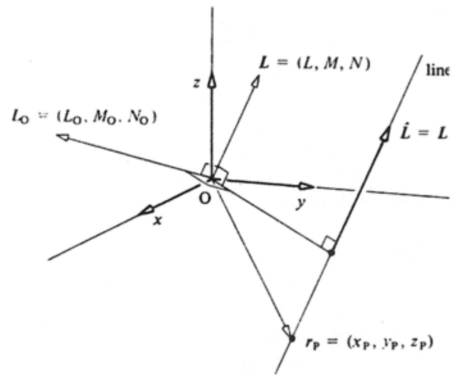

Fig. 4. The six Plücker coordinates (L, M, N; L_0, M_0, N_0) of a straight line in 3D space, represented by a unit dual vector $\hat{\mathbf{L}}$, where $\mathbf{L}$ defines the direction of $\hat{\mathbf{L}}$ and $\mathbf{L}_0$ is the moment of $\hat{\mathbf{L}}$ about the origin. (Source: J. Rooney, 1978a, p. 46)

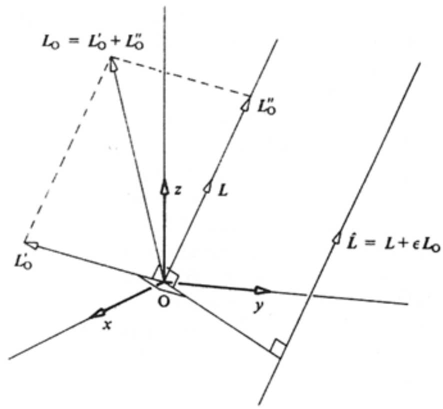

Fig. 5. A general dual vector $\hat{\mathbf{L}}$ (a motor), representing a screw in 3D space, where $\mathbf{L}$ defines the magnitude and direction of $\hat{\mathbf{L}}$, where $\mathbf{L}_0'$ is the moment of $\hat{\mathbf{L}}$ about the origin, and where $|\mathbf{L}_0''|/|\mathbf{L}|$ defines the pitch of the screw. (Source: J. Rooney, 1978a, p. 50)

If the line passes through the origin, its moment, $\mathbf{L}_0$, is zero, it is specified by a single vector $\mathbf{L}$, and it has only two degrees of freedom. Lines through the origin may therefore be put into one-one correspondence with the points on the surface of a unit sphere centred on the origin, and this forms part of the basis of the relationship between spherical (2D curved) geometry and spatial (3D flat) geometry.

The new location of a specific point under a line transformation is obtained by operating separately on any *two lines* which intersect in the point at its initial position, and then determining their new point of intersection after the transformation. This is analogous to the method used to find the new location of a line under a point transformation. In this case the procedure is to transform any *two points* lying on the initial line and then to determine the line joining their new positions.

The six Plücker coordinates (L, M, N; L_0, M_0, N_0) of a line, define the position and orientation of the line with respect to a point O, the origin. To describe the relative orientation of *two* directed skew straight lines in space a unique *twist angle*, α and a unique *common perpendicular distance*, d, are defined, although these two variables do not completely specify the configuration since the common perpendicular line itself must also be given. The situation is analogous to the case of two intersecting lines (Rooney, 1977). There the lines define a unique angle at which they intersect, but the normal line to the plane in which they lie is needed for a complete specification of the relative orientation. It is advantageous to combine the two real variables (scalars), α and d, into a type of 'complex number', known as a *dual number*. It is not widely known that the usual complex number may be generalised, and there are a further two essentially different types (Yaglom, 1968). All three are considered in (Rooney, 1978b) in the context of geometry and kinematics. These are:

the *complex number* $a + ib$, where $i^2 = -1$
the *dual number* $a + \varepsilon b$, where $\varepsilon^2 = 0$
the *double number* $a + jb$, where $j^2 = +1$

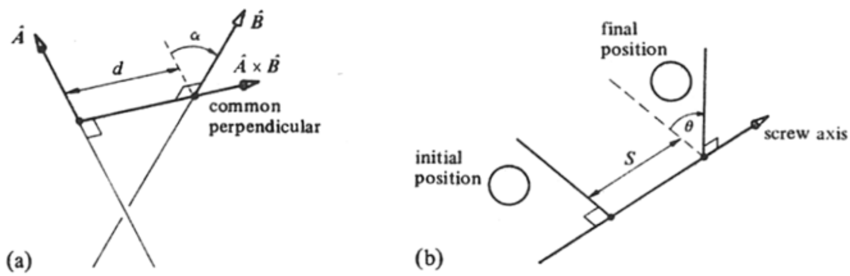

Fig. 6. Dual angles in 3D space: (a) the dual angle, $\alpha + \varepsilon d$, between two skew lines; (b) the dual angular displacement, $\theta + \varepsilon S$, of a rigid body. (Source: J. Rooney, 1978a, p. 47)

Algebraically, each of the three different types of complex number is just an ordered pair (a, b) of real numbers with a different multiplication rule for the product of two such ordered pairs. The symbol ε in the dual number is essentially the operator originally introduced by Clifford, (1873), although here it is an abstract algebraic quantity rather than an operator in mechanics. The usefulness of this type of abstract number derives from the work of (Study, 1901) who showed how the twist angle, α and common perpendicular distance, d, between two skew lines may be combined into a dual number of the form $\alpha + \varepsilon d$ (where $\varepsilon^2 = 0$). This is referred to as the *dual angle* between the lines (Figure 6a).

Dual angles also occur in the description of a general rigid-body spatial displacement, which involves a real angle and a real distance (Figure 6b).

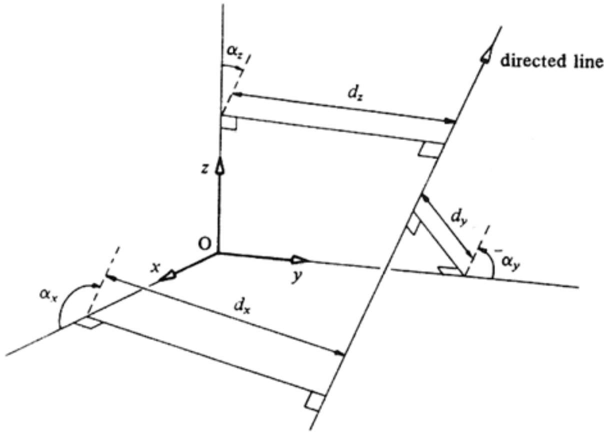

Fig. 7. The dual direction cosines, $\cos(\alpha_x + \varepsilon d_x)$, $\cos(\alpha_y + \varepsilon d_y)$, and $\cos(\alpha_z + \varepsilon d_z)$ of a directed line in space. (Source: J. Rooney, 1978a, p. 48)

It was Chasles (1830) who proved that such a displacement was equivalent to a combination of a *rotation* about and a *translation* along some straight line. Later Ball (1900) referred to this as a *screw displacement* about a *screw axis*. The motion thus defines a unique screw axis, a unique real angle θ (the rotation), and a unique real distance S (the translation). The variables θ and S may be combined into a dual number of the form $\theta + \varepsilon S$. This dual number is essentially a dual angle since the screw displacement may be specified by the initial and final positions of a line perpendicular to the screw axis, and these positions form a pair of skew lines (Figure 6b). Thus a spatial screw displacement can be considered to be a dual angular displacement about a general line (the screw axis) in space.

A given line in space, which does not pass through the origin, has three dual angles associated with it and they define it completely. These are the dual angles $\alpha_x + \varepsilon d_x$, $\alpha_y + \varepsilon d_y$, and $\alpha_z + \varepsilon d_z$, that it makes with the three coordinate axes (Figure 7). These three dual angles may be related to the six Plücker coordinates $(L, M, N; L_0, M_0, N_0)$, using rules for the expansion of (trigonometric) functions of a dual variable, and it is shown in Rooney (1978a) that the relationships are:

$$\cos(\alpha_x + \varepsilon d_x) = L + \varepsilon L_0,$$
$$\cos(\alpha_y + \varepsilon d_y) = M + \varepsilon M_0,$$
$$\cos(\alpha_z + \varepsilon d_z) = N + \varepsilon N_0.$$

The three dual numbers $L + \varepsilon L_0$, $M + \varepsilon M_0$ and $N + \varepsilon N_0$ are referred to as the *dual direction cosines* of the line and they may be considered to be the three components of a *unit dual vector*, $\hat{\mathbf{L}}$ in the same way that (L, M, N) forms a unit real vector, $\mathbf{L}$ whose components are three real direction cosines. The dual vector describing any line in space is written:

$$\hat{\mathbf{L}} = \mathbf{L}+\varepsilon\mathbf{L}_0 = (L, M, N)+\varepsilon(L_0, M_0, N_0) = (L+\varepsilon L_0, M+\varepsilon M_0, N+\varepsilon N_0).$$

Here the circumflex over a quantity does not indicate a unit quantity. It is referred to as the *dual symbol* and it is used always to signify a dual quantity (a dual number, dual vector, dual matrix, or dual quaternion). Thus, $\alpha + \varepsilon d$ would be written as $\hat{\alpha}$, and $\theta + \varepsilon S$ as $\hat{\theta}$. Similarly $\hat{L}$ would be $L + \varepsilon L_0$. The first component of the dual quantity ($\mathbf{L}$, α, θ, L, etc.) is referred to as the real or *primary* part and the second component ($\mathbf{L}_0$, d, S, L_0, etc.) is the dual or *secondary* part. Geometrically, the relationship between a real quantity, say α, and its corresponding dual quantity $\hat{\alpha}$ $(= \alpha + \varepsilon d)$ is essentially the relationship between the geometry of intersecting lines (spherical geometry) and the geometry of skew lines (spatial geometry).

Spherical geometry is partly concerned with subsets of points on the surface of a unit sphere. For example, three great-circle arcs define a *spherical triangle* (Todhunter and Leathem, 1932). But, since any point on the surface defines a unique (radial) line joining it to the centre, O, of the sphere, spherical geometry is also concerned with sets of intersecting straight lines in space (Figure 8a). The two viewpoints are equivalent and the length of a great-circle arc on the surface corresponds to the angle between the two intersecting lines defining the arc's endpoints. Three intersecting lines determine a spherical triangle.

Spatial geometry is partly concerned with the more general situation of non-intersecting or skew straight lines in space. For example, three skew lines define a *spatial triangle* (Yang, 1963), and Figure 8b illustrates these lines and their three common perpendiculars. For spatial rotations about a fixed point, O, the rotation axes all intersect in O and the geometry is spherical (Rooney, 1977). For screw displacements about skew lines the geometry is spatial (Rooney, 1978a).

The relationship between spherical geometry and spatial geometry was formalised by Kotelnikov (1895) in his *Principle of Transference*. The original reference is very difficult to obtain and consequently the precise statement of the principle and its original proof are not generally avail-

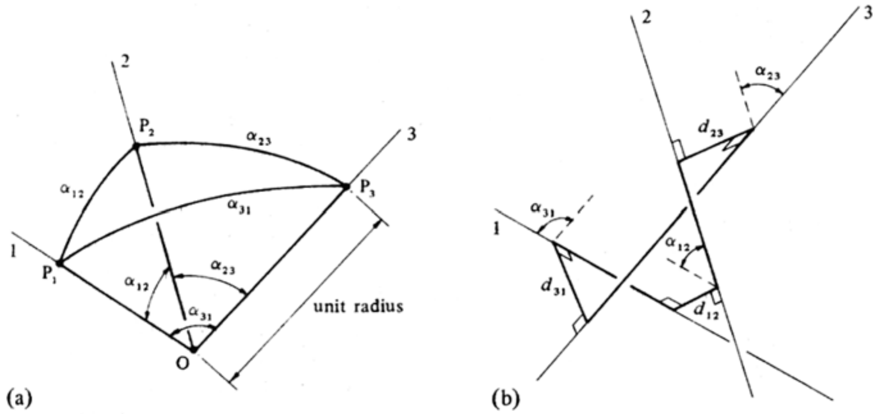

Fig. 8. The relationship between spherical geometry and spatial geometry: (a) a spherical triangle; (b) a spatial triangle. (Source: J. Rooney, 1978a, p. 51)

able (Rooney, 1975a). The one-many relationship may be expressed as $\alpha \leftrightarrow \alpha + \varepsilon d$ and $\theta \leftrightarrow \theta + \varepsilon S$. One version of the principle states that

> all laws and formulae relating to a spherical configuration (involving intersecting lines and real angles) are also valid when applied to an equivalent spatial configuration of skew lines if each real angle, α or θ, in the spherical formulae is replaced by the corresponding dual angle, $\alpha + \varepsilon d$ or $\theta + \varepsilon S$.

The real direction cosines, L, M and N, of a line through the origin involve the real angles α_x, α_y, and α_z, and the dual direction cosines, $L + \varepsilon L_0$, $M + \varepsilon M_0$ and $N + \varepsilon N_0$, of a general line involve the dual angles $\alpha_x + \varepsilon d_x$, $\alpha_y + \varepsilon d_y$, and $\alpha_z + \varepsilon d_z$. Thus, in applying the principle, real angles and real direction cosines must be replaced with dual angles and dual direction cosines respectively.

The dual vector $\hat{\mathbf{L}} = \mathbf{L} + \varepsilon \mathbf{L}_0$ representing a line, as in Figure 4, not passing through the origin is not the most general type of dual vector that may occur since, in Figure 4, $\mathbf{L}$ and $\mathbf{L}_0$ are orthogonal and $\mathbf{L}$ is a unit vector, so there $\hat{\mathbf{L}} = \mathbf{L} + \varepsilon \mathbf{L}_0$ is a *unit dual vector*. In the general case $\mathbf{L}$ need not be a unit vector and need not be orthogonal to $\mathbf{L}_0$, which is then not the moment of $\hat{\mathbf{L}}$ about the origin. What is obtained is a dual vector with six independent real components (L, M, N, L_0, M_0 and N_0), which is referred to as a *motor* (Clifford, 1873; Brand, 1947). This describes a line in space (as before) but with two extra magnitudes. The situation is illustrated by Figure 5, and the two extra magnitudes are the magnitude of $\mathbf{L}$ (this was previously a unit vector in Figure 4) and the component of $\mathbf{L}_0$ along $\mathbf{L}$, namely $\mathbf{L}_0''$ (this was previously zero in Figure 4). The direction of the line is still given by $\mathbf{L}$, and the component of $\mathbf{L}_0$ perpendicular to $\mathbf{L}$, namely $\mathbf{L}_0'$, now represents the moment of $\mathbf{L}$ about the origin, O.

A dot product and a cross product may be defined for general dual vectors in the style of Gibbs (1901). Thus, given two dual vectors $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$, where $\hat{\mathbf{A}} = \mathbf{A} + \varepsilon\mathbf{A}_0$ and $\hat{\mathbf{B}} = \mathbf{B} + \varepsilon\mathbf{B}_0$, the dot product, or *scalar product* (Brand, 1947), is defined as:

$$\hat{\mathbf{A}} \cdot \hat{\mathbf{B}} = (\mathbf{A} + \varepsilon\mathbf{A}_0) \cdot (\mathbf{B} + \varepsilon\mathbf{B}_0) = \mathbf{A} \cdot \mathbf{B} + \varepsilon(\mathbf{A} \cdot \mathbf{B}_0 + \mathbf{A}_0 \cdot \mathbf{B}).$$

This is a dual number in general and is independent of the location of O, the origin. It can be shown that if $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ are unit dual vectors defining two lines in space and if the dual angle between the lines is $\hat{\alpha} = \alpha + \varepsilon d$ then

$$\hat{\mathbf{A}} \cdot \hat{\mathbf{B}} = \cos\hat{\alpha} = \cos(\alpha + \varepsilon d) = \cos\alpha - \varepsilon d \sin\alpha.$$

This is in complete analogy with the relationship between two unit real vectors and the real angle between them: $\mathbf{A} \cdot \mathbf{B} = \cos\alpha$. If the scalar product of two non-parallel unit dual vectors is real (that is, if the dual part is zero) then the lines intersect. In addition if the scalar product is zero (that is, if both real and dual parts are zero) then the lines intersect at right angles (Brand, 1947).

In a similar way the cross product, or *motor product* (Brand, 1947) of two dual vectors in the style of Gibbs (1901) is defined as:

$$\hat{\mathbf{A}} \times \hat{\mathbf{B}} = (\mathbf{A} + \varepsilon\mathbf{A}_0) \times (\mathbf{B} \times \varepsilon\mathbf{B}_0) = \mathbf{A} \times \mathbf{B} + \varepsilon(\mathbf{A} \times \mathbf{B}_0 + \mathbf{A}_0 \times \mathbf{B}).$$

This is a motor in general and the line it defines is the common perpendicular line to $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ (Figure 6a). If $\hat{\mathbf{E}} = \mathbf{E} + \varepsilon\mathbf{E}_0$ is a unit line vector representing this common perpendicular, if $\hat{\alpha} = \alpha + \varepsilon d$ is the dual angle between $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$, and if $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ are unit dual vectors then it can be shown that

$$\hat{\mathbf{A}} \times \hat{\mathbf{B}} = \sin\hat{\alpha}\hat{\mathbf{E}} = (\sin\alpha + \varepsilon d \cos\alpha)\hat{\mathbf{E}}.$$

Again this is in complete analogy with the real vector case: $\mathbf{A} \times \mathbf{B} = \sin\alpha\mathbf{E}$. If the motor product of two unit dual vectors is a pure dual vector (that is, if the real or primary part is zero) then the lines are parallel. In addition if the dual part is also zero then the lines are collinear (Brand, 1947). Finally it is possible to define scalar triple products and motor triple products for dual vectors in complete analogy with the usual real vector case.

Now, because the general spatial screw displacement (Figure 6b) of a rigid body consists of a rotation through an angle θ about and a translation through a distance S along an axis in space (Chasles, 1830; Ball, 1900), a total of six parameters are necessary to define the displacement completely. Four parameters specify the axis (a line in 3D space), one parameter specifies θ, and one parameter specifies S. It thus appears that a single finite screw displacement may be represented by a general dual vector or *motor* (Clifford, 1873; Brand, 1947) since two magnitudes (θ and S) and a line having both direction and position are involved.

However, although this relatively simple representation is possible, it is not a very satisfactory one. The disadvantages arise in attempting to obtain the resultant of two successive screw displacements (this should itself be a screw displacement). One problem is that two screw displacements do not commute and the order in which they occur must first be specified. The resultant motor cannot therefore just be given by the sum (which is commutative) of the two individual motors, as it should be, if the screw displacements behaved as true motors. This situation is analogous to that encountered in attempting to use a simple vector representation for the sum of two finite rotations about a fixed point (Rooney, 1977). In that case the parallelogram addition law fails to give the resultant of two such rotations. As a consequence it is not possible to use a simple motor representation for screw displacements. Instead, a line transformation is used for the representation (Rooney, 1978a). The line transformation (representing a screw) is derived from a point transformation (representing a rotation) by replacing real angles and real direction cosines with dual angles and dual direction cosines in accordance with the Principle of Transference. The line transformation approach leads to the modern equivalent of Clifford's biquaternion, namely the *unit dual quaternion* representation, involving a combination of quaternions and dual numbers. The unit dual quaternion derives from a unit quaternion by replacing the four real components of the latter with four dual number components. Alternatively two real quaternions are combined as the primary and secondary parts of the resulting unit dual quaternion.

The concept of a quaternion, as introduced and developed by Hamilton (1844, 1899, 1901), was invented to enable the ratio of two vectors to be defined and thus could be used to stretch-rotate one vector, **r**, into another,

$\mathbf{r}'$, by premultiplying the first with a suitable quaternion. In this case $\mathbf{r}$ would be premultiplied by the product $\mathbf{r}'\mathbf{r}^{-1}$. The latter 'quotient' of vectors is a quaternion if the inverse $\mathbf{r}^{-1}$ of $\mathbf{r}$ is given an appropriate definition. The operation of premultiplying $\mathbf{r}$ by the quaternion $\mathbf{r}'\mathbf{r}^{-1}$ may be viewed as a point transformation operating on the point represented by the position vector $\mathbf{r}$.

The equivalent operation for line transformations requires an operator capable of operating on a line, and screw displacing it. A point transformation operates on the position vector, $\mathbf{r}$, of a point to give another position vector, $\mathbf{r}'$. A line is represented by a unit dual vector $\hat{\mathbf{L}} = \mathbf{L} + \varepsilon\mathbf{L}_0$, where $\mathbf{L} \cdot \mathbf{L} = 1$, and $\mathbf{L} \cdot \mathbf{L}_0 = 0$, so by analogy the problem is essentially one of transforming one unit dual vector $\hat{\mathbf{L}}_1$ into another, $\hat{\mathbf{L}}_2$. As with the quaternion ratio of two vectors, $\mathbf{r}'\mathbf{r}^{-1}$, this may be achieved if an appropriate ratio or quotient $\hat{\mathbf{B}}\hat{\mathbf{A}}^{-1}$ of two general dual vectors $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ can be defined. It was an analogous problem that led Clifford to invent the biquaternion as the ratio of two motors (Clifford, 1873). It transpires that the ratio of two general dual vectors is an operator formed from a combination of a quaternion and a dual number. Nowadays this is referred to as a *unit dual quaternion*, although it is essentially a biquaternion.

The relative spatial relationship of two general dual vectors $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ requires eight parameters for its specification. Four of these define the common perpendicular line between the axes of the motors; two more specify the dual angle between these axes; and finally two parameters are required to represent the ratios of the two magnitudes associated with the second motor to those associated with the first. So an operator to transform $\hat{\mathbf{A}}$ into $\hat{\mathbf{B}}$ must also have at least eight parameters in its specification.

A dual quaternion $\hat{q}$ is a 4-tuple of dual numbers of the form $\hat{q} = (q_1 + \varepsilon q_{01}, q_2 + \varepsilon q_{02}, q_3 + \varepsilon q_{03}, q_4 + \varepsilon q_4)$, where $\varepsilon^2 = 0$, and hence it has eight real components, q_1, q_2, q_3, q_4, q_{01}, q_{02}, q_{03} and q_{04}. It may be written alternatively, as with all dual quantities, in terms of primary and secondary parts as

$$\hat{q} = (q_1, q_2, q_3, q_4) + \varepsilon(q_{01}, q_{02}, q_{03}, q_{04}) = q + \varepsilon q_0,$$

where q and q_0 are real quaternions. This looks just like Clifford's biquaternion $q + \omega r$ where $\omega^2 = 0$. The operator for screw displacement is formed from a dual quaternion by providing the latter with an appropriate consistent algebra.

An algebra is imposed on dual quaternions by defining a suitable multiplication rule (addition and subtraction are performed componentwise). The rule that corresponds with that of Clifford (1873), for his biquaternions, is essentially equivalent to that for the real quaternions (Hamilton, 1844, 1899, 1901; Rooney, 1977) but with each real component replaced by the corresponding dual component. So, write two dual quaternions, $\hat{p}$ and $\hat{q}$, in the form

$$\hat{p} = (p_1 + \varepsilon p_{01}) + (p_2 + \varepsilon p_{02})i + (p_3 + \varepsilon p_{03})j + (p_4 + \varepsilon p_{04})k,$$
$$\hat{q} = (q_1 + \varepsilon q_{01}) + (q_2 + \varepsilon q_{02})i + (q_3 + \varepsilon q_{03})j + (q_4 + \varepsilon q_{04})k.$$

Then the dual quaternion product of $\hat{p}$ and $\hat{q}$ is defined by expanding the expression $\hat{p}\hat{q}$ using the standard rules of algebra together with the multiplication rules for products of quaternions, $i^2 = j^2 = k^2 = ijk = -1$ and $ij = k = -ji$, $jk = i = -kj$, $ki = j = -ik$, and finally using the rule $\varepsilon^2 = 0$, to give:

$$\begin{aligned}
\hat{p}\hat{q} = [&(p_1q_1 - p_2q_2 - p_3q_3 - p_4q_4)\\
&+ \varepsilon(p_1q_{01} - p_2q_{02} - p_3q_{03} - p_4q_{04} + p_{01}q_1 - p_{02}q_2 - p_{03}q_3 - p_{04}q_4)]\\
&+ [(p_1q_2 + p_2q_1 + p_3q_4 - p_4q_3)\\
&+ \varepsilon(p_1q_{02} + p_2q_{01} + p_3q_{04} - p_4q_{03} + p_{01}q_2 + p_{02}q_1 + p_{03}q_4 - p_{04}q_3)]i\\
&+ [(p_1q_3 - p_2q_4 + p_3q_1 + p_4q_2)\\
&+ \varepsilon(p_1q_{03} - p_2q_{04} + p_3q_{01} + p_4q_{02} + p_{01}q_3 - p_{02}q_4 + p_{03}q_1 + p_{04}q_2)]j\\
&+ [(p_1q_4 + p_2q_3 - p_3q_2 + p_4q_1)\\
&+ \varepsilon(p_1q_{04} + p_2q_{03} - p_3q_{02} + p_4q_{01} + p_{01}q_4 + p_{02}q_3 - p_{03}q_2 + p_{04}q_1)]k.
\end{aligned}$$

Division is defined (as an inverse of multiplication) for dual quaternions in terms of a *conjugate* and a *norm*. This is analogous to the division process for quaternions. The conjugate of $\hat{q}$ is defined as

$$\overline{\hat{q}} = (q_1 + \varepsilon q_{01}) - (q_2 + \varepsilon q_{02})i - (q_3 + \varepsilon q_{03})j - (q_4 + \varepsilon q_{04})k$$

and the norm of $\hat{q}$ is defined as the dual number

$$\begin{aligned}
|\hat{q}| &= (q_1 + \varepsilon q_{01})^2 + (q_2 + \varepsilon q_{02})^2 + (q_3 + \varepsilon q_{03})^2 + (q_4 + \varepsilon q_{04})^2\\
&= (q_1^2 + q_2^2 + q_3^2 + q_4^2) + 2\varepsilon(q_1q_{01} + q_2q_{02} + q_3q_{03} + q_4q_0).
\end{aligned}$$

The *inverse* or *reciprocal* of $\hat{q}$ is then

$$\hat{q}^{-1} = \frac{\bar{\hat{q}}}{|\hat{q}|}.$$

This is not defined if the primary part, q, of $\hat{q}$ is zero (that is, if $q_1 = q_2 = q_3 = q_4 = 0$) since the norm is then zero. It is easily checked that, for a non-zero norm, $\hat{q}\hat{q}^{-1} = \hat{q}^{-1}\hat{q} = 1$. If $|\hat{q}| = 1$ the dual quaternion is a unit dual quaternion.

In complete analogy with the real quaternions and real vectors considered in Rooney (1977), it is possible to use a dual quaternion to provide a dual vector algebra (Brand, 1947; Yang, 1963; Rooney, 1977). Thus a dual vector is identified with a dual quaternion having a zero first (dual number) component. Given two such dual vectors

$$\hat{\mathbf{A}} = (A_1 + \varepsilon A_{01})i + (A_2 + \varepsilon A_{02})j + (A_3 + \varepsilon A_{03})k,$$
$$\hat{\mathbf{B}} = (B_1 + \varepsilon B_{01})i + (B_2 + \varepsilon B_{02})j + (B_3 + \varepsilon B_{03})k,$$

their dual quaternion product is

$$\begin{aligned}\hat{\mathbf{A}}\hat{\mathbf{B}} = -[&(A_1 + \varepsilon A_{01})(B_1 + \varepsilon B_{01}) \\ &+ (A_2 + \varepsilon A_{02})(B_2 + \varepsilon B_{02}) + (A_3 + \varepsilon A_{03})(B_3 + \varepsilon B_{03})] \\ &+ [(A_2 + \varepsilon A_{02})(B_3 + \varepsilon B_{03}) - (A_3 + \varepsilon A_{03})(B_2 + \varepsilon B_{02})]i \\ &+ [(A_3 + \varepsilon A_{03})(B_1 + \varepsilon B_{01}) - (A_1 + \varepsilon A_{01})(B_3 + \varepsilon B_{03})]j \\ &+ [(A_1 + \varepsilon A_{01})(B_2 + \varepsilon B_{02}) - (A_2 + \varepsilon A_{02})(B_1 + \varepsilon B_{01})]k.\end{aligned}$$

This is expressed more concisely in terms of the scalar and motor products already defined earlier for dual vectors (Brand, 1947). It is then easily shown that the dual quaternion product of $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ is

$$\hat{\mathbf{A}}\hat{\mathbf{B}} = -\hat{\mathbf{A}} \cdot \hat{\mathbf{B}} + \hat{\mathbf{A}} \times \hat{\mathbf{B}}.$$

This product is in general a dual quaternion since the first component (the scalar product) is non-zero unless the lines associated with $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ intersect at right angles.

The ‘ratio’ of any two dual vectors $\hat{\mathbf{B}}$ and $\hat{\mathbf{A}}$ can now be formed as

$$\hat{\mathbf{B}}\hat{\mathbf{A}}^{-1} = -\frac{\hat{\mathbf{B}}\hat{\mathbf{A}}}{|\hat{\mathbf{A}}|},$$

where $\hat{\mathbf{A}}^{-1}$ is the (dual quaternion) inverse of $\hat{\mathbf{A}}$. This product $\hat{\mathbf{B}}\hat{\mathbf{A}}^{-1}$ is a dual quaternion and it will operate on the dual vector $\hat{\mathbf{A}}$ to give the dual vector $\hat{\mathbf{B}}$ since $(\hat{\mathbf{B}}\hat{\mathbf{A}}^{-1})\hat{\mathbf{A}} = \hat{\mathbf{B}}$. It is the modern form of Clifford's biquaternion. There are of course two ratios since dual quaternions do not commute, and it is equally possible to consider the 'ratio' $\hat{\mathbf{A}}^{-1}\hat{\mathbf{B}}$ in the above.

The operator $\hat{\mathbf{B}}\hat{\mathbf{A}}^{-1}$ operates on $\hat{\mathbf{A}}$ to produce $\hat{\mathbf{B}}$. But an operation is required which will screw displace *any* dual vector along a given line, and not just those intersecting the line orthogonally. For this reason, and by use of arguments similar to those considered in Rooney (1977), the following type of three-term product operation is needed to operate on any dual vector $\hat{\mathbf{A}}$ to screw displace it into $\hat{\mathbf{A}}'$:

$$\hat{\mathbf{A}}' = \hat{q}_{\hat{\mathbf{n}}}^{-1}(\hat{\theta})\hat{\mathbf{A}}\hat{q}_{\hat{\mathbf{n}}}(\hat{\theta}).$$

Here

$$\hat{q}_{\hat{\mathbf{n}}}(\hat{\theta}) = \cos\frac{\hat{\theta}}{2} + \sin\frac{\hat{\theta}}{2}\,\hat{\mathbf{n}}$$

is a unit dual quaternion, and $\hat{q}_{\hat{\mathbf{n}}}^{-1}(\hat{\theta})$ is its inverse (equal to its conjugate since its norm is unity). The dual angle $\hat{\theta} = \theta + \varepsilon S$ combines the screw displacement angle θ, and distance S, along the screw axis $\hat{\mathbf{n}}$, where

$$\hat{\mathbf{n}} = (l + \varepsilon l_0)i + (m + \varepsilon m_0)j + (n + \varepsilon n_0)k$$

represents the line of the screw axis, with direction cosines (l, m, n) and moment (l_0, m_0, n_0) about the origin, and where

$$(l + \varepsilon l_0)^2 + (m + \varepsilon m_0)^2 + (n + \varepsilon n_0)^2 = 1.$$

The trigonometric functions of the dual variable $\hat{\theta}$ are evaluated using the rules for expanding functions of a dual variable, namely:

$$\cos(\theta + \varepsilon S) = \cos\theta - \varepsilon S\sin\theta,$$

$$\sin(\theta + \varepsilon S) = \sin\theta + \varepsilon S\cos\theta.$$

The above operation, $\hat{\mathbf{A}}' = \hat{q}_{\hat{\mathbf{n}}}^{-1}(\hat{\theta})\hat{\mathbf{A}}\hat{q}_{\hat{\mathbf{n}}}(\hat{\theta})$, achieves the desired general screw transformation of any $\hat{\mathbf{A}}$ into a new position $\hat{\mathbf{A}}'$. It is equivalent to Clifford's

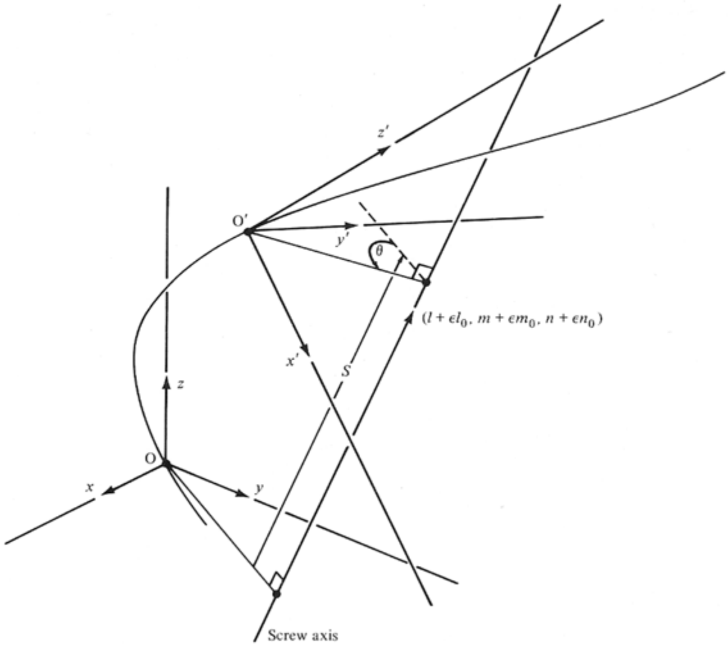

Fig. 9. The general spatial screw displacement of a coordinate system about a screw axis through angle θ and distance S. (Source: J. Rooney, 1984, p. 237)

tensor-twist, since it does not change the pitch of $\hat{\mathbf{A}}$. Although the operation is expressed in terms of the half dual angle

$$\frac{\hat{\theta}}{2} = \frac{\theta}{2} + \varepsilon \frac{S}{2},$$

it actually screw transforms $\hat{\mathbf{A}}$ into $\hat{\mathbf{A}}'$ through the full dual angle $\hat{\theta} = \theta + \varepsilon S$. The necessity for introducing the half dual angle into the unit dual quaternion echoes the situation that occurs with the representation of rotations about a fixed point using unit quaternions (Rooney, 1977). It was Rodrigues (1840) who first recognised this need when several rotations are performed consecutively (Baker and Parkin, 2003). It transfers naturally into the screw displacement situation. Because of the half dual angle the representation is double valued since $\hat{q}_{\hat{\mathbf{n}}}(\hat{\theta} + 2\pi) = -\hat{q}_{\hat{\mathbf{n}}}(\hat{\theta})$.

The form of the unit dual quaternion $\hat{q}_{\hat{\mathbf{n}}}(\hat{\theta})$ representing a general screw displacement of the xyz Cartesian coordinate system about a line with direction cosines (l, m, n) and moment (l_0, m_0, n_0) about the origin, through an angle θ and a distance S (see Figure 9), is expanded as:

$$\begin{aligned}
\hat{q}_{\hat{\mathbf{n}}}(\hat{\theta}) &= \cos\frac{\hat{\theta}}{2} + \sin\frac{\hat{\theta}}{2}\hat{\mathbf{n}} \\
&= \cos\frac{\theta+\varepsilon S}{2} + \sin\frac{\theta+\varepsilon S}{2}[(l+\varepsilon l_0)i + (m+\varepsilon m_0)j + (n+\varepsilon n_0)k] \\
&= \left[\cos\frac{\theta}{2} - \varepsilon\frac{S}{2}\sin\frac{\theta}{2}\right] \\
&\quad + \left[l\sin\frac{\theta}{2} + \varepsilon\left(l\frac{S}{2}\cos\frac{\theta}{2} + l_0\sin\frac{\theta}{2}\right)\right]i \\
&\quad + \left[m\sin\frac{\theta}{2} + \varepsilon\left(m\frac{S}{2}\cos\frac{\theta}{2} + m_0\sin\frac{\theta}{2}\right)\right]j \\
&\quad + \left[n\sin\frac{\theta}{2} + \varepsilon\left(n\frac{S}{2}\cos\frac{\theta}{2} + n_0\sin\frac{\theta}{2}\right)\right]k.
\end{aligned}$$

A unit dual quaternion $\hat{q}_{\hat{\mathbf{n}}}(\hat{\theta})$ is specified by only six (rather than eight) independent parameters because it has a unit norm, and so the operation $\hat{\mathbf{A}}' = \hat{q}_{\hat{\mathbf{n}}}^{-1}(\hat{\theta})\hat{\mathbf{A}}\hat{q}_{\hat{\mathbf{n}}}(\hat{\theta})$ screw transforms the dual vector $\hat{\mathbf{A}}$ without stretching it (its two magnitudes remain unchanged). It also transforms unit dual vectors $\hat{\mathbf{L}}$ into unit dual vectors.

The unit dual quaternion representation (the modern equivalent of Clifford's biquaternion, specifically his tensor-twist) for a screw displacement is elegant and economical compared with other representations. It is particularly useful when performing multiple screw displacements in succession, as is frequently required in the field of Mechanism and Machine Science. The representation is of course double- valued, so care must be taken in its use. It is considered to be one of the best representations of line transformations since it is so concise and is perhaps the most easily visualised of all the screw representations because the screw axis, $\hat{\mathbf{n}}$, and the dual angular displacement, $\theta + \varepsilon S$, enter so directly into its specification.

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NICOLAUS COPERNICUS (1473–1543)

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Abstract. Nicolaus Copernicus was an astronomer who provided the first modern formulation of a heliocentric (sun-centered) model of the solar system. Using logical arguments, available theory, and showing the weaknesses of the prevailing geocentric description of the universe, he elaborated a revolutionary model of the motion of the planets. His work is considered to be the most fundamental contribution ever made to the mechanics of celestial bodies. The paper introduces Copernicus' life, the history of his most important works, and some less well-known facts about his impact on the practice of general mechanics.

Biographical Notes

Nicolaus Copernicus in Latin, Mikołaj Kopernik in Polish, was born on the 19th of February 1473 in Poland in the town of Toruń. In the early XV century Toruń was part of the Prussian Confederation. With the Second Treaty of Toruń in 1466, Toruń and Prussia's western part, called "Royal Prussia", were connected to the Kingdom of Poland, while the eastern part remained under the administration of the Teutonic Order, later to become "Ducal Prussia".

Copernicus' father was probably a Germanised Slav, and a citizen of Cracow, then the capital of Poland. He was a trader, who in 1460 had migrated to Toruń and at the age of 40 married Barbara Watzenrode, daughter of a wealthy merchant. Copernicus with one brother and two sisters was the youngest child.

When Nicolaus Copernicus was a young boy of the age of ten his father died. Hence his uncle Lukas Watzenrode, the canon of the cathedral of Frombork (Frauenburg), who later held the position of Bishop of Varmia (Ermland), brought him up. Copernicus attended St. John's School in Toruń.

M. Ceccarelli (ed.), Distinguished Figures in Mechanism and Machine Science, 117–134.

From 1491 to 1495 he studied the so-called “free arts”, including mathematics with Euclidean geometry and astronomy taught by Albert Brudzewski (astronomer, mathematician and prominent professor), and canon law at the Cracow Academy (today the Jagiellonian University). At that time, Cracow Academy had already 100 years of history and was the best place to study astronomy in the northern part of Europe. Copernicus’ uncle Watzenrode financed his education in Cracow and wished him to become a bishop. Despite this expectation, after return to Frombork – at the age of 24 – Nicolaus was not sure what to choose for his future career.

In 1496, Lukas Watzenrode sent Copernicus to Bologna to study canon and civil law. While Nicolaus was in Bologna his uncle became in 1497 the Bishop of Warmia, and he proposed to Copernicus the position of a canon of Frombork Cathedral. Nicolaus refused to return and got permission to stay in Italy until 1500. With the end of his studies in Bologna he did not take his final exams and did not return to Frombork. Instead, together with his older brother, he went to Rome where the big celebrations of fifteen centuries of Christianity were organized. With a short return to Frombork in 1501, Copernicus got permission to continue his education for two more years under the condition that he would study medical science. In the years 1501–1503, he studied medicine at the University of Padua. The people believed that the zodiac symbols influenced different parts of the human body. Because of that, astronomy professors instructed medical students in predicting planet positions.

In 1502 Nicolaus was obliged to return, but two years study in a three years programme did not entitle him to the medical degree. To show to his uncle that the time and funds spent for his education were not lost, in May 1503, he passed doctoral exams in canon law at the University of Ferrara.

In 1503, he returned to Frombork where he first worked as a medical advisor and secretary to his uncle. Later his duties were extended by administration of the diocese of Frombork. In 1510, Nicolaus gave up the position of assistant to his uncle and kept only the responsibility as the canon of Frombork Cathedral. That ended his career as a high Church official, but offered more time for astronomical research.

In the years 1511–1513 he supervised financial transactions in the Frombork Church chapter and, as its representative, was involved in supervision of town taxes, the treasury system and judicial matters. In the summer of 1517, Nicolaus wrote a short work about a good monetary system. He formulated

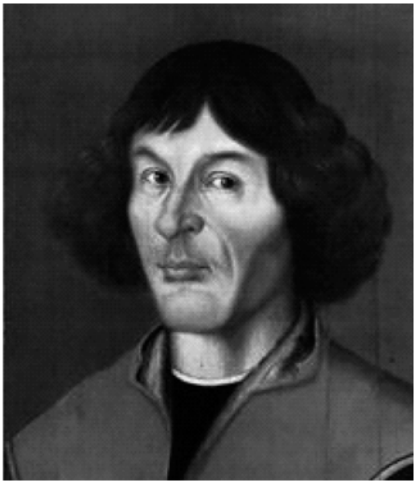


Fig. 1. Portrait of Nicolaus Copernicus from the museum of Toruń – the town near Frombork (beginning of XVI c., unknown artist).

the opinion that the "bad" money replaces "good" money. Good money is money that has little difference between its exchange value and its commodity value. Currently this rule is known as Thomas Gresham's law ("When there is a legal tender currency, bad money drives good money out of circulation") described 30 years after Copernicus.

Royal Prussia where Copernicus lived was an autonomous part of the Polish Crown land, but it had a separate monetary system and treasury. Nicolaus suggested that the king must supervise the minting of coins, and then their quality and quantity would be controlled. He proposed financial reforms preventing currency devaluation and elaborated a safe method of replacing the devaluated currency. Rules proposed by him assuring a "healthy" monetary system, were accepted in 1522 during the meeting of the Polish King's Prussian States. The reforms were introduced over the next six years and "healthy" coins minted by the Polish King Sigismundus the Old had a silver parity equal to its value.

From 1514 Copernicus started the description of his new astronomical theory. His main work *De Revolutionibus Orbium Coelestium* ("On the Revolutions of the Heavenly Spheres") explains the fundamentals of the theory that the Earth revolves around the Sun, contrary to the general belief that the Sun revolves around the Earth.

In 1543, Nicolas Copernicus suddenly became sick with a brain haemorrhage and after a short illness, he died on the 24th of May. The legend says that he received the first printed copy of his book on his deathbed.

Nicolaus Copernicus was buried in Frombork Cathedral. Archaeologists searched for his remains until on 3 November of 2005 it was announced that a few months earlier Copernicus' skull had been discovered. His face was reconstructed. The image of the elderly Copernicus shows a strong similarity to the XVI c. portrait of the younger astronomer, which possibly was painted from nature (Figure 1).

Copernicus spent most of his working life in Royal Prussia and his main language for written communication was Latin.

Astronomical Studies and Observations

While studying in Cracow (1491–1495), Copernicus read Euclid's book on geometry, which was first printed in 1482, moreover, he read the Latin translation of an Arabic text on astronomy. The students were taught e astrology, which required the interpretation of astronomical tables. The position of planets at the moment of birth was the basic information needed for a horoscope. Nicolaus obtained two sets of printed astronomical tables and, on the blank pages, put more details concerning the planet positions. This fact suggests that at that time he was already showing an interest in astronomy. Continuing his education in Bologna, Copernicus rented a room at the house of the famous astronomer Domenico Maria Novara. He attended Novara's lectures and became his assistant, helping him in astronomical observations. The first observations made together with Novara were later recorded in Copernicus' epochal book. At that time he read the astronomical works by Ptolemy (II c. AD) and started to compare this text with the theory of "ideal motion of celestial bodies" by Aristotle (IV c. BC).

Beginning in 1504 Copernicus began collecting observations and ideas pertinent to his famous theory. In 1504, Mars, Jupiter and Saturn were grouped in the Cancer constellation (big conjunction) which happens only each 20 years. Copernicus compared the observed and calculated positions and noticed the differences. Mars preceded the calculated position and Saturn lagged behind. Trying to explain this, he studied in more detail the works of Ptolemy. In 1515 Pope Leo X conceived the idea of modifying the Julian calendar, and asked Copernicus to become one of the experts proposing calendar modification. Unfortunately this correspondence was lost.

Copernicus spent at least 29 years (1514–1543) of his life on the descriptions and improvements of his theory. In those years Copernicus was focusing only on astronomy. He was mainly involved in observations, calculations and descriptions. The hand-written text of his famous manuscript consists of 212 double sided pages.

Review of Fundamental Manuscripts

In 1514 Copernicus wrote for his friends a short manuscript entitled *Commentariolus* ("Little Commentary") where he described his idea of the heliocentric system. Nicolaus continued gathering data for a more detailed description, which he started around the year 1529.

The major work *De Revolutionibus Orbium Coelestium* (usually translated as "On the Revolutions of the Heavenly Spheres", a more exact translation is "About Revolutions of Celestial Bodies") consists of six books called Book One, Book Two, etc. Book One includes 14 chapters. It presents the general vision of the heliocentric theory, and summarizes Copernicus' idea of the "universe". Book Two, also containing 14 chapters, is mainly theoretical; it presents the principles of spherical astronomy and introduces fundamentals for the arguments developed in the subsequent parts. Book Three, with 26 chapters, explains the apparent motion of the Sun and phenomena related to it. Book Four consists of 22 chapters and presents the description of the Moon's orbital motions. Book Five, in 36 chapters, contains concrete exposition of the new system explaining the planets' motion. Book Six, in 9 chapters, continues and summarizes the new heliocentric theory. Historians analysing Copernicus' manuscript noticed paper marks (water mark – philigran) – Figure 2, and changes in the color of ink. From this they concluded that the subsequencial pages were not written chronologically.

It is possible that the work was first written as a whole on the paper with philigrans C; then Copernicus improved and replaced selected parts. Paper with philigrans A and B was used only for manuscript packing. Philigrans D and E mark the pages which were replaced by Copernicus. Those philigrans appeared in the middle of Book Three and only the end of Book Six is marked again by philigrans C.

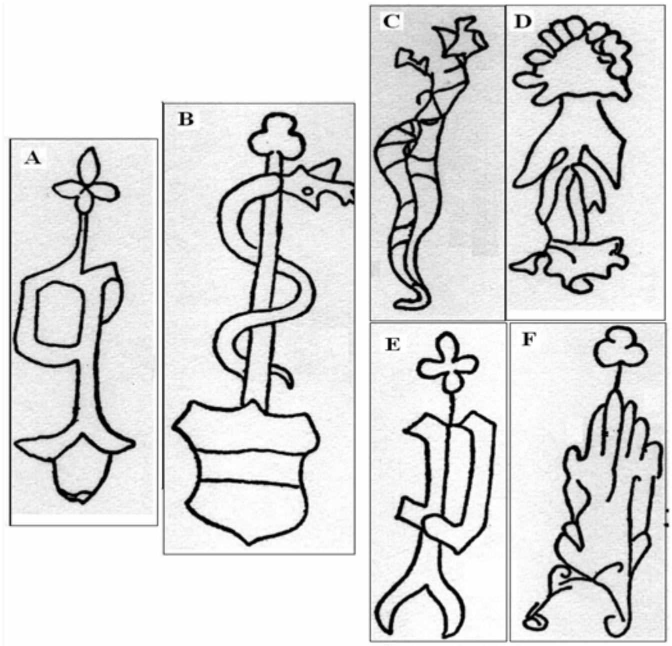


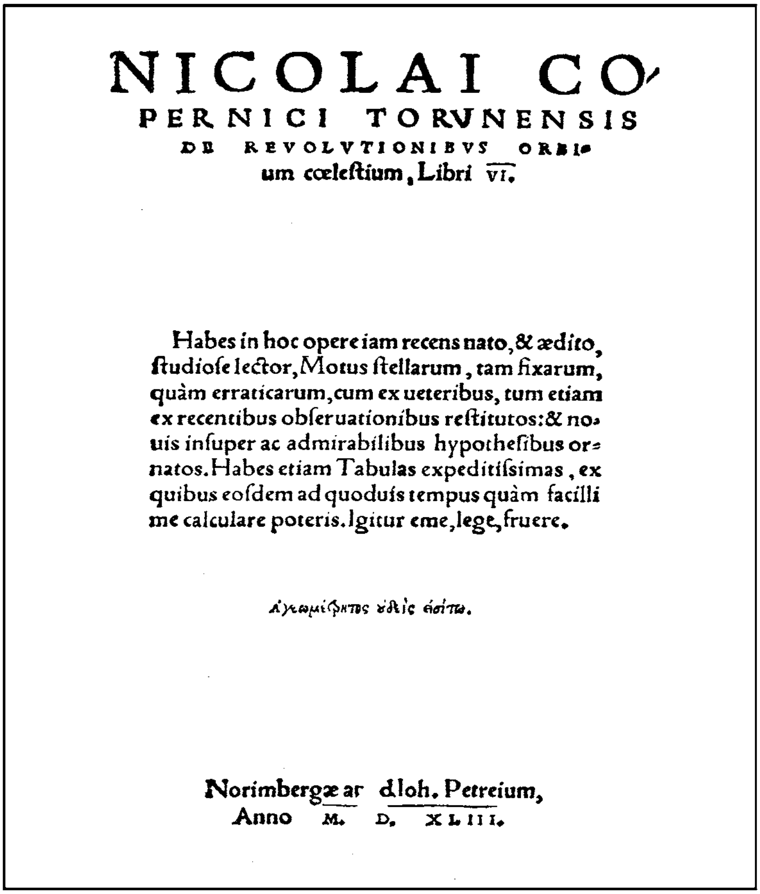
Fig. 2. Philigrans identified in Copernicus' manuscript (Birkenmajer, 1900).

Historians (Birkenmajer, 1900) suggest that the manuscript was revised two times and changes were written first on paper with philigrans D and the on paper with philigrans E. Less elaborated philigrans F, on thicker, worse quality paper, are only on pages 24 and 25 (Book One); the opinion is that those pages were added and are the last modification.

The main part of that manuscript was ready by the year 1530. In 1533, in the presence of several Catholic cardinals and Pope Clement VII, Johan Albrecht Widmannstetter delivered in Rome a series of lectures presenting a summary of Copernicus' theory. In 1536, Copernicus' manuscript describing the heliocentric theory was ready.

Despite urgings from many parts of Europe, he delayed publication, possibly worrying about criticism. In 1539 George Joachim Rheticus, a great mathematician from Wittenberg, arrived in Frombork and stayed with Copernicus for two years. During that time, he wrote a book *Narratio prima* outlining the Copernicus theory.

In 1542 Rheticus published a treatise on trigonometry elaborated by Copernicus (it was later included in the second book of *De Revolution-*


NICOLAI CO-
PERNICI TORVNENSIS
DE REVOLVTIONIBVS ORBI-
um cœlestium, Libri VI.

Habes in hoc opere iam recens nato, & ædito, studiose lector, Motus stellarum, tam fixarum, quàm erraticarum, cum ex ueteribus, tum etiam ex recentibus obseruationibus restitutos: & nouis insuper ac admirabilibus hypothesibus ornatos. Habes etiam Tabulas expeditissimas, ex quibus eosdem ad quoduis tempus quàm facillime calculare poteris. Igitur eme, lege, fruere.

ἀγεωμέτρητος οὐδεὶς εἰσίτω.

Norimbergæ apud Ioh. Petreium,
Anno M. D. XLIII.

Fig. 3. Title page of *De Revolutionibus* (1st edition), which is the only one with the note “Nicolai Copernici Torunensis . . . ” (Birkenmajer, 1900).

ibus). Under strong pressure from Rheticus, Copernicus finally agreed to give him his own manuscript; Rheticus then arranged with Johannes Petreius in Nuremberg to carry out the printing. The first printing (Figure 3) was finished in 1543 just before Copernicus’ death, and a second edition was printed in 1566 (in Basel). Originally the manuscript had an anonymous preface written by Andreas Osiander, a theologian and a friend of Copernicus. It is interesting that Osiander underscored that the theory presented and the trigonometric methods allow simpler and more accurate calculations of planet motions, but did not necessarily present the truth and did not have implications outside astronomy.

The title of the work was changed from the original "On the Revolutions of the Spheres of the World" to the "Six Books Concerning the Revolutions of the Heavenly Spheres" (*De Revolutionibus Orbium Coelestium*) – it appeared to mitigate the author's claim to describe the real universe.

As of our time Copernicus' work has had many editions. The manuscript is kept in the Library of Jagiellonian University in Cracow. The pages were scanned and the pictures can be seen on the Internet.

Copernicus' Contribution to Mechanics of Celestial Bodies Compared to the Existing Theories

Copernicus studied the ancient works on astronomy. One of the oldest texts that he obtained was written by Aristotle (IV c. BC). Aristotle wrote that the heavenly bodies are the most perfect realities, whose motions are ruled by simple principles. Their motions are perfect, which meant to him circular trajectories. Aristotle believed that the planets move with constant speed following ideal circles with the Earth being in the centre. Copernicus studied also the works of Ptolemy (II c. AD) who noticed that the motion of Mars differs much from Aristotle's ideal model. Trying to explain it, Ptolemy concluded that Mars moved with variable speed along a more complex than circular trajectory. Upon learning this conjecture, Nicolaus started careful observations of Mars and made a serious effort to find the rules governing its displacement. His famous *De Revolutionibus* is in great part dedicated to the model of Mars' revolutions. Copernicus studied the texts by Philolaus (V c. BC) and Aristarchus (III c. BC).

Philolaus defined a hypothetical astronomical object which he called the Central Fire. He claimed that the other celestial bodies including the Sun move around it. Two hundred year later Aristachus, impressed by this idea, proposed the first serious model of a heliocentric solar system. Unfortunately his description has not survived, possibly it was destroyed by the author himself who was afraid of objections. Luckily, several of his contemporaries managed to read the text and later cited it. Archimedes (III c. BC) wrote: "His (Aristarchus') hypotheses are that the fixed stars and the Sun remain motionless, that the Earth revolves around the Sun on the circumference of a circle, the Sun lying in the center of the orbit". Copernicus cited Aristarchus and

Philolaus in an earlier version of his famous work. He wrote: "Philolaus believed in the Earth's motion … Aristarchus from Samos too held the same view …" (deleted and not printed in the first edition, but incorporated in the later printings). The question arises why in the first print those references disappeared. One of the explanations is that, before publication, some theologian read the manuscript and he introduced the corrections with the aim of weakening a possible conflict with the Church supporting the geocentric theory. The theologian tried to convince the future reader that the heliocentric model is only a mathematical hypothesis and no references to philosophers were needed. The real inspiration for Copernicus were firstly the readings, but he also noticed the weakness of geocentric theory considering the observations. He explained his model using mathematics as well the results of long years of observations. Till the times of Copernicus the geocentric theory dominated in the version created by Ptolemy in his *Almagest* (about 150 AD). The Ptolemaic system dominated over many earlier theories considering the Earth being a stationary center of the universe. Stars were placed on the outer sphere rotating fast, the planets had smaller spheres, each planet with its own sphere. To explain the anomalies, such as the retrograde motion (appearing to move on the sphere opposite than that expected) observed in several planets, a system of epicycles was invented. It meant that a planet moves on a small circle, the centre of which moves around the circumference of the larger circle (with the Earth in the centre). The astronomers explained the anomalies using also the term of eccentrics – this meant that the orbit deviates from the circular and usually it was assumed to be elliptic. Ptolemy's unique contribution to the geocentric theory was the idea of an equant – a complex addition to motion description which resulted in the rotation of the Sun sometimes described using the central axis of the universe, but sometimes a different axis. This introduced a kind of "wobbling orbits", a fact that was seriusly confusing Copernicus – not all astronomers could get observations that conformed with the above theory.

De Revolutionibus Orbium Coelestium used a very advanced (for that time) trigonometry and geometry. Till now it has not been proved whether the geometric methods were invented by Copernicus himself or originated with Johannes Regimontanus (1426–1476). The fundamentals of trigonometry by Regimontanus were printed after his death in 1533 in Nuremberg, by which time the main part of Copernicus manuscript was already complete. One of the hypotheses says that Rheticus, when he came to Copernicus

(1539), brought some texts, and among them was the manuscript of Epilomat – the trigonometry by Regimontanus. However Copernicus claimed that only Ptolemy's work inspired him "Hoc autem sex theorematibus explicabimus et uno problemate, Ptolemeum fere secuti" (original text from *De Revolutionibus*). The researchers of Copernicus' life believed that the trigonometric methods used by the astronomer were of his own invention. His Commentariolus, written much earlier (in 1514) than *De Revolutionibus*, already used those methods. At that time there was no chance of learning Regimontanus' methods.

Ptolemy's equants were replaced by Copernicus with epicycles; he put the Sun near to the middle of the celestial spheres, but not exactly at the point which was considered an exact centre of the universe. Copernicus' system was not better confirmed experimentally than Ptolemy's. He was well aware of that. Until the XVIII c. only few astronomers were convinced by the Copernican system. It was important that his model attracted the attention of Galileo Galilei (1564–1642) and Johannes Keppler (1571–1630). Galileo's observation of the phases of Venus resulted in the first experimental evidence for the correctness of Copernicus' theory. Keppler expanded and improved the Copernicus' model.

Modern Interpretation of Copernicus' Contribution to General Mechanics and Mechanism Design

In his epochal book Copernicus underscored that basic technical disciplines are strongly related. He wrote "Arithmetica, Geometrica, Optice, Geodesia, Mechanica et si quae sint aliae: omnes ad illam sese conferant" (*De Revolutionibus*). Copernicus' contribution to mechanics is recognised as a contribution to the mechanics of celestial bodies; however reading his famous book one can notice that he focused not only on fundamentals of astronomy, but also developed methods of motion description and discussed the effects of motion superposition, which required an intutive feeling for the role of reference frames. He payed attention to the motion properties describing different motion trajectories and tried to explain the role of mass (gravity) center.

The term *gravity center* is introduced at the beginning of his famous manuscript. The most representative sentence says "Nec audenti sunt paripateticorum quitan, qui universam aquam decides tota terra maiorem prodiderunt atque aliud esse *centrum gravitatis*, aliud magiutudinis" ("land and water press upon a single center of gravity, the Earth has no other center of magnitude" – Book One, Chapter 2: *Moreover, the Earth is spherical*). Next he stated "... gravity is nothing but a certain natural desire. This impulse is present, we may suppose, also in the Sun, the Moon, and the other brilliant planets, so that through its operation they remain in that spherical shape which they display. Nerethless, they swing round their circuits in diverse ways" (Rosen, 2006). This suggests that Copernicus intuitively felt properly the essence of gravity. In Book One, Chapter 3: *How Earth forms a single sphere with water?*, he concludes that the Earth has the shape of a ball. He supports his opinion by referring to the observations of travellers and saying that Earth has its centre of gravity which attracts the soil and water. After remarks on the large size of oceans covering the Earth, he says that the Earth is round and "location of America compels us to believe that it is diametrically opposite to the Ganges district in India. From all these facts, finally, I think it is clear that land and water press upon a single center of gravity ..." (Rosen, 2006).

Copernicus noticed that *falling bodies are accelerating*: "... Praeterea quae sursum et dorsum aguntur ... non faciunt motum simplicem unifarem et aequalem. Levitate enim vel sui ponderis impetu neqeunt temperari. Et quaecumque decidunt, a principio lentum facientia motum, *velocitatem augent cadendo*" ("Whatever falls moves slowly at first but increases its speed as it drops" – Book One, Chapter 8: *The inadequacy of the previous ...*). It must be pointed out that the principles of motion of falling objects were described by Galileo Galilei, and published about 100 years after Copernicus' work.

Copernicus gave also an inuitive *explanation of relative motion*. He was the first person to distinquish "apparent" motion leading to the concept of relativity of motion. He claimed that apparent movement can be caused by a motion of the object itself, or the motion of observer or both. This is discussed in Book One, Chapter 5: *Does the circular motion suit the Earth?* and in Chapter 9: *Can several motions be attributed to the Earth? The center of the universe*. His considerations on relative motions resulted in proper understanding of the concept of *reference frame* "Such in particular is the daily rotation, since it seems to involve the entire universe except the Earth and what is around it. However, if you grant that the heavens have no part in this motion, but the Earth rotates from west to east, upon earliest consideration

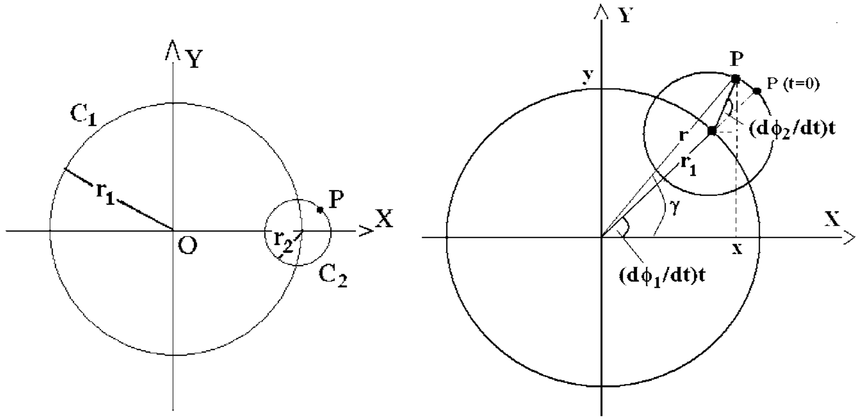


Fig. 4. Illustration of epicyclic movement: left – trajectories, right – coordinates of point P.

you will find that this is the actual situation considering the apparent rising and setting of the Sun, Moon, stars and the planets" (Rosen, 2006).

Nobody before Copernicus noticed the neccessity to introduce the "third movement of the Earth" – Earth's axis precession. Copernicus was in doubt how to explain the seasons. He explained in detail the "three motions" of the Earth. The first motion (Book One, Chapter 11: *Proof of the Earth's triple motion*) of the Earth is rotation causing the day and night. The second motion is the ecliptic motion around the Sun causing the seasons. The third motion is the change of inclination of Earth's axis of rotations – its precession.

Explaining the movements of the planets Copernicus focused on the *descriptions of different harmonic motions*. He was aware that the orbital motion of the Earth must cause apparent periodic oscillations of the stars. Epicycles visible in the motion of the other planets were the reflection of the Earth's orbital motion. First he analysed the periodic motion of suspended body due to the gravity force. This is in the part of Book Four where he describes the motion of Mercury.

Copernicus noticed that oscillations can be generated by superposition of two circular movements. This happens when one point having a circular trajectory C_2 moves on an eplicycle, i.e. that the center of circle C_2 moves on the circumference of another circle C_1 (Figure 4).

Copernicus analysed the following: on the circle C_1 (with radius r_1) moves the center of another circle C_2 (having radius r_2), with the constant angular velocity $d\phi_1/dt$. On the circle C_2 moves the point P with angular velocity

$d\phi_2/dt$. Nicolaus evaluated the coordinates of point P. Let t be the time measured from the moment when the point P is at the farthest distance from the centre O of circle C_1. If γ denotes the angle between axis OX and OP, then coordinates of point P are expressed by:

$$x = r\cos\gamma = r_1\cos(t \cdot d\phi_1/dt) + r_2\cos(t \cdot (d\phi_1/dt + d\phi_2/dt)),$$
$$y = r\sin\gamma = r_1\sin(t \cdot d\phi_1/dt) + r_2\sin(t \cdot (d\phi_1/dt + d\phi_2/dt)). \quad (1)$$

Assuming $d\phi_2/dt = -2d\phi_1/dt$, Copernicus obtained:

$$x = r\cos\gamma = (r_1 + r_2)\cos(t \cdot d\phi_1/dt),$$
$$y = r\sin\gamma = (r_1 - r_2)\sin(t \cdot \phi_1/dt). \quad (2)$$

With $r_1 = r_2$ he obtained:

$$x = 2r_1\cos(t \cdot d\phi_1/dt),$$
$$y = 0. \quad (3)$$

Relations (3) describe the harmonic motion along axis OX with magnitude $2r_1$. Simplifying relations (2), the formula for an ellipsoid is obtained. For example, computing $\cos(t \cdot d\phi_1/dt)$ from the first equality and substituting it into the second ($r_1 \neq r_2$), the ellipsoid formula is obtained:

$$x_2/(r_1 + r_2)^2 + y^2/(r_1 - r_2)^2 = 1. \quad (4)$$

Copernicus provided the above considerations and concluded that an ellipsoid describes the trajectory of periodic motion, which is a combination of two epicyclic movements (Birkenmajer, 1900).

Discussing precession in Book Six, Copernicus considered the result of superposition of two harmonic motions having different frequencies and obtained the formula for a 4-th degree curve which is of a Lissajous type.

Lissajoux curves, called also Bowditch curves, are produced by the composition of two sinusoidal motions at right angles. First studied by the American mathematician Nathaniel Bowditch in 1815, the curves were investigated independently by the French mathematician Jules-Antoine Lissajous in 1857–1858. The fact that Copernicus discovered the Lissajoux curve is not generally known. Copernicus gave the 4-th order formula describing the trajectory of equinox precession (Birkenmajer, 1900):

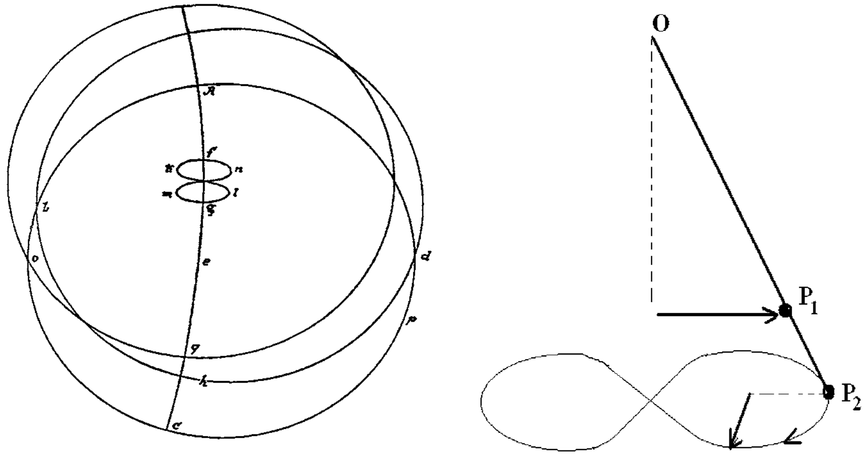


Fig. 5. Left: the copy of the picture illustrating precession (N. Copernicus – manuscript) (Birkenmajer, 1900). Right: motion trajectory of double spherical pendulum.

$$4a^2y^2(b^2 - y^2) = b^4x^2. \tag{5}$$

In the first edition, in Book Three, one can find a low quality diagram (produced by wood-print), where in its middle a figure resembling the number 8 can be noticed (Figure 5). This is followed by an explanation of the precession movement. In the Copernicus' manuscript (p. 74) this characteristic shape is even more visible.

In later editions the shape in this picture was printed differently and wrongly. Looking at this shape one can conclude that it represents a Lissajous curve. Let us assume that oscillations x have amplitude a and oscillations y have amplitude b and the phase shift between them is equal to ϕ:

$$x = a\sin(\omega_1 t + \phi),$$

$$y = b\sin(\omega_2 t).$$

Assuming $\omega_1 = 0.5\omega_2$, $\omega_2 = \omega$, and $\phi = n\pi$ $(n = 0, 1, 2, \ldots)$:

$$x = \pm a\sin(2\omega t), \quad y = b\sin(\omega t), \tag{6}$$

we fulfill relation (5).

It is interesting how Copernicus deducted the shape and obtained formula (5). One possible explanation is that he observed the motion of a double pendulum excited in two perpendicular directions. This is suggested by his repeated remarks on pendulum motion but does not explain the logic behind it.

If two mathematical pendulums OP_1 and P_1P_2 are attached one to the other, and the length l_2 of the lower pendulum (P_1P_2) is equal to one quarter of the length l_1 of the upper one ($l_2 = l_1/4$), and if pendulum ends are excited in perpendicular directions (spherical pendulum), the trajectory of the lower pendulum will move along a Lissajous curve (Figure 5). The period of oscillations of each pendulum is equal to:

$$T = 2\pi(l/g)^{0.5}. \tag{7}$$

For pendulums OP_1 and P_1P_2 the periods of oscillations are equal to $T_2 = T_1/2$, and $\omega_1 = 0.5\omega_2$ as is the discussed case described by the equations (5) and (6). It is quite possible that Copernicus used for experiments some church lamps suspended by long ropes (chains) to which he attached shorter pendulums.

In mechanism design the analysis of movements and its proper description in appropriate reference frames is very important. Copernicus created the fundamentals for complex motion analysis and illustrated it considering non-trivial examples of combination of periodic movements. In the design of specific machines (i.e. walking machines) the analysis of possible positions of center of mass (center of "gravity") is important for postural stability evaluation. Copernicus had an intuitive feeling that the center of gravity plays a role in stable movements: "... so that through its (center of gravity) operation they (planets) remain in that spherical shape which they display. Nevertheless, they swing round their circuits ...".

Summary

From his works Copernicus emerges a methodic and brilliant scientist. He presented his ideas clearly. Nobody before Copernicus felt the need to introduce the Earth's "third movement" – precession. The explanation of what causes the precession was incorrect, but it was logically and well summarized, referring to the need to balance all motions ("Cuius causam nemo forsitam meliorem afferet: quam axis terrae et polorum circuli aequinoctialis deflecxum quemdam ... Quoniam si motus axis terrae simpliciter et exacte

Fig. 6. Selection of postage stamps dedicated to Copernicus (courtesy of Dr. R. Babut – private collection).

convenirent cum motu centri, nulla penitus (ut diximus) appareret aequinoctiorum conversionumquae praeventio" – Book Three, Chapter 3: *Hypotheses by which the shift of equinoxes as well as the obliquity of the ecliptic and equator may be demonstrated*). The idea of three motions resulted in much more accurate than earlier evaluation of planet positions and their displacements. Currently the precession is explained using the law of gravitation.

The whole Copernicus theory was simple from the point of view of calculations and formal descriptions, but definitely was not simple as a complete model of the solar system.

Both his name and his contribution to the understanding of the cosmos surrounding us is known all over the world. Many countries commemorated him with postal stamps. Figure 6 shows three stamps, the first two of which were issued in Poland. The first was issued for the 500th anniversary of Copernicus' birthday, the second is a monochromatic copy of a famous oil paint showing Copernicus in his observatory. The big, color painting was made by a well-known Polish painter of historical events – Jan Matejko (1838–1893). The third stamp commemorating the 410th years after Copernicus death was issued in China.

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The author expresses her thanks to the personnel of the Institute of the History of Sciences and Technology, Polish Academy of Sciences and to the staff of the institute library who helped in the search for materials. This Institute has a long tradition in organizing “Colloquia Copernicana” and holds a rich selection of publications dedicated to Copernicus.

The advice and suggestions of the management of the Main Library of Jagiellonian University in Cracow is also highly appreciated.

The biographical notes were elaborated on the basis of the following books: Crombie (1959), Gingerich and MacLachlan (2005), Przypkowski (1972) and Sikorski (1995).

For preparation of the overview of Copernicus’ astronomical studies and a description of the history of his manuscripts, as well for a summary of his works on the mechanics of the celestial bodies, the author used the above listed books and the following documents: Copernicus (manuscript), Copernicus (1543), Rosen (2006) and Swerdlow (1973b).

The part describing Copernicus’ contribution to general mechanics was elaborated by the author by referring to original works of Nicolaus as well as to the publications: Birkenmajer (1900), Krajewski (1973), Michailov (1973), Rosen (2006), Sanding (1973) and Swerdlow (1973a).

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ALEXANDER YERSHOV (1818–1867)

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Abstract. Alexander Yershov was one of the founders of the Moscow Science School of Mechanisms and Machines. His book *Foundation of Kinematics or Elementary Theory about Motion in General and about Mechanisms of Machines Especially* was written for the Imperial Moscow University and the Imperial Moscow Technical Supreme School (IMTSS). It was the first Russian textbook on kinematics. The curriculum of the Moscow Educational Industrial School (MEIS and later IMTSS) which was one of the first higher engineering schools in Russia, is significantly connected with the name of Alexander Yershov. He began his career in 1845 at the institute as a teacher of practical mechanics, and then became a Professor, a Class Inspector and finally the Director. As a result of his activity as a Professor of Applied Mechanics and the Director of MEIS, the theoretical education curriculum was widened, the number of courses in practical mechanics was increased and standards were raised for qualification of teachers. Yershov orchestrated the reorganization of IMTSS and achieved equality of status between teachers and pupils at the St. Petersburg Institute of Technology. He also established the Mechanisms Collection of the Theory of Machines and Mechanisms Department at IMTSS.

Biographical Notes

Alexander Yershov (Figure 1) was born on the 2nd of July in 1818 in the village of Ivachevo, in Ryazan province, to the family of a poor nobleman Stepan L. Yershov.

When he finished gymnasia in Ryazan in 1835, Yershov entered the Physico-Mathematical Department of the Emperor’s Moscow University and graduated with honors as a candidate of science in 1839.

The same year Alexander Yershov was sent to St. Petersburg’s practical Technological Institute and Institute of Corps of Engineers of Routes, where

M. Ceccarelli (ed.), Distinguished Figures in Mechanism and Machine Science, 135–149.

Fig. 1. Alexander Yershov (1818–1867).

he studied process metallurgy, resistance of materials, technical drawing, descriptive geometry and other engineering disciplines.

At the same time he studied hard for his master's degree examinations and, after passing them in 1841, was granted a trip abroad to gather materials for scientific research. He left the country on the 13th of September. On his way to Paris, where he was offered the opportunity to take fundamental courses in practical mechanics, he learned about educational organization and gained necessary teaching skills at the Berlin Craft Institute, the Dresden Technical School and Freiberg Technical Educational Institutions.

The rest of 1841 and part of 1842 Yershov spent in Paris, where he attended courses on practical mechanics of Ponselet at the Paris Physico-Mathematical Faculty, listened to Morin's lectures at the School of Bridges and Roads, and studied with Bellange, who taught applied mechanics and hydraulics. He learned descriptive geometry from Olivier at the Conservatoire of Arts and Crafts. He visited machine works and factories in Paris and neighbouring towns on the summer and autumn vacations in 1842. He learnt the organization and methods of teaching at technical school in Zurich and visited engineering plants in Mulhouse.

After that Alexander Yershov went to England, where he acquainted himself with Leeds and Manchester textile factories in details.

Yershov returned home in July, 1843 and taught practical mechanics and descriptive geometry from 1843 till 1853 in a senior technical class of the 3rd Moscow gymnasium, which had been opened in 1839. He quickly organized an excellent laboratory of machine models and tools.

He also continued working on his dissertation on the subject of water as an engine, which he had begun while abroad. He defended his thesis on the 26th of August in 1844 at Moscow University and won the master's degree in fundamental and applied mathematics, after which he began his educational work. Yershov's course on practical mechanics involved kinematics, dynamics, theory of engines, dynamical theory of machines, and studies of materials resistance.

In 1844 Yershov was confirmed in his position as adjunct professor, and in 1853 was given the acting post of extraordinary professor of the Physico-Mathematical Department of the University.

From 1845 on he taught practical mechanics and descriptive geometry at the MEIS; he was dedicated for the rest of his life to these topics and to teaching them. In 1855 the post of Class Inspector was established and Yershov was appointed to it.

On the 5th of July in 1859 Yershov was confirmed in his position as director of MEIS. It was impossible to combine the duties of director of a first-rate technical educational institution and those of a professor at the University, so Yershov chose to leave the University. As the director of MEIS he proposed to transform it into a Technical Secondary School, organized as a technical institute – Moscow Technical Secondary School. The first draft of the IMTSS's regulations was composed by Yershov in 1857. In 1861 he presented to the Moscow board of trustees a detailed proposal for the envisioned reformation. A long drawn-out struggle with the board of trustees followed and the new regulations were approved only in 1868.

By that time, the years of hard work and struggle with the Board of Trustees, as well as family miseries such as his son's death and financial problems, had undermined his health. Alexander Yershov died on the 21st of February in 1867.

List of Main Works

1. "About Water as the Engine", 1844.
2. The program of practical mechanics with Chebyshev's response.
3. *Foundation of Kinematics or Elementary Theory about Motion in General and about Mechanisms of Machines Especially*, 1854.
4. *About Higher Technical Education in Western Europe*, 1857.
5. Regulations' project of the IMTSS, 1867.

Review of Main Works on Mechanism Design

About Water as the Engine

The argument "About Water as the Engine", being A. Yershov's master's dissertation, was published in 1844. It was the first work in Russia on the theory of water engines. The work is basically devoted to theoretical consideration of work of bulk wheels and turbines. It is possible to judge the contents of this work in Figure 2.

The work of wheels and turbines is divided into three parts. In the first part, bases of a hydromechanics are considered. In the second part power parameters of hydromechanical systems are considered. In the third part, various types of hydraulic machines are considered and their comparative analysis is given.

The value of this work is that each theoretical thought is accompanied with the experimental data. In this work, the unity of theory and practice is discussed and the advantage is given to geometrical and skilled investigation in the mechanics. The scientific and practical work of question theory and construction of turbines has completely confirmed the justice of these thoughts. This was the first work in Russia to present the theory of water engines. However it was not widely popular at first; its recognition came only later. References to it can be found in the modern literature.

About the Program of Practical Mechanics

This first course in practical mechanics was based on achievements of schools of Germany, France, England, and on features of the Russian mentality and traditions of the Russian technical and classical universities.

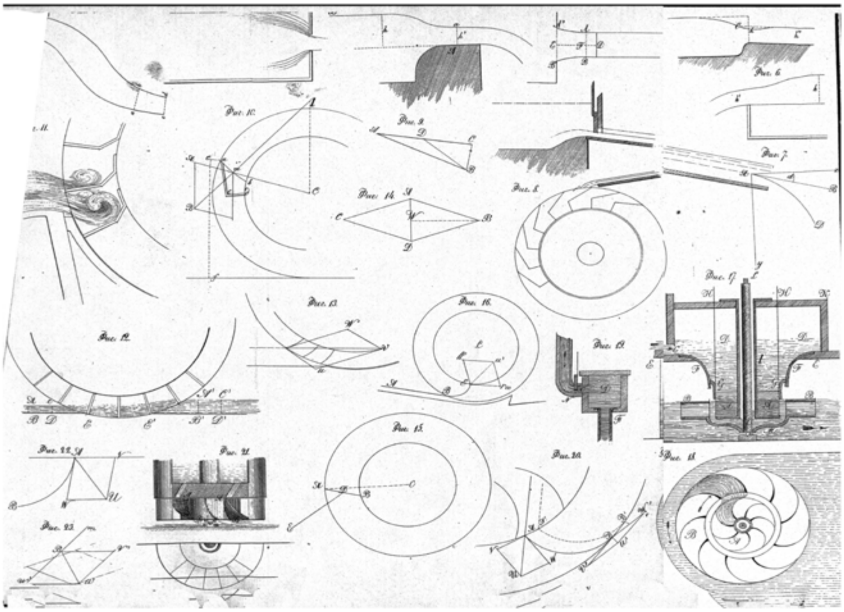

Fig. 2. "About water as the engine".

The course of practical mechanics consists of five sections: (1) movements and the machines considered (examined) irrespective of forces; (2) dynamic theory of machines; (3) engines and the machines acceptable movement; (4) construction of machines; (5) mechanical technology.

In the first section, preliminary concepts about movement are given and the geometrical description of the mechanisms serving for transfer and change of movement is given.

In the second section, methods of definition and calculation of work of constants and variable forces are stated; experiences for definition of laws of friction are described; research of loss of work on an example of friction of cogwheels and friction of a belt is discussed

In the third section, the theory of engines is given. "Alive" engines are considered; the theory of windmills; construction of water wheels (in research on water as the engine); definition of work of the steam machine by Ponselet's method.

In the fourth section, the properties of the most common materials in machines and methods of their processing are considered.

In the fifth section, two manufactures are considered: mechanical spinning and weaving. At the end of the course, results were given of Moraine's experiment – calculation of work of the moving force for the working of other machines in spinning and weaving mills.

The course of Practical Mechanics was reviewed P. Chebyshev. He offered to make the following additions.

The first section: to the mechanisms serving for transfer and change of movement to add a wedge, the block, rack-wheel and the infinite screw; to consider cylindrical and conic wheels; to state the theory of the infinite screw.

The second section: to consider: (1) friction of a wedge; (2) friction of axes; (3) friction of ordinary screw; (4) friction of sliding body. Except for friction Chebyshev offered to include in the program loss of work from rigidity of cords and belts, loss of "alive" force at impact and absorption of work by friction at the moment of impact.

The third section: to include the theory of construction of wings, with an explanation of their benefits and inconveniences, constructions of water wheels, methods of calculation of work of the water, delivered by any source. To exclude definition of work of the steam machine based on erroneous theory of Ponselet.

The fifth section: Chebyshev suggested mentioning articles of the third section.

Chebyshev's remarks have concrete character. As a whole, Chebyshev approved Yershov's installations concerning tasks and purposes of the course and the program as a whole.

About the Program of the Course in Descriptive Geometry

The course is divided into two parts: the first part contains all material relating to a point, to a direct line and to a plane; in the second – data on curves and on surfaces are considered. Except for these two general parts, construction of a sundial and mechanical drawing are taught.

Foundation of Kinematics or Elementary Theory about Motion in General and about Mechanisms of Machines Especially

Yershov's textbook (Figure 3) was the first standard Russian textbook of TMM. The work was issued in 1854 by the printing-house of the Moscow University for students of the Imperial Moscow University and IMTSS. The

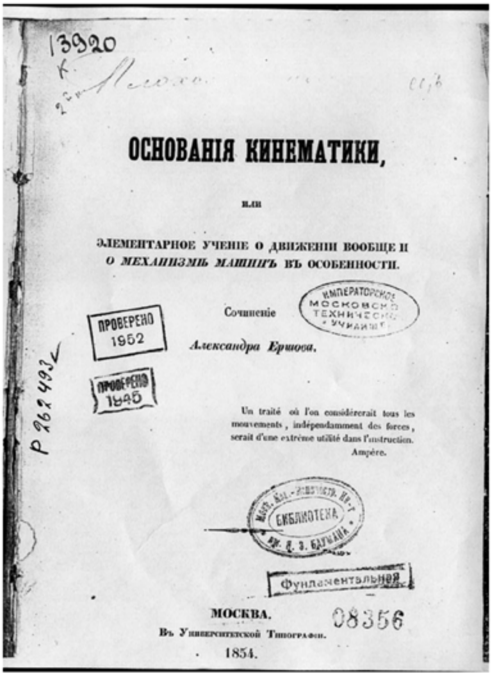

Fig. 3. *Foundation of Kinematics or Elementary Theory about Motion in General and about Mechanisms of Machines Especially*, 1854.

textbook could be used in conjunction with the newest practical mechanics manuals of that time. In structure it reminds one very much of a course of Poncelet: "Mecanique physique et experimentale" (1841), a monograph of E. Bur, *Cours mecanique et machine* (1862), and *Traite de cinematique* (1849).

Yershov's work contains 292 pages; 10 pages are devoted to the foreword, 50 pages to kinematics; the remainder of the volume addresses the question of transformation and transfer of movement by means of various mechanisms (the theory of mechanisms and machines).

It is possible to judge the scientific predilections of Yershov by the epigraph to his work:

"Un Traite ou l'on considererait tous les mouvements,
independamment des forces,
serait d'une extreme utilite dans l'instruction."
Ampere

In the foreword Yershov gives a brief historical review, and expresses appreciation to his predecessors: to Ampere, Poncelet, Monge, Willis, etc.

Classification of mechanisms of machines in Yershov's textbook is based on concepts of Monge, i.e. on distinction of kinds of movements by form and direction, and representations of Ampere about independence of geometrical and kinematic properties of the mechanism of forces. In Willis's classification, Yershov sees weaknesses of various parts of a mechanism, depending on comparative speeds.

Yershov mainly uses geometrical constructions by means of which he managed to simplify the theory of relative movement of two cylinders (§27), rod and crank (§27, §129), sliding of cogs (§88), etc.

In the part "Preliminary concepts". Yershov defines as a basic subject in mechanics the study of laws and the reasons of movement of bodies, and gives basic kinematic and some dynamic definitions. Further he investigates a question about inertia. He refers to Euler's letters "Lettres à une princesse d'Allemagne" (Paris, 1843), to Carnot's work "Principes fondamentaux de l'equilibre et du mouvement" in which inertia is nothing else than "the resistance arising by change of a condition of bodies". He notes that Laplace and D'Alembert' rejected the concept about inertia as against Ostrogradsky, Ponselet, Morin, etc.

The first part discusses elementary mechanics and, particularly, kinematics of a point and a body. Moreover , it contains information about reference sources for machining for different materials, making it appropriate for a book addressed to students of the "Educational Industrial Institution".

The second part contains citations to Monge and Ampere and gives a general plan of the book.

Chapter 1. "Moving transmission" (pp. 54–179): rectilinear continuous moving, wobbling rectilinear moving, circular continuous moving, circular wobbling moving.

Chapter 2. "Moving transformation" (pp. 180–256): rectilinear continuous to circular continuous and vice versa, circular continuous to wobbling rectilinear and vice versa, wobbling rectilinear to circular wobbling and vice versa, circular continuous to circular wobbling and vice versa.

Chapter 3. "Moving differential and combined" (pp. 257–266).

Moving transmission (Figure 4)

1. Rectilinear continuous moving. Moving transmission by means of flexible and liquid substances. Moving transmission through direct contact (blocks and polyspasts, wedge, hydrotransmission).
2. Wobbling rectilinear moving. Guides, rails, connecting slider with coupler, poppet heads including those for steam, Watt's straightlines mechanism, Evanse's straightlines mechanism.
3. Circular continuous moving. Endless cords, straps, chains. Transmission between parallel and nonparallel axes. Gear ratio. Pulley with variable diameter. Joint's parallelogram. Gears. Cycloid and evolvent cogs. Clearance in gearing. Internal gearing. Pin gearing. Bevel gearing. Gearing with skew axes. Worm-gearing. Hooke's joint. Coupler and friction clutch. Compound gearing (with fixed axes, planetary, clockwork).
4. Circular wobbling moving. Linkages and gearing-linkages.

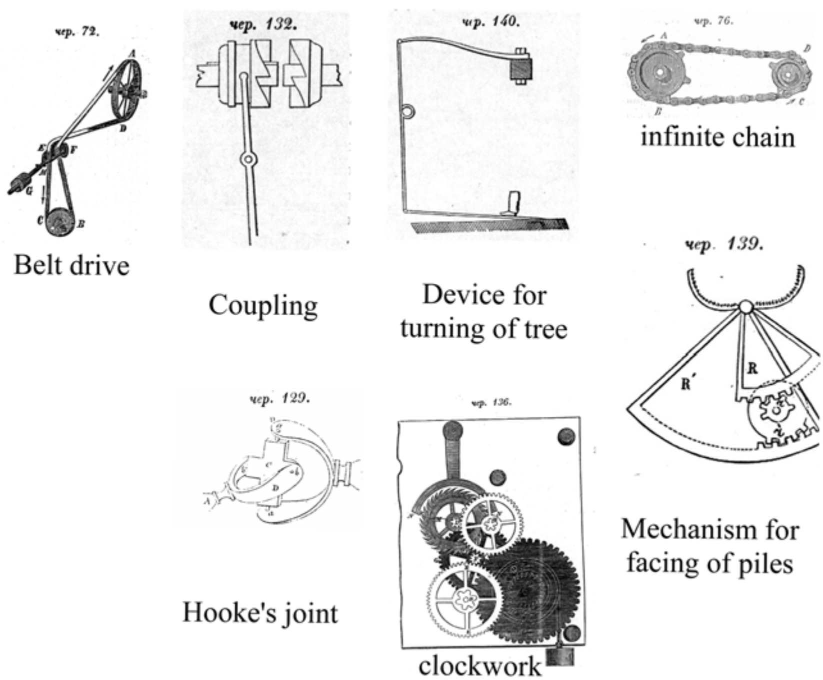

Fig. 4. Examples of Moving transmission.

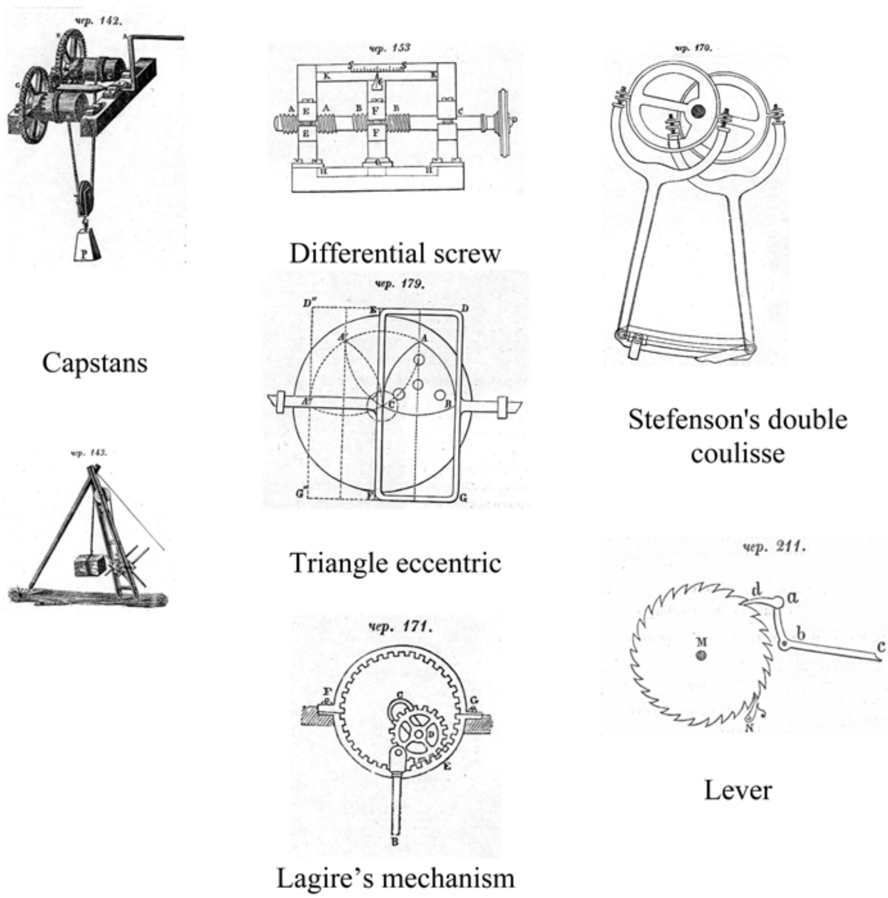

Fig. 5. Examples of moving transformation.

Moving transformation (Figure 5)

1. Transformation moving circular to rectilinear continuous. Capstans simple and complex, screw and nut.
2. Transformation moving circular to wobbling rectilinear. Coupler and crank, eccentric, double eccentric of Stefenson. Planetary train of Lagire. Cams planar and space. Friction hummer. Rack-and-gear drive, differential rack-and-gear drive.

3. Transformation moving circular to circular wobbling. Crank-slider and crank-coulisse mechanisms. Moving transmission from contact (cams for hammer head lift). Ratchet mechanism.
4. Transformation moving circular wobbling to wobbling rectilinear. Moving differential and composite.

Moving differential and combined

In these chapters, planetary mechanisms and, in particular, differentials are considered. The examples taken from practice well illustrate the theoretical beginnings of planetary mechanisms.

Thus, this textbook carries a strongly pronounced, highly applied character, not relying on a formal definition of theoretical mechanics as a whole. Yershov was appraised for his work as an alphabet on "… TMM without it we could not understand nor composition nor how the machines work and also it is in the number of the elementary work based on elementary mathematics …".

Foundation of Collection of Mechanisms in IMTSS

It is possible to assume that Yershov was the founder of the collection of mechanisms in IMTSS. At present we know only two models of mechanisms that were created during Yershov's epoch. The first model is the original orthogonal spatial gear transmission. The input wheel of the transfer is a flat wheel on the front of which is only one tooth, made like Archimedes's spiral (Figure 6a). The inscription "Ivan Naumov 1861 year" was engraved on the model. The second model is a centrifugal regulator with inertial element as a massive ring. On the base of the model a tablet with the inscription "Pavel Ivanov, 1862 year" was fixed (Figure 6b). These models with small differences agree visually with the drawings in Redtenbacher's catalogue.

About Higher Technical Education in Western Europe

In 1857 Alexander Yershov wrote a scientific work *Higher Technical Education in Western Europe*. In this work he analyzed and compared higher educational technical institutions in France, Germany and England. Yershov

Fig. 6a. Orthogonal spatial gear transmission (1861).

Fig. 6b. Centrifugal regulator (1862).

drew a conclusion that there was a necessity for the development of technical education and forming a technical intelligentsia in Russia.

In the 1830s–1850s there were great changes in Russia: hand work was replaced by machines, manufactory turned into factory, and industrial revolution began. However the development of the industrial society was moving on very slowly. This fact became clearly visible during the Crimean war (1853–1856). The supremacy in technical equipment of one of the warring sides won the victory in this battle, not the armies or generals. It is possible to assume that it was a consequence of lack of technically competent experts for development of industry, industrial engineering and, hence, military engineering.

Development of an industrial society demanded not only abolishing serf slavery and developing industry, but also providing elementary education to the population of the country, and increasing the number of educated people.

Alexander Yershov understood that it was very difficult to prepare the necessary number of specialists for the country. He stood for the organization of technical schools and paid a lot of attention to this question in his work. Yershov thought that secondary technical education should not aim only at preparation for entering higher educational technical institutions, because industry had a necessity for specialists with secondary education too.

Yershov attached a lot of importance to general education that was given in gymnasiums. In his opinion, this education was enough for entering the Universities, but not enough for entering higher educational technical institutions. He wanted gymnasiums to give students knowledge that would be enough for entering the technical educational institutions too. Yershov suggested organizing some special courses, such as technical drawing and mechanics, and decreasing teaching of ancient languages.

Yershov represented two different systems of education to readers – French and German.

In the first one there were identical rules for entering schools and attending lectures; on the contrary, the second one had variety in both.

The French considered rather general education for students of special schools. Furthermore, they considered all applied sciences as parts of the whole that was compulsory for every technical specialist. The difference would be only in practical studies.

On the contrary, the Germans accepted students without sufficient preparation and rather often divided subjects superfluously. They even mixed high and general school.

Alexander Yershov valued French technical school very highly because its graduates had excellent theoretical and practical knowledge. However his follower F. Orlov (1872–1892) took many ideas from his experience of German schools, especially schools of F. Reuleaux.

Yershov thought that special technical educational institutions should be organized in Russia. He did not rule out the possibility of preparing technical specialists at the Physico-Mathematical Faculties of the Universities after general education during the first and second courses. But he considered a wide perspective to be creation of special open technical institutions.

He pointed out the importance of organization and subsidizing of scientific journeys for teachers of higher technical schools in order to liberalize their experience in science, technique and educational systems.

Regulations' Project of Imperial Moscow Technical Supreme School

Alexander Yershov undertook the project of formulating the first regulations of IMTSS in common with the leading professorate of the IMTSS.

In 1868, a year after Yershov's death, MEIS was officially reorganized into a high educational technical school. The general aim of the school was to "educate civil-engineers and mechanical engineers". The whole educational course lasted six years. There were changes in the plan of education. New subjects like botany, zoology, mineralogy, applied physics, science of railway and etc. were entered.

Scientific studies of the educational institution were based on a combination of theory and practice. It was a fitting tribute to Yershov and his foresight.

The graduates were given some rights, such as: educated foremen were excused from a recruiting obligation, mechanics-builders, engineers- mechanics and engineers-technologists were ranked among the estate of personal honorary citizens and had all the rights of this estate. All students were excused from recruiting obligations during the time of their education.

At the time of graduation all of the publicly supported students were given books, necessary tools and clothes.

IMTSS had several advantages. It was excused from some taxes. The buildings that belonged to the IMTSS were excused from billeting and duties. IMTSS had the right to order different books abroad and they could be imported without any duty or checking on the source. An essential achievement was that professors and teachers were equated to nobility. The professors were hereditary nobility, teachers were personal gentry nobility.

Modern Interpretation of Main Contributions to Mechanism Design

A summary of Yershov's two basic contributions to development of a technical education:

1. He prepared a plan for transformation of MEIS to a technical educational institution of a higher level.
2. He was one of founders of the Moscow school of the theory of mechanisms. The further development and perfection of this school was carried

out by such scientists as P. Chebyshev, F. Orlov, N. Mertsalov, L. Smirnov, I. Artobolevsky, L. Reshetov, and V. Gavrilenko.

Acknowledgements

The authors wish to thank the museum of BMSTU in Moscow, the archive of BMSTU in Moscow, the fund of the Institute of Scientific Information of Social Sciences of Lomonosov's Moscow State University, St. Petersburg State Archive, and the Russian State Library in Moscow, in particular their Dissertational Department.

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FERDINAND FREUDENSTEIN (1926–2006)

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Abstract. Ferdinand Freudenstein is considered the father of modern kinematics in America. He made his mark early with his seminal PhD dissertation in which he developed what is known as Freudenstein's Equation. During a long career at Columbia University, he and his students produced outstanding research results in every area of modern kinematics. At the time of his death there were over 500 academic descendents belonging to the Freudenstein family tree. His progeny are teachers in many different countries, and his research results have shaped the teaching and practice of mechanism and machine theory throughout the world.

Biographical Notes

Ferdinand Freudenstein (Figure 1), was born into a Jewish family, on May 12, 1926, in Frankfurt am Main, Germany. He was the son of George Freudenstein, an imaginative and successful merchant, and Charlotte Rosenberg, a beautiful and wise woman whose family included prominent art historians.

When Ferdinand was ten years old, he, his parents and two sisters fled the Nazis for safety in Holland. In the spring of 1937, after six months in Amsterdam, the family moved to England where they joined his brother who was studying there. They lived in London during the blitz, moved briefly to Cambridge, and then spent several years in Llandudno, North Wales. During this period his father and brother were sent into exile in Australia, since the British Government regarded all adult male German citizens, even victims of Nazi anti-Semitism, as enemy aliens.

In 1942, when he was 16 years old, Ferdinand, his mother and his two sisters sailed on an old British cargo boat from England to Trinidad. They remained there for six weeks until a distant cousin, Walter Kahn, arranged for their visas to the United States.

M. Ceccarelli (ed.), Distinguished Figures in Mechanism and Machine Science, 151–181.

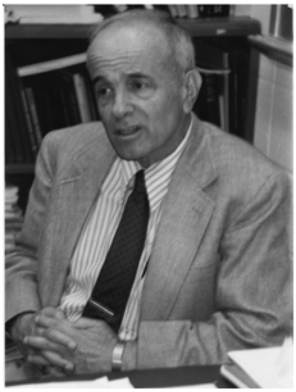

Fig. 1. Ferdinand Freudenstein in his office at Columbia University.

They arrived in New York harbor in March of 1942. Ferdinand had a high school equivalency certificate from Wales and was able to enter college at New York University (NYU). He spent two years studying there and then, at age 18, joined the U.S. Army.

He was in the Army for about a year and a half, during which time he was able to graduate from the Army Specialized Training Program in Engineering, at Texas A&M (1945). After the Army, with financial assistance from the GI-Bill, he went to Harvard University and earned an M.S. Degree in Mechanical Engineering in 1948.

As a young man, Ferdinand was a fine pianist and an accomplished player of the xylophone. He also was a good tennis player and enjoyed athletics. He had unusual powers of concentration that allowed him to concentrate on his studies even under the adverse conditions that accompanied the family's transition from Frankfurt to New York. Ferdinand's father had wanted his son to join him in business, but Ferdinand had a desire to be a pure mathematician; mechanical engineer was a family compromise.

After receiving his M.S., he worked as a Development Engineer in the Instrument Division of the American Optical Company in Buffalo, New York. He was there for approximately two years, and then left to study for his PhD at Columbia University. He had always had an interest in mechanisms, and his work experience furthered his interest. This was also stimulated by summer jobs as a development engineer at Ford Instrument (1951) and American Machine and Foundry (1952), and as a Member Technical Staff, at Bell Laboratories (1954).

At Columbia he had one major problem: there was no faculty member who did research in kinematics of mechanisms. Fortunately, Professor H. Dean Baker, a specialist in combustion, agreed to be Freudenstein's thesis supervisor, and to allow him to work on mechanism kinematics, even though Baker himself knew very little about the subject. For the rest of his life, Freudenstein was extremely grateful for what he considered as an act of great kindness and generosity on Baker's part.

Freudenstein was awarded financial support through the Du Pont Fellowship in Mechanical Engineering for the 1951–1952 and 1952–1953 academic years. In 1954 he received his PhD and was appointed to an assistant professorship in Columbia University's Mechanical Engineering Department. His career up the academic ladder in the Department was meteoric. In less than three years he was promoted to Associate Professor (1957). Then one year later he became the Chairman of the Department of Mechanical Engineering (1958), a post he held for six years (1958–1964). After only two years as an associate professor, he was promoted to the rank of Professor (1959).

In the same year, at the age of 33, he married Leah Schwartzchild. Their first child, David, was born on February 3, 1961, and their second child, Joan, was born on February 6, 1964. The young family took up residence in the Riverdale section of the Bronx where they purchased a comfortable three story brick house on a quiet residential street. Ferdinand lived in that house for the rest of his life, and he died in it on March 30, 2006.

At the end of the 1950s and the beginning of the 1960s, Freudenstein's Kinematics program at Columbia started to get world-wide recognition. He brought Rudolf Beyer, from Munich, to Columbia to offer a special graduate course on spatial mechanisms.

Following that, there was a highly publicized visit of a three person cultural delegation from the Soviet Union. This was a "first" in the troubled Cold War relations between the Soviet Union and the United States. One of the three was the head of the All-Union Society "Znanie." He was Academician Ivan Ivanovich Artobolevskii, who was a member of the Supreme Soviet of the U.S.S.R. and the foremost figure in mechanisms research in the U.S.S.R. Artobolevskii visited Freudenstein, and an important connection was made with the mechanisms establishment in the Soviet Union.

The mathematician Oene Bottema, from Delft, visited while returning from a workshop at Yale where he had introduced the groundbreaking work, of his student Veldkamp, on instantaneous invariants. Then Freudenstein invited the British mathematician Eric Primrose, first to work on studying the coupler curves of geared five-bar mechanisms, and then for a visit of several weeks to join in collaborative research on the algebraic geometry of six-bar coupler curves.

In the midst of all Freudenstein's professional success, tragedy struck when his wife died in May 1970. This created a huge challenge for Ferdinand who had not had any real experience with domestic details such as cooking and running a household. Fortunately he was a quick learner. For the next ten years, he ran the household and was both mother and father to his children. At the same time he remained a highly productive and world-renowned Columbia professor.

In May 1980, ten years after the death of his first wife, Ferdinand married Lydia Gersten. Lydia was a teacher who was widowed, had grown children and was caring for her elderly mother. Lydia and her mother moved into the house in Riverdale. Lydia took over the domestic management of the household, and became a close and loving partner in Ferdinand's life. Over the next years, Ferdinand's children grew up, Lydia's mother died, and Lydia and Ferdinand became the sole occupants of the house. Their marriage flourished for nearly twenty-six years, until Ferdinand's death, and was the greatest gift in the last half of his life.

Two years after he remarried, Ferdinand was made Stevens Professor of Mechanical Engineering. He held this chair for two years (1982–1884), and then in 1985 was made Higgins Professor, a chair which he held until his retirement. In addition to these honors, he was elected as a member of the National Academy of Engineering, an Honorary Member of IFToMM and a Fellow of the New York Academy of Sciences; he became an Honorary Life Fellow of the ASME. He accumulated the following list of awards:

ASME Junior Award, 1955, for the paper "Approximate Synthesis for Four-bar Linkages"; Guggenheim Fellow for "Studies in the Kinematics of Mechanisms", 1961–1962, 1967–1968; Great Teacher Award of Society for Older Graduates of Columbia University, 1966; ASME Machine Design Award, 1972; Mechanisms Committee Award, ASME, 1978; OSU South Pointing Chariot Rotating Trophy (for contributions to mechanisms), 1980–1981; Best Paper Award, ASME Mechanisms Conference, 1970, 1980, 1982, 1984, 1986; ASME Charles Russ Richards Award, 1984; Applied Mechanisms Conference Award for "A Lifetime of Contributions to Mechanisms", 1989; The Egleston Medal for distinguished engineering achievement, Columbia University, 1992.

Freudenstein held membership in the Harvard Engineering Society, the Columbia Engineering Society, Sigma Xi, Pi Mu Epsilon and the VDI (Verein Deutscher Ingenieure). He was active within the American Society of Mechanical Engineers and held the following elected positions: Chairman, Mechanisms Committee (1964–1965); Chairman, Mechanisms Conference (1964); Executive Committee Design Engineering Division (1972–1977); Chairman, Design Engineering Division (1976-1977). In addition he served on advisory panels for the National Science Foundation and the Army Research Office.

Throughout his career he was involved as an industrial consultant. He very much valued these contacts and the insights they afforded into "real-world" engineering problems. His main consulting activities were with Bell Telephone Laboratories, Designatronics, IBM, The Singer Company, Foster Wheeler, Gulf and Western and General Motors. The General Motors consulting activities went on for over fifteen years. Several of his consulting activities led to technical publications in the open literature. In addition he served as an expert witness on several cases involving engineering issues.

The Mechanical Engineering Department at Columbia University relied heavily on a part-time student body many of whom were not born in America. Most of the graduate courses started at about 4PM or 5PM, since many students held part-time or full-time jobs in one of the many colleges or industrial firms in the vicinity of New York City. So, for example, his first PhD student was George Sandor, a Hungarian refugee, who was Chief Engineer for Time, Inc. His second student, Bernard Roth, a native New Yorker whose parents had emigrated from Eastern Europe, was employed as a lecturer at the City College of New York. His third and fourth students were Ronald Philipp, who was an Army Captain stationed at West Point, and An Tzu (Andy) Yang, a refugee from China, who worked at American Machine and Foundry, Inc.

After graduation, many of his PhD students went on to academic positions at other universities. His academic family grew rapidly, and by the time of his death there were over 500 descendants belonging to the Freudenstein academic family tree. The tree is reproduced at the end of this section. It gives the name of each of Freudenstein's PhD students and the names of each of their PhD students, and then subsequent generations. The year and school of each person's PhD is included. Certainly some names have been missed.

The sheer size of the "tree" does not fully reveal Freudenstein's immense influence. He was always available to advise anyone on a research problem. Through his advice, his frequent guest lectures at other universities, and his dedication and example, he influenced all the leading figures in the United States mechanism research community. Ultimately his influence became world-wide.

Starting in 1960 and continuing over his long career, he was able to obtain financial support for his research on an almost continuing basis. He never applied for large grants, nor did he generally have a large research group of students. He tended to have several students at a time, often spaced about one or two years apart in their progress over the various stages in the PhD process. Due to special circumstances, there were several periods where three or more students graduated in the same year, but these were deviations from his normal pattern. His principle supporters were the National Science Foundation, the Army Research Office and to a lesser degree the General Motors Corporation.

The National Science Foundation funded grants with the titles: Burmester theory in the kinematics of mechanisms; Kinematic analysis and synthesis of six-link chains; Kinematic analysis of spatial linkages; Surface generation in plane and spatial mechanisms; Gross-motion kinematic characteristics of mechanisms; Determination and Minimization of Forces in Statically Indeterminate High-Speed Linkage Mechanisms; Kinematics and dynamics of high-speed mechanical components; Kinematics and Dynamics of Basic Mechanical Components; On the Kinematics, Dynamics and Design Optimization of Basic mechanical components and systems.

The Army Research Office funded: Combinatory topological analysis of kinematic structure; Computational kinematics; Optimization in the kinematic and functional design of mechanisms and mechanical systems; An integrated approach to the optimization of high-speed cam-follower systems; Creation of mechanisms according to the separation of kinematic structure and function; Development of an Expert System for the Creative Design of Mechanisms; Instrumentation for ... (the previous contract); The further development and refinement of an expert system approach to the creative design of mechanisms and mechanical systems.

The General Motors Corp. funded work titled "Research on high-speed mechanisms and robotic mechanisms" and "Conceptual design of optimum mechanism configurations."

Through his career he was in demand as a lecturer. He lectured at companies, government laboratories, universities and at academic conferences and workshops. He presented a plenary lecture at the Fourth World Congress on Theory of Machines and Mechanisms, in Newcastle-upon-Tyne.

Ferdinand Freudenstein was a kind and soft spoken individual. He was extremely modest. He avoided controversy and academic politics. He admired mathematics and mathematicians. Yet, he also had a strong interest in applied engineering, and producing analytical models of machines and devices. He was an accessible professor, and was always pleased to assist his professional colleagues at Columbia and throughout the world. He was rather informal, and often signed his letters F.F. or F^2, and suggested his PhD students call him F^2 (pronounced ef-square).

His benevolent influence was felt, both directly and indirectly, by practically everyone who taught or did research in the field of kinematics or machines and mechanisms. Over his career, he wrote a great number of evaluation letters for his many students and their academic descendants, as well as for individuals whom he knew primarily through their research publications. This was a great chore, but he felt it to be his duty to the profession and he undertook it without complaint.

In 1991, in recognition of Freudenstein's 65th birthday, Professor Arthur Erdman, a Freudenstein academic grandchild, organized a conference in Brainard, Minnesota. The event produced an exceptional book titled: *Modern Kinematics: Developments in the Last Forty Years*. The conference was an academic family reunion, filled with excellent science and engineering. Ferdinand and Lydia enjoyed it tremendously (Figure 2).

The following year Ferdinand was awarded the coveted Egleston Medal for distinguished engineering achievement from Columbia University. However, somewhere in his brain the amyloid protein plaque that is suggestive of Alzheimer's disease was starting to build up. The once brilliant and incisive mind was starting to forget, and in normal conversation he often repeated himself without realizing he had done so. Ferdinand was able to keep up his work and teaching at a reduced scale until in 1996, at the age of 70, Freudenstein retired from his position at Columbia University with the title of Higgens Professor Emeritus.

At the time of his retirement, he typed a letter to his former students, which he never sent, apparently defeated by the task of tracking down all

Fig. 2. Ferdinand and Lydia Freudenstein with academic family and friends at his 65th Birthday.

the addresses. After his death, Lydia found the letter (Figure 3) among his possessions.

As he mentions in this letter, Freudenstein had considered writing a book after he retired from teaching. This pleasure was denied to him by his illness. However, he was able to maintain his family life for several years after retirement. Inexorably, however, his difficulties became progressively more debilitating. Throughout his illness he was cared for at home by his wife Lydia, who devoted her entire attention to his care and comfort. He passed away peacefully in his Riverdale home on March 30, 2006.

Five months after Freudenstein's death, Professor Pierre Larochelle, of the Florida Institute of Technical, compiled a list of Freudenstein's academic descendents. The family tree he generated reached into the fifth generation and contained 500 names. A slightly modified version of the "tree" is presented below, in Table 1. The school and year for the award of the PhD degree follow each name. The names of Freudenstein's students are in the left-most part of each column. Subsequent generations have their names offset to the right, each name is offset two spaces from the thesis advisor's name. The table runs continously, so all names in the first column preceed the names in the second column.

Columbia University in the City of New York | *New York, N.Y. 10027-6699*

DEPARTMENT OF MECHANICAL ENGINEERING

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July 1, 1996

To my dear students and co-workers:

How quickly the years have flown! After 42 years of teaching and research at Columbia University, I am planning to retire on July 1, 1996.

I am writing to let all of you know how you have enriched my life through the years. A teacher teaches his students, but a teacher also learns from his students, and I have learned much from you.

I hope we shall continue to be in touch for many years. I look forward to hearing from you and learning about your careers and your achievements. I shall try to attend meetings and conferences so that I do not grow stale "kinematically." I may even try my hand at writing as book - who knows?

I look upon you all as an extension of myself, as members of the far-flung Freudenstein family.

With all good wishes, my dear students, for continued health and success.

I remain,

Your Professor

Ferdinand Freudenstein

Fig. 3. Letter by Freudenstein to his former students.

Table 1. The Freudenstein family tree.

Sandor, George (Columbia 1959)	Surender Reddy, B. (II Sc. Bangalore 2001)
Kaufman, Roger (RPI 1969)	Ranganath, R. (II Sc. Bangalore 2003)
Erdman, Art (RPI 1971)	Bandyopadhyay, S. (II Sc. Bangalore 2006)
Midha, Ashok (Univ. of Minn.1977)	Zi, Zhiming (Stanford 1987)
Turcic, David (Penn State 1982)	Burdick, Joel (Stanford 1988)
Her, Innchyn (Purdue 1986)	Chirikjian, Greg (Cal. Tech 1992)
Farhang, Kambiz (Purdue 1989)	Ebert-Uphoff, I. (Johns Hopkins 1997)
Nahvi, Hassan (Purdue 1991)	Bosscher, Paul (Georgia Tech 2004)
Howell, Larry (Purdue 1993)	Voglewede, Phil (Georgia Tech 2004)
Lyon, Scott (BYU 2003)	Dang, Anh (Georgia Tech 2002)
Lusk, Craig (BYU 2005)	Kozak, Kris (Georgia Tech 2003)
Wittwer, Jonathan (BYU 2005)	Wolff, Sebastien (Georgia Tech 2006)
Mahyuddin, Andi (Purdue 1993)	Stein, David (Johns Hopkins 2001)
Murphy, Morgan (Purdue 1993)	Wang, Yunfeng (Johns Hopkins 2001)
Khanuja, Sukhwant (Purdue 1996)	Lee, Sangyoon (Johns Hopkins 2002)
Mettlach, Gregory (Prdue 1996)	Suthakorn, Jackritt (Johns Hopkins 2003)
Hagen, David (Univ. of Minn. 1982)	Kim, Moon Ki (Johns Hopkins 2004)
Chase, Tom (Univ. of Minn. 1983)	Zhou, Yu (Johns Hopkins 2004)
Ling, Zhi-Kui (Univ. of Minn. 1990)	Schuyler, Adam (Johns Hopkins 2005)
Mirth, John (Univ. of Minn. 1990)	Kim, Jin Seob (Johns Hopkins 2006)
Hable, Mary (Univ. of Minn. 1996).	Chen, I-Ming (Cal Tech 1994)
Holte, Jennifer (Univ. of Minn. 1996)	Yang, Guilin (Nanyang Tech. U. 1999)
Langlais, Timothy (Univ. of Minn. 1999)	Dash, A. K. (Nanyang Tech. U. 2004)
Peterson, Steven (Univ. of Minn. 1984)	Xing, S. (Nanyang Tech. U. 2004)
Liou, Feu-Wen(Frank) (Univ. of Minn. 1987)	Pham, Huy H. (Nanyang Tech. U. 2005)
Fang, Yong (U. of Miss.-Rolla 1995)	Theingi (Nanyang Tech. U. 2005)
Yeh, J.H., (U. of Miss.-Rolla 1997)	Yan, Liang (Nanyang Tech. Univ. 2006)
Huang,Chia-Pin (U. of Miss.-Rolla 2000)	Jin, Yan (Nanyang Tech. U. 2006)
Zhang, Jun (U. of Miss.-Rolla 2001)	Choset, Howie (Cal Tech 1996)
Ruan, Jianzhong (U. of Miss.-Rolla 2003)	Acar, Ercan (Carnegie Mellon U. 2002)
Han, Lijun (U. of Miss.-Rolla 2004)	Lee, JiYeong (Carnegie Mellon U. 2003)
Eiamsa-Ard, K. (U.Miss.-Rolla 2005)	Zhang, Y. (Carnegie Mellon Univ. 2003)
Kota, Sridhar (Univ. of Minn. 1987)	Grabowski, R(Carnegie Mellon U. 2004)
Lee, Chun-Liang (Univ. of Mich. 1992)	Atkar, Prasad (Carnegie Mellon U. 2005)
Ullah, Irfan (Univ. of Mich. 1993)	Shammas, E. (Carnegie Mellon U. 2006)
Chuenchom, Thatchai (U.. of Mich. 1994)	Goodwine, Bill (Cal. Tech 1997)
Chiou, Shean-Juinn (U. of Mich. 1994)	Wei, Yejun (Notre Dame 2002)
Ananthasuresh, G.K. (U. of Mich. 1994)	McMickell, M. Brett (Notre Dame 2003)
Krovi, Venkat (U. Penn. 1998)	Lin, Qiao (Cal. Tech 1998)
Saxena, Anupam (U. Penn. 2000)	Wang, Yi (Carnegie Melon U. 2005)
Kim, Moon (U. Penn. 2001)	Yang, Bozhi (Carnegie Melon U. 2006)
Wang, Xiaoye (U. Penn. 2002)	Murphy, Todd (Cal. Tech 2002)
Mankame, Nilesh (U. Penn. 2004)	Vela, Patricio (Cal. Tech 2003)
Koh, Sung (U. Penn. 2005)	Cai, Chunsheng (Stanford 1988)
Frecker, Mary (Univ. of Mich. 1997)	Raghavan, Madhusudan (Stanford 1989)
Goulbourne, Nakhiah (Penn State 2005)	Wang, Harrison (Stanford 1989)
Ramrakhyani, D. (Penn State 2005)	Agrawal, Sunil (Stanford 1990)
Saggere, Laxman (Univ. of Mich. 1998)	Annapragada, M. (Univ. of Delaware 1999)
Kalnas, Ronald (Univ. of Mich. 1999).	Xu, Xiaochun (Univ. of Delaware 1999)
Hetrick, Joel (Univ. of Mich. 2000)	Faiz, Nadeem (Univ. of Delaware 1999)
Joo, JinYong (Univ. of Mich. 2000).	Veeraklaew, T. (Univ. of Delaware 1999)
Moon, Yong-Mo (Univ. of Mich. 2001)	Ferreira, A. (Univ. of Delaware 2001)
Moon, Sangku (Univ. of Mich. 2003)	Pledgie, Stephen (Univ. of Delaware 2002)
Lu, Kerr-Jia (Univ. of Mich. 2004)	Zhang, Yuhong (Univ. of Delaware 2004)
Kim, Charles (Univ. of Mich. 2005)	Hao, Yongxing (Univ. of Delaware 2004)
Lin, Chuen-Sen (Univ. of Minn.1987)	Pathak, Kaustubh (Univ. of Delaware 2005)

Table 1. (Continued)

Chiou, Huang (Univ. of Minn.1987).	Mankala, K. (Univ. of Delaware 2006)
Olson, Daniel (Univ. of Minn.1987)	Oh, S. (Univ. of Delaware 2006)
Faik, Salaheddine (Univ. of Minn.1988).	James, Paul (Stanford 1991)
Rekow, E. Diane (Univ. of Minn.1988)	Kolaroy, Krasimir (Stanford 1991)
Schendel, Mike (Univ. of Minn.1990)	Huang, Chintien (Stanford 1992)
Zine-Eddine, Boutaghou (U. of Minn.1991)	Mavroidis, Constantinos (Univ. of Paris 1993)
Lee, Ming (Univ. of Minn.1991)	Lee, Joseph (Rutgers Univ. 2000)
D'Costa, Joseph (Univ. of Minn.1993)	Badescu, Mircea (Rutgers Univ. 2003)
Holte, Jenny (Univ. of Minn.1996)	Lee, Eric (Rutgers Univ. 2004)
Mlinar, John (Univ. of Minn.1997)	DeLaurentis, Kathryn (Rutgers Univ. 2004)
Rizq, Raed (Univ. of Minn.1997)	Matone, Ricardo (Stanford 1997)
Johnson, Todd (Univ. of Minn.1998).	Nielsen, James (Stanford 1997)
Hong, Boyang (Univ. of Minn. 2000)	Chatterjee, Goutam (Stanford 1998)
Swigart, John (Univ. of Minn. 2000)	Zinn, Michael (Stanford 2005)
Ho, Chen Ta (Charlie) (Univ. of Minn. 2000)	Philipp, Ronald E. (Columbia 1964)
Hartfel, Marge (Univ. of Minn. 2001)	Yang, A.T. (Andy) (Columbia 1964)
Guion, Marie (Univ. of Minn. 2003)	Kirson, Y. (U.C. Davis 1975)
Kramer, Steven (RPI 1973)	Hsia, Lih-Min (U.C. Davis 1980)
Scott, Patsy (Univ. of Toledo 2002)	Pennock, Gordon (U.C. Davis 1983)
Sadler, J. Peter (RPI 1973)	Stanisic, Michael (Purdue 1986)
Dimarogonas, Andrew (RPI 1975)	Remis, Steven (Notre Dame 1994)
Reinholtz, Charles (Univ. of Florida 1983)	Lorenc, Steven (Notre Dame 1995)
Williams, Bob (Virginia Tech 1988)	Nesnas, Issa (Notre Dame 1995)
Wu, Jianhua (Ohio University 2004)	Ryuh, Beom Sahng (Purdue 1989)
Shooter, Steven (Virginia Tech. 1995)	Shun, John Bun (Purdue 1982)
Tidwell, Paul (Virginia Tech 1995)	Mattson, Keith (Purdue 1995)
Canfield, Steve (Virginia Tech 1997)	Lee, G. (U.C. Davis 1987)
Roth, Bernard (Columbia 1962)	Schaaf, J. (U.C. Davis 1989)
Chen, Paul (Pictiaw) (Stanford 1968)	Dil Pare, Armand (Columbia 1965)
Pieper, Donald (Stanford 1968)	Haddad, Wassim (Florida Tech 1987)
Waldron, Kenneth (Stanford 1969)	Huang, H.-H. (Florida Tech 1992)
Seeger, B.R. (Univ. of NSW 1973)	Ying, S. (Florida Tech 1993)
Yeo, B.P. (Univ. of NSW 1974)	Moser, R. (Florida Tech 1994)
Baker, J.E. ((Univ. of NSW 1976)	Williams, D. (Florida Tech 1995)
Strong, R.T. (Univ. of Houston 1978)	Kapila, V. (Georgia Tech 1996)
Ahmad, A. (Univ. of Houston 1979)	Chellaboina, V. (Georgia Tech 1996)
Sun, J.W.H. (Univ. of Houston 1979)	Fausz, J. (Georgia Tech 1997)
Kumar, A. (Univ. of Houston 1980)	Leonessa, A. (Georgia Tech 1999)
Huang, J.C. (Univ. of Houston 1980)	Corrado, J. (Georgia Tech 2000)
Song, Shin-Min (Ohio State 1984)	Hayakawa, T. (Georgia Tech 2003)
Cope, Ralph (Ohio State 1986)	Nersesov, S. (Georgia Tech 2005)
Wang, Shih-Liang (Ohio State 1986)	Buchsbaum, Frank (Columbia 1967)
Tsai, M.J. (Ohio State 1986)	Dobrjansky, Leo (Columbia 1967)
Reidy, John (Ohio State 1986)	Wallace, Donald (Columbia 1968)
Kumar, Vijay (Ohio State 1987)	Dratch, Ralph (Columbia 1970)
Ulrich, Nathan (U. Penn. 1990)	Dubowsky, Steven (Columbia 1971)
Kim, Jung-Ha (U. Penn 1990)	Gardner, T.N. (U.C. Los Angeles 1975)
Wang, Yin Tien (U. Penn 1993)	Maatuk, J. (U.C. Los Angeles 1976)
Sarkar, Nilanjan (U. Penn. 1993)	Perreira, N.D. (U.C. Los Angeles 1977)
Ouerfelli, Mohamed (U. Penn. 1994)	Doydum, Cemal (Lehigh Univ.)
Howard, William (U. Penn. 1995)	Tucker, Michael (Lehigh Univ.)
Wang, Chau-Chang (U. Penn. 1995)	Kuntz, Herbert (Lehigh Univ.)
Zefran, Milos (U. Penn. 1996)	Sunada, W.H. (U.C. Los Angeles 1981)
Desai, Jaydev (U. Penn. 1998)	Bobrow, J. (U.C. Los Angeles 1982)
Kennedy, Christopher (Drexel 2004)	Casale, Malcolm (U.C. Irvine 1989)
Hu, Tie (Drexel 2006)	McDonell, Brian (U.C. Irvine 1995)

Table 1. (Continued)

Chanthasopeephan, T. (Drexel 2006)	Martin, Bryan (U.C. Irvine 1996)
Krovi, Venkat (U. Penn. 1998)	Ploen, Scott (UC Irvine 1997)
Sugar, Tom (U. Penn. 1999)	Norton, Jeffery (U.C. Irvine 1998)
Wang, Zheng (Arizona State U. 2005)	Sohl, Garett, (U.C. Irvine 2000)
Hollander, K. (Arizona State U. 2005)	Wang, Eric Chia-Yu (U.C. Irvine 2001)
Song, Peng (U. Penn. 2002)	Michaelis, Matthew (U.C. Irvine 2005)
Esposito, Joel (U. Penn. 2002)	Aoyagi, Daisuke (U.C. Irvine 2006)
Belta, Calin (U. Penn. 2003)	Leavitt, John (U.C. Irvine 2006)
Das, Aveek (U. Penn. 2004)	Vargas Albro, Juanita (U.C. Irvine 2006)
Rao, Rahul (U. Penn. 2004)	Shiller, Zvi (MIT 1987)
Chitta, Sachin (U. Penn 2005)	Fiorini, Paolo (U.C. Los Angeles 1995)
Parikh, Sarangi (U. Penn. 2005)	Botturi, Debora (Univ. of Verona 2004)
Kim, Jongwoo (U. Penn 2006)	Satish, Sundar (U.C. Los Angeles 1995)
Gardner, John (Ohio State 1987)	Vafa, Zia (MIT 1987)
Huang, Mingzen (Ohio State 1988)	Papadopoulos, E. (MIT 1990)
Nair, Satish (Ohio State 1988)	Moosavian, Ali (McGill 1996)
Murthy, Vasudeva (Ohio State 1990)	Rastegari, R. (Toosi Univ. of Tech 2006)
Mukherjee, Sudipto (Ohio State 1991)	Martin, Eric (McGill 1999)
Suhaib, Mohammad (IIT Delhi 2004)	Dupuis, Erick (McGill 2001)
Gawade, Tusher (IIT Delhi 2004)	Vlachos, C. (Nat. Tech. U. of Athens 2004)
Dutta, V.P. (IIT Delhi 2005)	Hootsmans, N. (MIT 1992)
Nanua, Probjot (Ohio State 1992)	Oppenheimer, C. (MIT 1992)
Husain, Muqtada (Ohio State 1993)	Deck, J. (MIT 1992)
Chin, Pei-Chieh (Ohio State 1993)	Torres, M. (MIT 1993)
Chung, Wen-Yeuan (Ohio State 1994)	Gu, P.-Y. (MIT 1994)
Sreenivasan, S.V. (Ohio State 1994)	Farritor, Shane (MIT 1998)
Owen, Frank (U. of Texas-Austin 1998)	Rentschler, Mark (U. of Neb.-Lincoln 2005)
Choi,B.-J. (U. of Texas-Austin 1998)	Dumpert, Jason (U. of Neb.-Lincoln 2006)
Gonzalez, L. J (U. of Texas-Austin 1999)	Gao, Xinbao (U. of Neb.-Lincoln 2005)
Schuetze, K. T. (U. of Texas-Austin 2000)	Huang, J. (U. of Neb.-Lincoln 2006)
Park, Jaihun P. (U. of Texas-Austin 2001)	Meggiolaro, M. (MIT 2000)
Choi,Y.-J. (U. of Texas-Austin 2003)	Iagnemma, Karl (MIT 2001)
Raines, Allen (U. of Texas-Austin 2006)	Sujan, V. (MIT 2002)
Yu, Y.C. (Ohio State 1994)	Yu, H. (MIT 2002)
Venkataraman, Shankar (Ohio State 1997)	Lichter, M. (MIT 2004)
Yang, Po-Hua (Ohio State 1999)	Spenko, Matthew (MIT 2005)
Schmiedeler, Jim (Ohio State 2001)	Plante, J.S. (MIT 2005)
Nichol, Jamie (Stanford 2005)	Yuan, Mark (Columbia 1972)
Singh, Surya (Stanford 2006)	Soylemez, Eres (Columbia 1974)
Kahn, Michael (Stanford 1969)	Tutak, Bulent (Firat Univ. 1988)
Tsai, Lung-Wen (Stanford 1972)	Lee, Ting (Columbia 1975)
Chang, Sun-Lai (Univ. of Maryland 1991)	Tanaka, Hitoshi (Columbia 1976)
Chen, Dar-Zen (Univ. of Maryland 1991)	Datseris, Phillip (Columbia 1977)
Lee, Jyh-Jone (Univ. of Maryland 1991)	Lin, Paul (Univ. of Rhode Island 1985)
Lin, Chen-Chou (Univ. of Maryland 1991)	Wu, Yin-Min (Univ. of Rhode Island 1987)
Tahmasebi, Farhad (Univ. of Maryland 1993)	Yang, Bi Wu (Univ. of Rhode Island 1989)
Ou, Yeong-Jeong (Univ. of Maryland 1994)	Ziegert, John (Univ. of Rhode Island 1989)
Shing, Tai-Kang (Univ. of Maryland 1994)	Tian, Huwawei (Univ. of Rhode Island 1994)
Hsieh, Hsin-L (Univ. of Maryland 1996)	Mankame, Anil (Univ. of Rhode Island 1997)
Stamper, Richard (Univ. of Maryland 1997)	Almesallmy, M. (Univ. of Rhode Island 2006)
Schultz, Gregory (Univ. of Maryland 2002)	Berzak, Nir (Columbia 1979)
Wiederich, James (Stanford 1973) FMC Corp.	Chew, Meng-Sang (Columbia 1980)
Sutherland, George (Stanford 1973)	Gill, Gurdev Singh (Columbia 1981)
Gupta, Krishna.C. (Stanford 1974)	Pisano, Albert (Columbia 1981)
Tinubu, S.O. (U. of Ill. Chicago 1983)	Chan,C.-Y. (U.C. Berkeley 1988)
Kazerounian, K (U. of Ill. Chicago 1984)	Lin, Yuyi (U.C. Berkeley 1989)

Table 1. (Continued)

Nedungadi, Ashok (Unic. Of Conn. 1988)	Wen, G. (U. of Missouri Columbia 1998)
Wang, Zhaoyu (univ. of Conn. 1990)	Cai, X. (U. of Missouri Columbia 2005)
Qiann, Zhihong Gene (U. of Conn. 1993)	Harby, D. (U. of Missouri Columbia 2006)
Tylaska, Timothy (Univ. of Conn. 1993)	Cho,Y.-H. (U.C. Berkeley 1990)
Bozorgi, Jamshid (Univ. of Conn. 1995)	Hodges, P. (U.C. Berkeley 1991)
Vahidi, Siamak (Univ. of Conn. 1995)	Kim,C.-J. (U.C. Berkeley 1991)
Jovanovic, Vojin (Univ. of Conn. 1997)	Simon, Jonathan (U.C. Los Angeles 1997)
Sun, Yunquan (Univ. of Conn. 2003)	Sherman, Faiz (U.C. Los Angeles 1998)
Alvarado, Carlos (Univ. of Conn. 2004)	Tseng, Fan-Gang (U.C. Los Angeles 1998)
Pimsarn, Monsak (Univ. of Conn. 2005)	Huang, Long-Sun (U.C. Los Angeles 1999)
Mu, Zongliang (Univ. of Conn. 2005)	Lee, Jung-Hoon (U.C. Los Angeles 2000)
Cheng, Harry (Univ. of Ill. Chicago 1989)	Yi, Taechung (U.C. Los Angeles 2000)
Samak, S.M. (Univ. of Ill. Chicago 1989)	Yao, Da-Jeng (U.C. Los Angeles 2001)
Mirman, C.R. (Univ. of Ill. Chicago 1990)	Kim, Joonwon (U.C. Los Angeles 2003)
Singh, Vinod (Univ. of Ill. Chicago 1992)	Fan, Shih-Kang (U.C. Los Angeles 2003)
Ma, Rufei (Univ. of Ill. Chicago 1995)	Lu, Yen-Wen (U.C. Los Angeles 2004)
Beloiu, A.S. (Univ. of Ill. Chicago 1996)	Yi, Ui-Chong (U.C. Los Angeles 2004)
Li, J. (Univ. of Ill. Chicago 1997)	Shen, Wenjiang (U.C. Los Angeles 2004)
Shimano, Bruce (Stanford 1978)	Meng, Desheng (U.C. Los Angeles 2005)
McCarthy, J. Mike (Stanford 1979)	He, Rihui (U.C. Los Angeles 2005)
Ge, Jeff (U.C. Irvine 1990)	Moon, Hyejin (U.C. Los Angeles 2005)
Srinivason, L. (SUNYStony Brook 1997)	Tsai, Jane Gin-Fai (U.C. Los Angeles 2005)
Kang, D. (SUNY Stony Brook 1997)	Hsu, W. (U.C. Berkeley 1992)
Mattice, M. (SUNY Stony Brook 2000)	Lee, A. (U.C. Berkeley 1992)
Chang, C.F. (SUNY Stony Brook 2000)	Lin, Liwei (U.C. Berkeley 1993)
Xia, Jun (SUNY Stony Brook 2001)	Brennen, R. (U.C. Berkeley 1993)
Purwar, A. (SUNY Stony Brook 2005)	DeVoe, Don (U.C. Berkeley 1997)
Bodduluri, R.Mohan.C. (U.C. Irvine 1990)	Panchapakesan, B.(U. of Maryland 2001)
Dooley, John (U.C. Irvine 1993)	Darabi, Jafar (Univ. of Maryland 2001)
Larochelle, Pierre (U.C. Irvine 1994)	Sniadecki, N. (Univ. of Maryland 2003)
Tse, David (Florida Tech 2000)	Kimball, Chris (Univ. of Maryland 2003)
Ketchel, John (Florida Tech 2006).	Piekarski, Brett (Univ. of Maryland 2003)
Murray, Drew (U.C. Irvine 1996)	Cheng, Wei-Jen (Univ. of Maryland 2005)
Hao, Fangli (U.C. Irvine 1996)	Li, Lihua (Univ. of Maryland 2005)
Soriano, Bernard (U.C. Irvine 1996)	Liu, Jing (Univ. of Maryland 2005)
Collins, Curtis (U.C. Irvine 1997)	Zhu, Likun (Univ. of Maryland 2005)
Perez-Gracia, Alba (U.C. Irvine 2003)	Juneau, T. (U.C. Berkeley 1997)
Su, Hai-Jun (U.C. Irvine 2004)	Ljung, P. (U.C. Berkeley 1997)
Nayak, Janshwhare (Stanford 1979)	Brosnihan, T. (U.C. Berkeley 1998)
DeSa, Subhas (Stanford 1979)	Cohn, M. (U.C. Berkeley 1998)
Ohwovoriole, Ejevo(Morgan) (Stanford 1980)	Horsley, D. (U.C. Berkeley 1998)
Henandez-Gutieerrez, Ignacio (Stanford 1980)	Lebouitz, K. (U.C. Berkeley 1998)
Ravani, Bahram (Stanford 1982)	Lemkin, M. (U.C. Berkeley 1998)
Mark L. Hornick (U. of Wisc.-Madison 1985)	Roessig, T. (U.C. Berkeley 1998)
Chen, Yih-Jen (U. of Wisc.-Madison 1986)	Evans, J. (U.C. Berkeley 1999)
Wang, Li-Chun (U. of Wisc.-Madison 1986)	Talbot, N, (U.C. Berkeley 1999)
Ku, Tai-Son (U. of Wisc.-Madison 1987)	Muller, L. (U.C. Berkeley 2000)
Sardis, Robert (U.. of Wisc.-Madison 1987)	Davis, W. (U.C. Berkeley 2001)
Kashani, A. R. (U. of Wisc.-Madison 1989)	Kuan, N. (U.C. Berkeley 2001)
El-Sinawi, Ameen (Univ. of Dayton 1998)	Papavasiliou, A. (U.C. Berkeley 2001)
Hamad, Ibrahim (Univ. of Dayton 2002)	Stupar, P. (U.C. Berkeley 2001)
Mandura, Tallal (Univ. of Dayton 2004)	Bellew, C. (U.C. Berkeley 2002)
Al-Halwa, Khalid (Univ. of Dayton 2005)	Dougherty, G. (U.C. Berkeley 2002)
Lee, Shi (U.C. Davis 1991)	Seward, K. (U.C. Berkeley 2002)
Wang, Jin-Wu (U.C. Davis 1992)	Vestel, M. (U.C. Berkeley 2002)
Loduha, T. A.(U.C. Davis 1993).	Knobloch, A. (U.C. Berkeley 2003)

Table 1. (Continued)

Yang, C-N (U.C. Davis 1994)	Frank, J. (U.C. Berkeley 2004)
Choi, Jungho (U.C. Davis 1994)	Hobbs, E. (U.C. Berkeley 2004)
Hwang, Kwun-Ying (U.C. Davis 1995)	Bircumshaw, B. (U.C. Berkeley 2005)
Huang, H-T (U.C. Davis 1995)	Fischer, Ian (Columbia 1985)
Liptai, L. L. (U.C. Davis 1996)	Wu, Yeou-Kai (NJIT 1994)
Nederbragt, W. W. (U.C. Davis 1997)	Rahman, Sahidur (NJIT 1996)
Ghaida, H. A. (U.C. Davis 1999)	Mayourian, Moez (Columbia 1985)
Sprott, K. S. (U.C. Davis 2000)	Hanachi, Shervin (Columbia 1986)
Gabibulayev, M. (U.C. Davis 2004)	Sohn, Wayne (Columbia 1987)
Eberharter, H. L. (U.C. Davis 2005)	Chen Chin-Kong (Columbia 1989)
Salisbury, J. Kenneth (Stanford 1982)	Fang, Wenchung (Eugene) (Columbia 1989)
Townsend, William (MIT 1988)	Bernard, Judd (Columbia 1990)
Brock, David (MIT 1993) MIT	Fabien, Brian (Columbia 1990)
Eberman, Brian (MIT 1995)	Pennestri, Ettore (Columbia 1991)
Morrell, John (MIT 1996) Yale	Bucknor, Norman (Columbia 1991)
Leveroni, Susanna (MIT 1997)	Veikos, Nicholas (Columbia 1991)
Madhani, Akhil (MIT 1997)	Shim, Jae Kyung (Columbia 1991)
Ottensmeyer, Mark (MIT 2001)	Kim, Seon Pyung (Korea Univ. 2000)
Hong, Jesse (MIT 2001)	Chung, Woo Jin (Korea Univ. 2001)
Barbagli, Federico (Stanford 2002).	Lee, Tae Young (Korea Univ. 2001)
Diolaiti, Nichola (Stanford 2005)	Choe, Kang (Columbia 1994)
Morris, Dan (Stanford 2006)	Lin, Don (Columbia, 1994)
Kerr, Jeffrey (Stanford 1984)	Sun, Yungao (Columbia 1994)
Craig, John (Stanford 1986)	Wasfy, Tamer (Columbia 1994)
Ghosal, Ashitava (Stanford 1986)	Bard, David (Columbia 1995)
Dwarakanath, T.A. (II Sc. Bangalore 1993)	Lozancic, Dragan (Columbia 1995)
Theodore, Rex J. (II Sc. Bangalore 1995)	

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Non Circular Drive, U.S. Pat. # 4,865,577, 1989.

Review of Main Works on Mechanism Design

Freudenstein's initial publications (Freudenstein, 1954, 1955) stemmed from his PhD research. In the second of these papers he developed the equation which later bore his name. The Freudenstein Equation is an elegantly simple scalar equation representing the closure constraint on a planar four-bar chain; it is based on figure 1 of his paper, shown here as Figure 4.

In his paper, the length of the fixed link is normalized to unity, the input crank length is b, the coupler length is c, the output link's length is d, and the input and output angles, measured from the line of the fixed link, are ϕ and φ, respectfully.

Using:

$$R_1 = 1/d, \quad R_2 = 1/b, \quad R_3 = (1 + b^2 - c^2 + d^2)/(2bd).$$

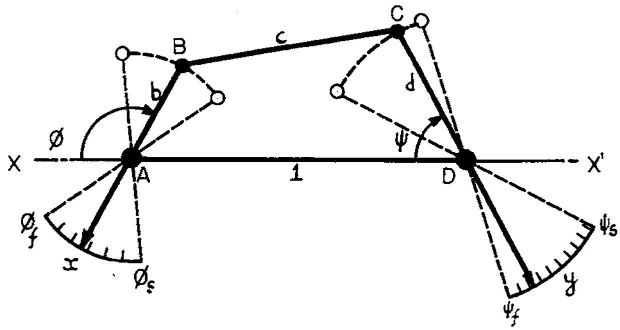

FIG. 1 FOUR-BAR LINKAGE

Fig. 4. The figure used to derive Freudenstein's Equation.

Freudenstein obtained his equation in the form:

$$R_1 \cos \phi - R_2 \cos \varphi + R_3 = \cos(\phi - \varphi).$$

It is useful for designing and analyzing planar four-bar function generating mechanisms.

If three input angles and their corresponding output angles are specified, writing this equation once for each input-output angle pair yields three linear equations for the R's. Once R_1, R_2 and R_3 are known, the link lengths b, c and d are easily determined. The resulting four-bar chain, with the calculated link lengths b, c, d, will be a closed loop when the input link is placed in any one of the three specified positions (ϕ) and the output link is at the corresponding output angle (φ). This is the so-called three precision-point design.

Furthermore, by measuring the angles from unspecified arbitrary starting positions, ϕ_s and φ_s, two additional design variables can be introduced. This is done by setting $\phi = \phi_s + p_i$ and $\varphi = \varphi_s + q_i$, where p_i and q_i are the angular rotations for, respectively, the input and output links measured from their starting positions. With these substitutions Freudenstein's equation can be used as the basis for a four or five precision-point design. For this case, the equation takes the form:

$$R_1 \cos(\phi_s + pi) - R_2 \cos(\varphi_s + q_i) + R_3 = \cos[(\phi_s + p_i) - (\varphi_s + q_i)],$$

$$i = 1, 2, 3, 4, 5.$$

Of course, now it is necessary to solve non-linear equations. In his paper, Freudenstein develops a detailed solution for four and five precision-position designs.

Finally, by introducing scale factors r_ϕ and r_φ for the conversion of the crank angles to some functional variables, such as x and y, in $y = f(x)$, p_i and q_i can be replaced in Freudenstein's equation by $p_i = r_\phi(x_i - x_s)$ and $q_i = r_\varphi(y_t - y_s)$. By leaving the scale factors unspecified, Freudenstein added two new design variables, and was able to achieve seven precision-point designs. In addition to the finitely separated precision points, this paper also treats order approximations, i.e., the type of approximation where there is only one precision point but there are a number of derivatives specified at this precision point.

He later followed up his great success with this function generation paper by publishing a paper incorporating the point-position reduction method into function generation (Freudenstein, 1957a, 1957b) and a tabulation of optimized function-generation solutions (Freudenstein, 1958b).

In the same year as his landmark function generation paper, he described the design and construction of a novel pantograph type mechanism for use in an instrumentation laboratory at Columbia University (Freudenstein and Calcaterra, 1955).

This was the first of many papers describing mechanisms he designed and/or analyzed. The list includes: recording mechanisms (Freudenstein, 1965b), chain type mechanisms (McLarnan and Freudenstein, 1966; Freudenstein and Chen, 1986; Chen and Freudenstein, 1988; Bucknor and Freudenstein, 1992, 1994; Veikos and Freudenstein, 1992), tone arm mechanisms (Freudenstein and Soylemez, 1973), track mechanisms (Freudenstein and McLarnan, 1973), roller mechanisms (Freudenstein and Soylemez, 1975), lifting rigs (Freudenstein and Longman, 1975; Fabien, Longman and Freudenstein, 1991), multiport levers (Freudenstein and Soylemez, 1977), locks (Soylemez and Freudenstein, 1980), spatial cranks and rockers (Freudenstein and Soares, 1980), variable stroke engines (Freudenstein and Maki, 1983, 1984; Freudenstein, Maki and Tsai, 1988), variable motion cam shaft drives (Freudenstein, Tsai and Maki, 1983), Cardan type universal joints (Fischer and Freudenstein, 1984), wrench grips (Freudenstein, 1985, 1986), scissors (Freudenstein, 1990), paper cutters (Freudenstein and Chen, 1992) and cassette loaders (Freudenstein, McCandless and Mawhirt, 1993). The list of Freudenstein's patents and patent disclosures at the end of the previous section includes the fruits of some of these studies.

Another of his early contributions was an elegant paper on the extreme values of velocity and acceleration in four-bar output cranks. In this paper he showed that, in a four-bar, at an extreme value of the velocity ratio the collineation axis is perpendicular to the coupler link (Freudenstein, 1956). Twenty years later he published a broader study on this same topic where he and his co-authors considered the velocity variations in various mechanisms (Freudenstein, Primrose, Ray and Laks, 1976).

In his early years, he published two articles on gear design and accuracy (Freudenstein, Wartham and Watrous, 1957; Freudenstein, 1958a). These were the forerunners of several research studies on gear design and analysis. His topics included the study of epicyclic gear trains (Freudenstein, 1971; Freudenstein and Yang, 1972; Yang and Freudenstein, 1973; Bernard and Freudenstein, 1990), noncircular gears (Freudenstein and Primrose, 1975), gear train inertia (Freudenstein and Mayourian, 1982) and force and power-flow in gear trains (Freudenstein, Longman and Chen, 1984; Pennestri and Freudenstein, 1990a, 1990b, 1993a, 1993b).

In 1959 Freudenstein published three additional landmark papers. The first (Freudenstein, 1959a) considered the effect of precision-point spacing on the resulting error curves for function generating mechanisms. In this paper he developed a method to choose precision points, based on Chebyshev polynomial concepts, that lead to minimum maximum-errors. (A related paper (Freudenstein, 1961c), on interval interpolation, was published two years later.)

The second was a paper on the synthesis of path-generating mechanisms (Freudenstein and Sandor, 1959). This paper marked two big beginnings for Freudenstein: it was the first paper he co-authored with one of his PhD students, and it marked his transition away from the function generation synthesis problem into the area of path generation. In this new paper he reinforces the utility of complex numbers and introduced the important concept of using a virtual linkage named the "compatibility four-bar" to determine the ranges of real points on the circle- and center-point curves.

This paper was the starting point for the important series of papers he published on the vector-loop methods in synthesis and the modern development of classical Burmester theory (Freudenstein and Sandor, 1961; Freudenstein, 1961a; Roth, Freudenstein and Sandor, 1962; Primrose, Freudenstein and Sandor, 1964; Freudenstein, Bottema and Koetsier, 1969; Freudenstein and Primrose, 1981).

The third work of 1959 (Freudenstein, 1959c) marked his first foray into the dynamic aspects of mechanisms; he derived the harmonic analysis of four-bar crank-and-rocker mechanisms. This paper is now regarded as a classic work. In it he used his analytic results to find crank-and-rocker length ratios that minimize the higher harmonics while optimizing the transmission angle.

This was then followed by another paper on harmonic analysis (Freudenstein and Mohan, 1961) and on papers on the dynamics of springs and cams (Freudenstein, 1960a, 1960b). In this latter paper he observed that cam dynamics could be improved by minimizing the harmonic content of the driving function, and he developed a family of curves with low harmonic content in order to minimize the peak acceleration in the system.

Throughout his career he returned to the dynamic analysis and design of mechanical components, cams and linkage systems. A brief outline of this extensive body of work is given below.

In Freudenstein, Vitaglio et al. (1969) he considered the dynamic response of mechanical systems. In Freudenstein (1970) he modelled shock absorbers in freight cars. In Woo and Freudenstein (1971) he introduced the use of screw coordinates as a tool in the dynamic analysis of spatial mechanisms.

In Dubowsky and Freudenstein (1971) he and his PhD student modelled a pin joint with clearance as a spring-damper system. They studied the contact forces in the joint and the behaviour of what they termed an impact pair. In Freudenstein (1971, 1972) he presented tutorials for practicing engineers on the issue of vibrations in machines. In Freudenstein (1973b) he studied the effect of dynamic models where machine elements are represented by a combination of lumped and distributed parameters. In Funabashi and Freudenstein (1979) he, and a visiting professor from the Tokyo Institute of Technology, synthesized planar and spherical four-bar linkages for quality high-speed operations. In Berzak and Freudenstein (1979) he and his student determined important properties of polynomial cam curves.

In Freudenstein, Macey and Maki (1981), he and two engineers at GM, studied optimal balancing in high speed machinery. In Gill and Freudenstein (1983) he and his PhD student studied the minimization of inertia forces in spherical four-bars. In Chew, Freudenstein and Longman (1983) they applied optimal control theory to cam design, and showed that it is not possible to optimize simultaneously for all significant design criteria.

In Pisano and Freudenstein (1983), with his PhD student, he developed new models for automotive valve trains. These models, combined with experimental investigations, yielded increased understanding of the significance of friction, spring dynamics and hydraulic tappets in automotive valve trains. In

Freudenstein, Maki and Mayourian (1983) he developed criteria for energy efficient cam follower systems.

In Freudenstein (1984) he outlined his thoughts on possible research initiatives for the area of machine dynamics. In Chen and Freudenstein (1986) they presented a dynamic analysis of a universal joint, and included manufacturing tolerances in their model. In Hanachi and Freudenstein (1986) they developed a predictive model for the optimization of high-speed cam-follower systems. In Freudenstein and Macy (1990) they derived the inertia torques in a Hooks joint. In Fabien, Longman and Freudenstein (1990) they studied the control of an electromagnetic suspension and in Fabien, Longman and Freudenstein (1994) they studied the design of high-speed cams using optimal control theory.

Another topic he returned to several times was the determination and optimization of the transmission angle as a means of improving static force transmission and kinematic performance. He considered planar four-bars (Roth, Freudenstein and Sandor, 1962; Freudenstein and Chew, 1979), geared five-bars (Lee and Freudenstein, 1978), spherical four-bars (Freudenstein and Primrose, 1972) and spatial four-bars (Soylemez and Freudenstein, 1982).

With his second PhD student, he worked on obtaining numerical solutions to the sets of nonlinear algebraic equations that arise in the synthesis of linkages. In the course of solving the nine-precision point synthesis for four-bar coupler curves, they developed a method they named the "Bootstrap" method (Roth and Freudenstein, 1963). This was the earliest versions of what is now known as polynomial continuation.

In a follow up paper they called their method the parameter-perturbation procedure (Freudenstein and Roth, 1963) and they invented a numerical example to illustrate that it could converge to a solution without starting from a good initial guess. This example led to the development of a widely used test function for nonlinear equation solvers and optimization routines known as the Freudenstein–Roth function.

An application of numerical methods to two-degree-of-freedom linkages followed four years later (Philipp and Freudenstein, 1966). Freudenstein later went on to consider the path generation synthesis of spatial mechanisms (Alizade, Freudenstein and Pamidi, 1976; Alizade, Novruzebekov, Freudenstein and Sandor, 1980). In two subsequent PhD theses he and his students introduced the idea of using heuristics to perform mechanism synthesis (Lee and Freudenstein, 1976; Datseris and Freudenstein, 1979). This is an iterative approach that improves a set of arbitrarily selected feasible solutions. The advantage of this method is that it is not dependent on the mathematical properties of either the objective or constraint functions.

With his fourth PhD student, he showed how to apply dual-number quaternions to the analysis of spatial linkages (Yang and Freudenstein, 1964). The success with this work had continued benefits in two areas: it awakened an interest in the kinematics community in the use of dual numbers, dual vectors and dual quaternions, and it furthered Freudenstein's interest in the position analysis of multi-link closed-loop spatial chains. This increased interest led to a series of advances in the analysis of spatial and spherical mechanisms (Freudenstein, 1968; Wallace and Freudenstein, 1970, 1975; Woo and Freudenstein, 1970; Yuan and Freudenstein, 1971; Pamidi and Freudenstein, 1975; Freudenstein and Primrose, 1976).

Freudenstein had a special interest in the algebraic geometry and differential geometry of coupler curves. He was interested in knowing what a mechanism could or could not do. He published a study which showed that linked mechanisms could not generate transcendental curves (Freudenstein, 1963) and in it also studied their singularity conditions. He also enlisted the collaboration of Eric Primrose, a Senior Lecturer in the Department of Mathematics at the University Leicester in England, who was an expert on classical algebraic geometry. Together they analyzed the coupler curves of planar geared five-bar linkages (Freudenstein and Primrose, 1963; Primrose and Freudenstein, 1963), planar six-bar linkages (Primrose, Freudenstein, and Roth, 1967) and simple spatial linkages (Freudenstein and Primrose, 1969).

Freudenstein's first publication on curvature theory was a short pedagogical redevelopment of R. Mueller's work on finding Burmester points with higher order approximations to straight line (Freudenstein, 1961a). This was followed by what is arguably the classic modern paper on the subject of coupler curve curvature (Freudenstein, 1965). In this paper he shows how to study any coupler curve's curvature as well as first, second and nth change of curvature, all in a systematic manner related to the kinematics of a planar motion. He then co-authored a paper showing how to extend the classical instantaneous, positions of a plane, Burmester theory to conics (Sandor and Freudenstein, 1967). Freudenstein later presented a very elegant treatment of the curves associated with the synthesis of the instantaneous kinematics of planar motions (Woo and Freudenstein, 1969).

With his sixth PhD student, Freudenstein entered the area of graph theory and pioneered its application to type synthesis (Dobrjanskyj and Freudenstein, 1967; Freudenstein, 1967b, 1977). From this time forward, the topic of type synthesis and its elaborations became one of the major themes of his research, and he continued to work on this topic until his retirement

With Frank Buchsbaum, his seventh student, in what was a major breakthrough in the study of gear trains, he introduced graphs and so-called colored graphs to represent planetary gear trains (Buchsbaum and Freudenstein, 1970). These deduced fundamental rules and provided a definition for graph isomorphism. Freudenstein (1971) further extended graph theory to include definitions such as rotation and displacement graphs, and the corresponding concepts of isomorphism for graph rotation and displacement. Furthermore, he showed that the displacement equation for any epicyclic gear train can be obtained by inspection from the kinematic structure. These techniques were elaborated on, using dual numbers, dual vectors and dual transformations, so as to extend the fundamental epicyclic circuit theory to gear trains with bevel gears and hypoid gears (Yang and Freudenstein, 1973; Freudenstein and Woo, 1974). In Freudenstein and Maki (1979) he introduced the concept of obtaining new devices by studying the structure. In Mayourian and Freudenstein (1984) he used his graph theory developments to show how to create an atlas of the kinematic structure of mechanisms.

In a prize winning paper he and his PhD student Wayne Sohn introduced the concept of dual graphs, and showed how to use them to deal with contracted graphs (Sohn and Freudenstein, 1984). In Fang and Freudenstein (1986, 1990) he introduced the concepts of a stratified representation and a hierarchical representation, respectively, to facilitate the automatic computer generation of mechanisms. In Vucina and Freudenstein (1991) he showed how to do synthesis using a combination of graph theory and nonlinear programming.

In addition to the above mentioned topics, Freudenstein published important works on the analysis and design of robot arms (Freudenstein and Primrose, 1984; Lin and Freudenstein, 1986; Tsai and Freudenstein, 1989), on classical Cardonic motion and its generalizations (Freudenstein, 1960, 1975; Freudenstein, Primrose and Chen, 1994), on type determination of spherical (Freudenstein, 1964c) and skew four-bars (Freudenstein and Kiss, 1969), on the kinematic analysis of the human knee joint (Freudenstein and Woo, 1969), on cam design and analysis (Freudenstein, 1960b; Freudenstein and Buchsbaum, 1973; Barzak and Freudenstein, 1979; Pisano and Freudenstein, 1983; Freudenstein, Tsai and Maki, 1983; Hanachi and Freudenstein, 1986; Fabien, Longman and Freudenstein, 1994), on general constraints (Freudenstein and Alizade, 1975) and on Chebyshev polynomials in synthesis (Freudenstein, 1976b).

Modern Interpretation of Main Contributions to Mechanism Design

In North America, Freudenstein is regarded as the father of modern kinematics. He is credited with leading the study and practice of the kinematics of mechanisms into the digital computer age. Freudenstein brought a scientific approach to the subject and introduced mathematical tools that had not heretofore been used in kinematics. He was among the first to utilize the power of digital computation in solving kinematics problems. Through his exemplary research and teaching, he created a large family of engineers, teachers and researchers that carried his ideas and methods to universities and corporations throughout North America and many other parts of the world. These Freudenstein disciples were both his own students and the countless others with whom Freudenstein and his students had various forms of professional interactions.

Freudenstein's work has had a lasting impact on many areas of the theory and practice of machines and mechanisms. It is impossible to overstate his influence on university education and research. The list of 500 PhD descendants in Table 1 gives only a partial indication of the vast spread of Freudenstein's influence.

In July of 1991, in celebration of Ferdinand Freudenstein's 65th birthday, a special meeting was held near Minneapolis, Minnesota to honor him. In connection with this event, in 1993 a book was published titled *Modern Kinematics: Developments in the Last Forty Years* (edited by Arthur G. Erdman; published by John Wiley & Sons, Inc., New York). This book is a review of the significant developments in the 40 years since the influence of the digital computer was first felt on the field. It contains over 600 pages, and was the work of 13 chapter editors and 60 authors, all organized by a Freudenstein academic grandchild, Professor Arthur Erdman of the University of Minnesota. This book gives a thorough assessment of the state of the art in the early 1990s. Since it covers a period that coincides essentially with all of Freudenstein's professional life, it is an excellent and unique resource to help assess the impact of Freudenstein's work and personality on the broad field of kinematics and mechanism design.

What is clear, when one looks at the full scope of the development presented in *Modern Kinematics: Developments in the Last Forty Years*, is that Freudenstein's work was at the starting point of many of the big problems that have been dealt with in the course of the field's development during the period that started in the early 1950s and has continued through the 1990s into the present day.

Acknowledgements

The author is very grateful to Freudenstein's wife, Lydia, and to his brother-in-law, Dr. Felix Wimpfheimer, for providing material related to the family history. I am indebted to Prof. Pierre Larochelle for allowing me to include a version of the Freudenstein academic family tree, which he so painstakingly assembled in homage to Freudenstein's memory.

KURT HAIN
(1908–1995)

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Abstract. Dr.-Ing. E. h. Kurt Hain was a pioneer of Applied Kinematics in Germany. He started his scientific career in his native town Leipzig. Between 1936 and 1994 he wrote 13 books and 380 scientific papers and designed 176 mechanism models. He worked in nearly all subfields of kinematics and machinery. There are notions, procedures and methods that are strongly connected with his name. His kinematic heritage for future generations of mechanical engineers is now preserved in the framework of a national research project in Germany, named "DMG-Lib".

Biographical Notes

Kurt Hain (Figure 1) was born in Leipzig (Saxony), Germany, on May 24, 1908. After having finished his apprenticeship as a mechanic and turner he studied mechanical engineering at the "Höhere Maschinenbauschule" (Higher School of Mechanical Engineering) in his native town.

Fig. 1. Dr.-Ing. E. h. Kurt Hain.

M. Ceccarelli (ed.), Distinguished Figures in Mechanism and Machine Science, 183–215.

The then very well-known expert in kinematics Paul Knechtel taught him and evoked in him an interest in mechanism theory as basic knowledge for the fast developing mechanization period. This was a key experience for Kurt Hain and fixed his future professional career. After his studies he worked from 1931 on with different industrial companies in Leipzig, Dresden and Dessau until he moved in 1939 together with his family to Braunschweig where he started to work as a research and test engineer in the "Luftfahrt-Forschungsanstalt" (Aviation Research Institution). This institution changed after World War II into the "Forschungsanstalt für Landwirtschaft" (FAL) (Research Institution of Agriculture). In 1948 Kurt Hain became group leader and in 1961 departmental leader of kinematics in the "Institut für landtechnische Grundlagenforschung" (Institute of Agro-Technical Basic Research) where he first made major improvements in the design of tractor-drawn farm implements. He retired in 1973.

In summarizing Hain's list of publications, we note that he presented his first paper in 1938 to the community of mechanism theory at the age of 30. It was titled "Geschwindigkeitsverhältnisse sämtlicher Koppelpunkte eines gegebenen Gelenkvierecks" (Velocity ratios of all coupler points of a given four-bar) and appeared in the German journal *Maschinenbau – Reuleaux-Mitteilungen/Archiv für Getriebetechnik (RM-AfG)* (Hain, 1938c). This journal was edited by the "Reuleaux-Gesellschaft" (Reuleaux Society) being part of the "Fachgruppe Getriebetechnik" (Expert Group Kinematics) within the "Verein Deutscher Ingenieure" (VDI) (Association of German Engineers). Editor-in-chief was Rudolf Beyer from Zwickau (Saxony), at that time spokesman of the German kinematicians. So it was a great honour for Kurt Hain, who had not received a university education, to be given this chance for prominent publication. Actually, Kurt Hain had written and published four papers before (Hain, 1936, 1937, 1938a, 1938b), but he never included them in his list of publications. Nevertheless – in the author's opinion – they are interesting enough to be mentioned and discussed here later.

To Hain's last publication in 1994 titled "Getriebe für die Massenfertigung – Getriebe-Bewertungen durch einfache und eindeutige Kennwerte" (Mechanisms in Mass Production – Assessments of Mechanisms by Simple and Clear Characteristics) (Hain, 1994) he assigned his own number 361; but there are more, because he omitted reviews of lecture notes and books from colleagues and of conferences as well as papers for teaching courses. So, looking back at his lifework (Kerle et al., 2006) we have 380 papers, 13 books,

Table 1. List of books written by Kurt Hain.

No.	Title	Publisher	Year
1	Angewandte Getriebelehre	Schroedel-Verlag, Hannover	1952
2	Die Feinwerktechnik	Fachbuchverlag Pfanneberg, Gießen	1953
3	Angewandte Getriebelehre, 2. Aufl.	VDI-Verlag, Düsseldorf	1961
4	Getriebelehre – Grundlagen und Anwendungen, Teil I: Getriebe-Analyse	Hanser-Verlag, München	1963
5	Applied Kinematics, 2nd ed.	McGraw-Hill Company, New York	1967
6	Getriebe-Atlas für verstellbare Schwing-Dreh-Bewegungen	Vieweg-Verlag, Braunschweig	1967
7	Einflüsse von Gelenkspiel und Reibung auf die im Getriebe wirkenden Kräfte (Fortschritt-Berichte VDI-Z, Reihe 1, Nr. 17)	VDI-Verlag, Düsseldorf	1969
8	Atlas für Getriebe-Konstruktionen	Vieweg-Verlag, Braunschweig	1972
9	Getriebebeispiel-Atlas – Getriebebeispiele für den Konstrukteur	VDI-Verlag, Düsseldorf	1973
10	Getriebetechnik – Kinematik für AOS- und UPN-Rechner	Vieweg-Verlag, Braunschweig	1981
11	Gelenkgetriebe-Konstruktion mit HP Serie 40 und 80 (together with H. Schumny)	Vieweg-Verlag, Braunschweig	1984
12	Gelenkgetriebe für die Handhabungs- und Robotertechnik	Vieweg-Verlag, Braunschweig	1984
13	Getriebeberechnungen für hohe Ansprüche mit Ausnutzung der Koppelkurven-Krümmungen (Fortschritt-Berichte VDI-Z, Reihe 1, Nr. 155)	VDI-Verlag, Düsseldorf	1987

2 contributions to handbooks, 164 reports or summaries and at least 23 sets of lecture courses in kinematics that he had written or composed. The list of his books is given in Table 1.

Two lists of Hain's publications are found in the German literature of mechanism theory (Hain, 1979; Kerle, 1988) from which we can conclude that Kurt Hain did research work on almost every field of kinematics: planar and spatial linkages and cam mechanisms, systematics, kinematic and dynamic analysis and synthesis of mechanisms. His fields of research can be broadly classified into some 26 special categories according to Table 2.

Moreover, starting in 1959 and ending in 1973 with his retirement, he designed 176 mechanism models. Most of them were actually built, but not all of them could be kept or saved. The number of models still existing and

Table 2. Categories of research activities of Kurt Hain.

General category	Special category/method
A. Systematics (type and number synthesis)	1. Mechanisms with more or less than one degree of freedom (d.o.f.)
	2. Band or belt mechanisms
	3. Equivalent mechanisms
	4. Sliding-pair mechanisms
	5. Screw mechanisms
B. Kinematic analysis	6. Mechanisms with gears
	7. Ratchet mechanisms and index mechanisms
	8. Radius of translation
	9. Following-the-drawing-calculation method
C. Dynamic analysis	10. Mechanisms with springs
	11. Pole force method
	12. Transmission angle
	13. Effects of joint backlash
D. Kinematic synthesis	14. Point position reduction
	15. Dwell and pilgrim-step linkages
	16. Point-dwell mechanisms
	17. Cognates
	18. Drag-link mechanisms
E. Cam mechanisms	19. Cam-linkage design
	20. Cams with pure rolling contact
F. Applications (applied kinematics)	21. Computing mechanisms
	22. Mechanisms for use in agricultural machines and transport devices
	23. Mechanisms for use in machine tools
	24. Mechanisms for use in clamping and handling devices and robots
G. Catalogues and models	25. Design charts, mechanism catalogues and atlases
	26. Mechanism model collection

preserved now at the Professorship of Kinematics of the Technical University of Dresden runs up to 76.

There are notions and methods that originate from Kurt Hain, for example the *Drehschubstrecke* (*radius of translation*), a *Punktrastgetriebe* (*point-dwell mechanism*), the *Polkraftverfahren* (*pole force method*), the *Punktlagenreduktion* (*point position reduction*) and the *Zeichnungsfolge-Rechenmethode* (*following-the-drawing-calculation-method*). In 1965 during the course of development of Konrad Zuse's invention, large mainframe computers were introduced in Germany and Kurt Hain seized the opportunity to prepare the publication of his first "Getriebeatlas" (mechanism atlas) (Hain, 1967a) in cooperation with some of his institute research employees. This first catalogue concentrates on the possibilities of the well-known four-bar linkage. Two atlases followed (Hain, 1972, 1973). In 1969 Hain himself bought a desk micro-computer of the type "Tektronix 31", put it on the desk in his home and started to write simple programs for the analysis and synthesis of mechanisms. The Tektronix machine could repeat steps that had been preprogrammed, using the alphanumeric and functional buttons of the keyboard. The programs could be stored on a magnetic card and data could be printed out and read on a small paper roll printer. In 1975 Hain replaced the Tektronix machine by the pocket computers HP 67 and HP 97 from Hewlett-Packard and TI 59 from Texas Instruments with the possibility to plug in ROM-modules for programs. Again five years later Hain bought the top pocket computer product of Hewlett-Packard, the HP 41C, which were soon followed by the upgraded HP 41CV and HP 41CX. All these pocket computers could be upgraded by plugging in memory or ROM-program modules. The types HP 41CV and HP 41CX additionally could be connected with a small magnetic tape device.

The micro- and pocket computers formed the geometrical base for Kurt Hain to demonstrate the efficiency of his *Zeichnungsfolge-Rechenmethode* and to improve it, which he used for the analysis (Hain, 1975) as well as for the synthesis of mechanisms (Hain, 1983). On this basis, Hain could check his numerical results from time to time by simply using pencil, ruler and set square and thus going back to his early years of research. An interesting historic overview of the development from purely graphical methods to the *Zeichnungsfolge-Rechenmethode* is partly written and explained by Hain himself in one of his papers (Hain, 1988).

In spite of being convinced of the efficiency of numerical methods in mechanism theory Kurt Hain refused to deal with the latest computer generation, the personal computers (PCs). This is due on the one hand to the necessity of writing long and tedious program lines in a proper language – at least at that time; on the other hand he distrusted the PC which in his opinion executed too many uncontrollable operations beyond handling mathematical algorithms. In his book *Applied Kinematics* (Hain, 1967b) we read his statements of the matter on page 379:

1. *Computer programmers, while well-versed in the capabilities of the computer, cannot be expected to comprehend the complexities of other fields;*

the problem must be defined and formulated by the designer before it can be handed to the programmer.

2. *There is a common misconception that the introduction of computers will reduce the need for specialists. This is not the case, particularly in the field of mechanisms, since it is not yet possible to express the best design in theoretical terms for the computer's use; there is still a need for the practical consideration.*

The first statement is still valid today; the second statement is not in its entirety, because for example there are powerful optimization procedures today which help to find among many possible solutions a mechanism to be adapted in the best way to a given kinematic task.

On October 6, 1957 Kurt Hain left Braunschweig, starting by plane in Hamburg and arriving in New York City via London, following an invitation of some of his American professional colleagues, the professors Allen S. Hall from Purdue University, Lafayette (IN), Ferdinand Freudenstein from Columbia University, New York City (NY), Richard S. Hartenberg from Northwestern University, Evanston/Chicago (IL) and Joseph S. Beggs from University of California, Los Angeles (CA) (Crossley, 1988). Hain made a coast-to-coast tour, gave lectures at several American universities, visited many industrial companies and discussed with technical executives and development engineers problems where solutions could be found on the basis of mechanisms. On this occasion Kurt Hain also participated in the 4th Conference on Mechanisms at Purdue University and presented two papers (Hain, 1957a, 1957b), the latter one being a report on German research activities on mechanism theory since 1945, the end of World War II. This report contains a list of 520 items (books and papers) from kinematicians in both former east and west parts of Germany. Hain returned to Braunschweig on January 10, 1958. An overview of the stations of his first travel to the USA is given in Figure 2.

Three further stays in the USA as visiting professor followed: 1961 at Yale University, New Haven (CT), 1965 and 1966 at Massachusetts Institute of Technology (MIT) in Cambridge (MA). With the already mentioned book *Applied Kinematics*, the English version of the German 2nd edition (Hain, 1961), Kurt Hain became very well-known in the USA. The book with 2412 reference items was edited by McGraw-Hill Corporation and Hain could rely on the help of his American colleagues and friends Douglas P. Adams from MIT, Thomas P. Goodman from Northwestern University, Frank E. Crossley,

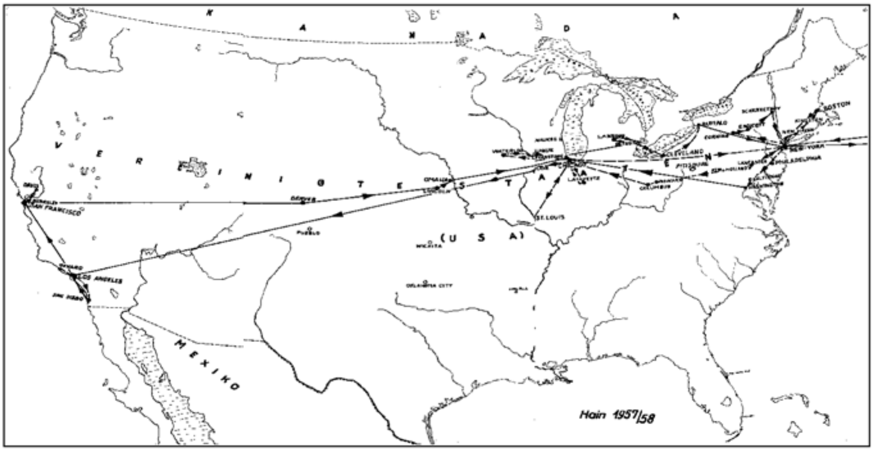

Fig. 2. Kurt Hain's itinerary of his visit to the USA in 1957/58.

Georgia Institute of Technology (GIT), Ferdinand Freudenstein again, Bruce L. Harding (Livermore Corporation) and Daniel R. Raichel (Maxson Electronics Corporation).

In addition, Hain temporarily gave guest lectures at the universities in Bologna (Italy), Delft and Eindhoven (Netherlands) and at the International Centre for Mechanical Sciences (CISM) in Udine (Italy). From 1967 till 1977 he belonged to the permanent teaching staff in Mechanical Engineering of the Technical University of Braunschweig.

Kurt Hain received many honours and awards. In 1963 the "Max-Eyth-Gesellschaft zur Förderung der Landtechnik" (Max-Eyth-Society for the Promotion of Agro-Technique) gave him the Max Eyth commemorative coin. The VDI paid tribute to Hain's merits in the field of kinematics and to his activities in promoting the VDI goals as regards the transfer of knowledge between universities and industrial companies: "VDI-Ehrenzeichen in Gold" (Golden Medal of Honour) in 1964, "VDI-Ehrenplakette" (Badge of Honour) in 1967, and "VDI-Ehrenmitglied" (Honorary Member) in 1981. The most important national awards were the Honorary Doctorate (Dr.-Ing. E. h.) of the Darmstadt Polytechnic in 1970 and the Cross of Merit 1st Class of the Federal Republic of Germany in 1971. In 1979 Kurt Hain became Honorary Member of the International Federation for the Theory of Machines and Mechanisms (IFToMM).

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Review of Main Works on Mechanism Design

Kurt Hain's books can be regarded as summaries of his previous work, results and discoveries. So, one possible way to review his main works (on the base of his main ideas) is by grouping them under the following headings:

- On the truly first published papers.
- Methods originating from Kurt Hain.
- Design charts, catalogues and atlases.
- Mechanism model collection.
- Three examples of selected highlights in research activities.

On the Truly First Published Papers

After Kurt Hain's death in 1995, four papers were found in his private archive. They were published in the *FTV-Nachrichten* (FTV News) of the "Freie Technische Vereinigung" (FTV) (Free Technical Union) at the "Höhere Maschinenbauschule" in Leipzig where Hain started his scientific education (Figure 3).

Today we can take these four papers as some exercise work for Hain as a newcomer before entering the expert group of kinematics of the famous Reuleaux-Society of the VDI. The papers are most probably unknown to the mechanism community of either yesterday or today.

First example (Hain, 1936). A simple four-bar linkage with link lengths a, b, c, d (fixed link d) is driven by a tangential input force P_a at (joint) point 2 (Figure 4).

This point as well as the points 3 and 5 belong to the coupler triangle with lengths b, e, f. The question is for the tangential output forces $P_c \equiv P_3$ and P_5 that act at points 3 and 5, respectively, and can be balanced by $P_a \equiv P_2$. Hain gives two different solutions, both are based on graphical versions (similar to the *Joukowsky-lever-method*) of the power theorem

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Kräfteübertragung in Kurbeltrieben

Von Kurt Hain

Fig. 3. Header of the *FTV-Nachrichten* in Leipzig presenting Kurt Hain's first publication.

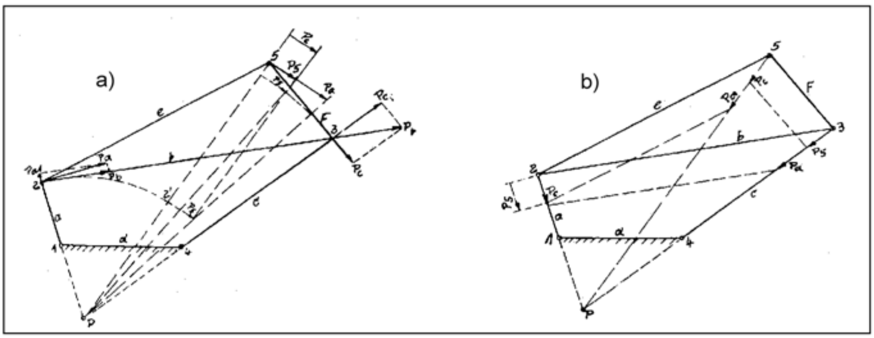

Fig. 4. Two possible graphical solutions of the power theorem.

$$\sum_i = (\vec{P}_i \cdot \vec{v}_i) = 0, \tag{1a}$$

as a sum of vector scalar products coupling forces P_i and velocities v_i. This product can be expressed as a sum of vector cross products

$$\sum_i = (\vec{\overline{v}}_i \times \vec{P}_i) = \vec{0}, \tag{1b}$$

(Figure 4a), if instead of the velocities $\vec{v}_i$ of the force acting points the perpendicular velocities $\vec{\overline{v}}_i$ are considered – the perpendicular velocities now appear as distances of the points in question to the common instantaneous velocity

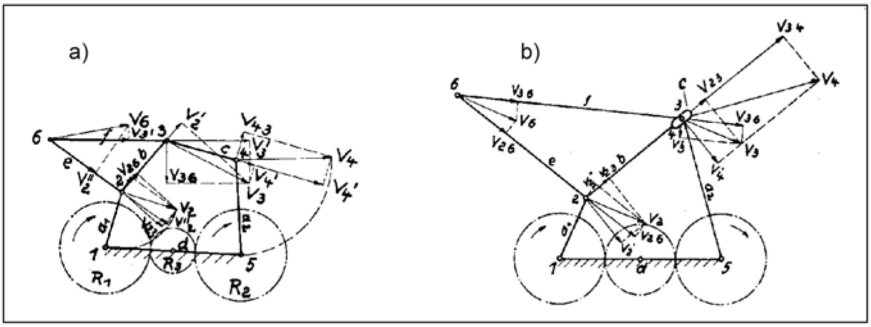

Fig. 5. Five-bar linkage with two gear-coupled cranks: (a) Five turning pairs, (b) four turning pairs, one sliding pair.

centre P of the coupler triangle 2-3-5. On the other hand, the power theorem may be written and treated as

$$\sum_i (\vec{P}_i \cdot \vec{v}_i) = 0, \tag{1c}$$

which leads to an alternative graphical solution (Figure 4b).

Second example (Hain, 1938a, 1938b). A five-bar linkage with link lengths a_1, b, c, a_2, d (fixed link d) is driven by the two cranks a_1 and a_2 with constant angular velocities ω_{a_1} and ω_{a_2}, respectively (Figure 5). The cranks are connected by gear wheels with pitch radii R_1 and R_2, respectively, thus we have a constant ratio $i = \omega_{a_1}/\omega_{a_2} = R_2/R_1$ of the two angular velocities, and the linkage has only one d.o.f. Link b belongs to the coupler triangle with lengths b, e, f. The question is for the linear velocities of the points 3 and 6.

Hain now investigates the following cases and discusses different possible solutions for (α) $i = +1$, $\omega_{a_1} = 1$ rad/s, (β) $i = -1$, $\omega_{a_1} = 1$ rad/s and (γ) $i < 0$, but $\neq -1$, $\omega_{a_1} = 1$ rad/s on the basis of the superposition theorem, paying respect to the fact that two points of a rigid plane have the same velocity in direction of the connecting line of the two points regarded as the first component; the second component is perpendicular to the first (Figure 5a). In a second step Hain replaces the link c by a slider and thus introduces a sliding pair between links b and a_2 (Figure 5b) and presents and explains solutions for the cases (α), (β), (γ).

We must keep in mind that kinematicians in the past had to be able to do good and clear drawings, because the solutions of kinematic problems

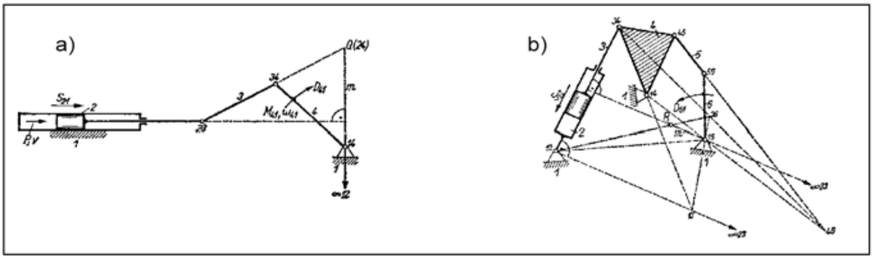

Fig. 6. Determination of the radius of translation m with a four-link slider-crank (a) and a six-link mechanism of the Watt chain type (b).

were essentially based on graphical methods and it must be emphasized again that especially Kurt Hain in his further scientific career never really left the graphical world because of his characteristic imaginative power. With his eyes closed he could see a mechanism running!

Methods Originating from Kurt Hain

Radius of translation. The *radius of translation m – Drehschubstrecke* in German – is given as a unit of length and is a special form of the transmission ratio between two moving links of a mechanism, in case one of the two links performs rotary motion with angular velocity ω and the other one translational motion with linear velocity v. Additionally, if we assign a torque M to the rotating link and a force P to the sliding link, we can derive from the power theorem

$$m = \frac{v}{\omega} = \frac{M}{P}\,. \tag{2}$$

A simple example is given with the slider-crank mechanism of Figure 6a (Hain, 1967b). Symbols S_{21} and D_{41} correspond to the motions of the links 2 (slider) and 4 (crank) with reference to the fixed link 1 (frame) of the mechanism. These symbols may be arranged as follows:

$$\left.\begin{matrix} S_{21} \\ D_{41} \end{matrix}\right] 24.$$

The vertical order of the index subscripts yields pole $Q \equiv 24$, the (ideal) instantaneous centre of rotation of link 2 relative to link 4. The pole Q serves to find the *radius of translation* $m \equiv m_{21-41}$ being the (directed) distance 14-24 of the two poles 14 and 24.

Far more interesting and useful for analytic purposes is the determination of the *radius of translation* in cases where there are relative velocities between moving links. In the six-link mechanism of Figure 6b, translation S_{32} of input piston 3 relative to cylinder 2 is executed with rotation D_{61} of output link 6 relative to frame 1. The vertical arrangement of the index subscripts yields

$$\left.\begin{matrix} S_{32} \\ D_{61} \end{matrix}\right] \text{36-12}$$

which defines a so-called *collineation axis* 36-12 that must be located following the *Kennedy–Aronhold theorem* of three corresponding poles all lying on a straight line. One possible solution is drawn in Figure 6b: the *collineation axis* 36-12 intersects with another one ∞23-16 given by S_{32} and D_{61} in point R; the distance 16-R is the (directed) *radius of translation* $m \equiv m_{32-61} = v_{32}/\omega_{61} = M_{61}/P_{32}$, according to equation (2). The *radius of translation* may also profitably be applied with the determination of angular accelerations in mechanisms and for synthesis purposes of four-bar linkages (Hain, 1963b, 1966a).

Pole force method. The graphical *pole force method* is also based on the power theorem and relates input and output forces or torques in a mechanism with one d.o.f. The method results in a very compact diagram of forces, normally makes use of the *Culmann intermediate resultant* and can lead in steps thereafter to an easier and faster determination of joint forces.

To begin with, the six-link mechanism in Figure 7a is loaded by a weight force G_0 on link 6; on the other hand we have an opposing force P on link 4 (Hain, 1967b). Because G_0 acts from link 1 (frame) to link 6, it is written as $G_{0/16}$ and P is written as P_{14}. Now, the force notations are arranged as follows:

$$\left.\begin{matrix} G_{0/16} \\ P_{14} \end{matrix}\right] 46.$$

The vertical order of the different subscript digits results in pole 46 through which an arbitrarily positioned "K-axis" k_{46} is drawn. Pole 46 occurs as the instantaneous centre of links 4 and 6. Pole 14 about which force P generates a torque $M_P = P \cdot r$ is the fixed pivot of link 4. Weight force G_0 produces a torque $M_G = G_0 \cdot h$ about the instantaneous pole 16. A force

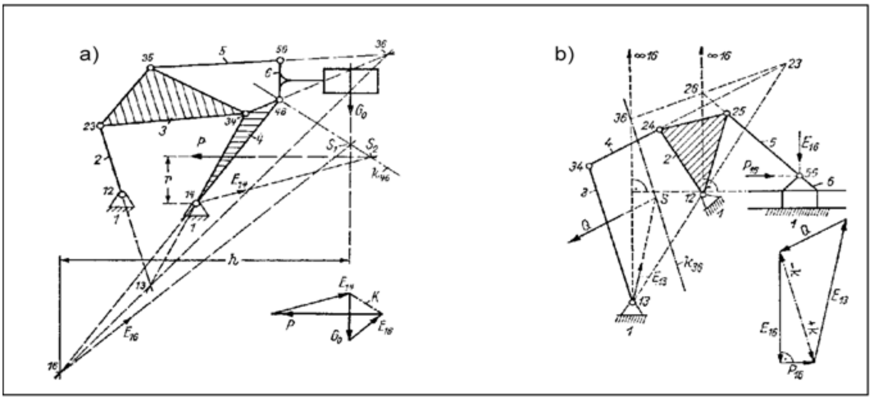

Fig. 7. Two six-link mechanisms (Watt chain types) loaded by external forces and their corresponding force polygons: with turning pairs only (a) and with one output slider (b).

transmitted from link 6 to link 4 through pole 46 must be resisted by an equal and opposite force transmitted from link 4 to link 6. This is the reason why we can draw an arbitrary axis k_{46} through 46 and resolve G_0 into a force K through 46 along k_{46} and a force E_{16} through pole 16. The desired force P is then the resultant of $-K$ (equal and opposite to K) and a force E_{14} through pole 14. The direction line of force G_0 intersects axis k_{46} at point S_1; the direction line of the desired force P intersects axis k_{46} at point S_2; forces P and E_{14} can now be determined from K in the force polygon, in which E_{14} runs parallel to line S_2-14. The four forces G_0, E_{16}, P and E_{14} must form a closed polygon, where K assumes the role of a *Culmann intermediate* or *auxiliary line*. Forces E_{14} and E_{16} are therefore auxiliary forces which are assumed to produce a state of equilibrium between G_0 and P at poles 16 and 14. It should be noted that these auxiliary forces do not correspond to the actual joint forces; for example E_{14} does not denote the real pivot force G_{14}; this is due to the arbitrary choice of the K-axis k_{46} through pole 46.

The counter force P may also be determined from G_0 by making use of moment relations. In conformity with pole notations of forces – $G_{0/16}$ and P_{14} – torques about poles 16 and 14, respectively, can be expressed as

$$\frac{M_G}{M_P} = \frac{G_0 \cdot h}{P \cdot r} = \frac{16 - 46}{14 - 46} \,. \tag{3}$$

This equation can easily be solved for P.

Another example is given in Figure 7b with a slider as output link 6. Force Q, specified in magnitude and direction, is exerted by frame 1 on input crank 3, and the opposing force P_{16}, exerted by frame 1 on slider 6, is to be determined. The two forces may be arranged as follows:

$$\left.\begin{matrix} Q_{13} \\ P_{16} \end{matrix}\right] 36.$$

The vertical order of the differing subscript digits yields the pole 36. An arbitrary line k_{36} is drawn through this pole, intersecting at point S the direction line of force Q. A force triangle is then constructed with sides Q, a force $+K$ in the direction of k_{36}, and a third force E_{13} with direction parallel to line segment S-13. Then a second force triangle is constructed from $-K$ and forces E_{16} and P_{16} (the latter two normal to each other); the direction line of P_{16} runs parallel to the linear path of link 6, and the direction line of E_{16} is towards pole 16 which is at infinity. Thus the magnitude of P_{16} is obtained.

Because of the fact that the *pole force method* locates almost all possible instantaneous centres of velocity at the beginning, it is a very convenient method in case more than one external forces are acting on a given mechanism and their effects must be superimposed (Hain, 1964a). Intermediate forces in the mechanism normally need not be considered. The *pole force method* can also be used to determine the dimensions of a mechanism for a prescribed state of force.

Point position reduction. In earlier treatments of dimensional synthesis, quantitative requirements clearly were rather restricted. The *point position reduction* method is a tool for the dimensional synthesis of four-bar linkages A_0ABB_0. We know from the Burmester theory that by means of the *centre point* and *circle point curves* it is possible to control four desired positions of the coupler and, depending upon the intersection of two of these curves, sometimes five positions. But it takes a considerable amount of pains to make the Burmester theory practicable.

Already in 1943 Hain found a far simpler graphical method making use of the idea to let coincide two or more points of the given linkage positions in one and the same point (Hain, 1943). The *point position reduction* method requires us to draw circles only for the link end points A or B, respectively, through three points, which is always a definite task. The goal of the method is therefore to be able to pass a circle through three of four points where these few points correspond to a considerably greater number of positions. Later

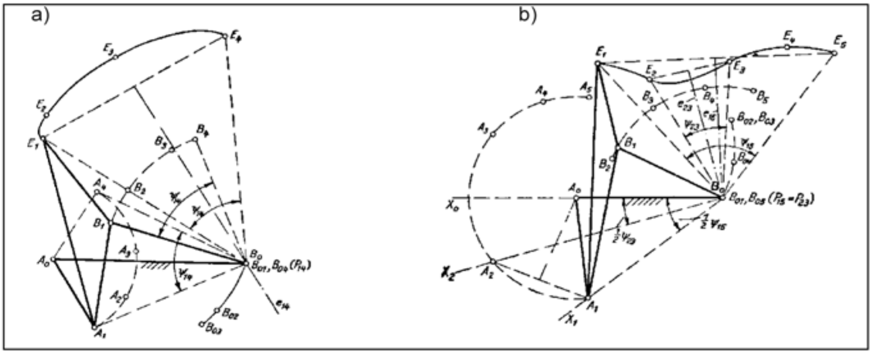

Fig. 8. Point position reduction with a four-bar linkage for four (a) and five point positions (b).

on, Hain extended his method by writing programs for pocket computers up to six given coupler points (Hain, 1980). In special cases it is possible to give even eight coupler points. The highest theoretical number of point positions that the four-bar linkage can be made to correspond to is nine.

In Figure 8a four point positions E_1, E_2, E_3, E_4 are given which are to be traversed by coupler point E of a four-bar linkage (Hain, 1967b). We let fixed point B_0, the centre for rocker B_0B, be assumed anywhere on the perpendicular bisector of line E_1E_4. Now, if the two crank positions A_0A_1 and A_0A_4 are made symmetric to fixed link A_0B_0, with A_1 and A_4 on a circular arc around B_0, then B_0 represents the (finite) turning pole P_{14} of the two coupler positions A_1B_1 and A_4B_4. All correlated points of the two positions of the coupler plane, for example E_1 and E_4, A_1 and A_4, B_1 and B_4, lie on a circular arc, respectively, around B_0 and enclose with this point an angle ψ_{14}.

Hence, a four-bar linkage for the given points E_1 to E_4 can be constructed in the following way: B_0 is assumed to be a point on anyone of the six possible perpendicular bisectors of these points. The intersection of a circular arc of any size ($B_0A_1 = B_0A_4$) around B_0 with a circular arc of any size ($A_1E_1 = A_4E_4$) around E_1 and E_4 yields points A_1 and A_4. Point A_0 can be assumed as any point on the perpendicular bisector of A_1A_4, and thus radius A_0A is established. Points A_2 and A_3 are determined as intersections of the crank circle and circular arcs around E_2 and E_3 with radius A_1E_1. Further, the triangle $E_1A_1B_{02}$ is made congruent to the triangle $E_2A_2B_{01}$, and the triangle $E_1A_1B_{03}$ congruent to the triangle $E_3A_3B_{01}$. Points B_{01} and B_{04} coincide because triangles $E_1A_1B_{04}$ and $E_4A_4B_{01}$ are congruent due to the special choice of B_0 as P_{14}. The centre of the circular arc through the three points $B_{01} = B_{04}$, B_{02}, B_{03} gives point B_1 on the rocker B_0B. Because of the many assumptions in the described method, there is an infinite number of four-bar linkages of the required kind from which the most favourable can be selected.

For five given point positions E_1, E_2, E_3, E_4, E_5, an infinite number of four-bar linkages can be derived in an equally simple way, say, by choosing B_0 as the intersection point of two perpendicular bisectors of line segments joining two given point pairs. If the order of the points is given (Figure 8b) in which the positions of coupler point E will be traversed in the plane, there are five possibilities for point B_0 as the intersection point of two perpendicular bisectors, namely of E_1E_5 and E_2E_3, E_1E_5 and E_2E_4, E_1E_5 and E_3E_4, E_1E_4 and E_2E_3, E_2E_5 and E_3E_4. In Figure 8b, B_0 has been chosen as the intersection of the perpendicular bisectors e_{15} and e_{23}; thus, B_0 is simultaneously pole P_{15} of coupler positions 1 and 5 and pole P_{23} of coupler positions 2 and 3. All correlated points of positions 1 and 5 of the coupler plane, for example, E_1 and E_5, A_1 and A_5, B_1 and B_5, enclose the angle ψ_{15} with B_0. All correlated points of the coupler positions 2 and 3 enclose the angle ψ_{23} with B_0. Any bundle of rays B_0X_0, B_0X_1, B_0X_2 is drawn through B_0 such that angle $X_0B_0X_1 = \psi_{15}/2$ and angle $X_0B_0X_2 = \psi_{23}/2$. These angles are described in the same sense as angle $E_5B_0E_1$ and $E_3B_0E_2$, respectively. We now strike circular arcs of any radius $A_1E_1 = A_2E_2 = AE$ around E_1 and E_2 cutting rays B_0X_1 and B_0X_2 in A_1 and A_2. The perpendicular bisector to A_1A_2 cuts B_0X_0 in A_0. The radius $A_0A_1 = A_0A_2$ thus defines crank length A_0A. We find A_3, A_4, A_5 on the crank circle as was shown above. The crank positions A_1 and A_5 as well as A_2 and A_3 lie symmetrically to position A_0B_0 according to the choice of point B_0. Now, if the triangle $E_1A_1B_{05}$ is made congruent to the triangle $E_5A_5B_{01}$, the triangle $E_1A_1B_{02}$ congruent to the triangle $E_2A_2B_{01}$, the triangle $E_1A_1B_{03}$ congruent to the triangle $E_3A_3B_{01}$ and the triangle $E_1A_1B_{04}$ congruent to the triangle $E_4A_4B_{01}$, then B_{01} coincides with B_{05}, and B_{02} coincides with B_{03}. The point B_1 on the rocker B_0B is again the centre of a circle through the three points $B_{01} = B_{05}$, $B_{02} = B_{03}$, B_{04}. An infinite number of four-bar linkages result here also through the arbitrary choice of the bundle of rays B_0X_0, B_0X_1, B_0X_2 and the arbitrary choice of length AE.

Point-dwell mechanism. Following Kurt Hain a *point-dwell mechanism* is a six-link mechanism in which a point of a special coupler plane stands approximately still for a finite period during continued motion of the mech-

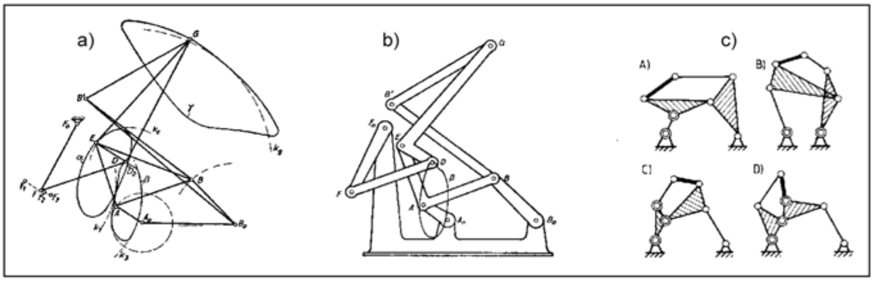

Fig. 9. Design of a six-link point-dwell mechanism: (a) points and curvature circle segments, (b) practical form, (c) possible structures.

anism. In Hain (1967b), he states that this condition is only possible with the mechanism shown in Figure 9a: a six-bar linkage of the Watt chain type based on a four-bar A_0ABB_0 with frame link A_0B_0 and a point E on the coupler link AB is driven by the crank A_0A. Links EG and $B'G$ are then added, where B' is a point on link B_0B. Choosing proper dimensions one can make a point D on the coupler plane EG be at rest for a while. The coupler plane EG is not connected to any link which has a fixed pivot; thus, it is possible that point D at position D_2 can be the centre of a curvature circle k_e to the coupler curve α of point E and, simultaneously, the centre of another curvature circle k_g to coupler curve γ of point G. If an arbitrary coupler link DF is connected at point D and then to output crank F_0F, these links will remain at rest when point D is at position D_2. Because of the arbitrary lengths of links DF and F_0F an infinite number of point-dwell mechanisms are possible, all driven by point D. The mechanism as a whole has eight links and subsequently one d.o.f. A practical form of it is shown in Figure 9b.

In 1981 Kurt Hain completed the structure of the special six-bar linkage with two fixed pivots for use as a point-dwell mechanism by differing between binary input links (cranks) (linkages A and B in Figure 9c) and ternary input links (linkages C and D in Figure 9c) on the one hand and, on the other hand, between revolving and only swinging turning pairs, marked by double circles and single circles in Figure 9c, respectively (Hain, 1981b).

Design Charts, Catalogues and Atlases

In 1951 J. A. Hrones and G. L. Nelson published their voluminous design chart book *Analysis of the Four-bar Linkage* with Wiley & Sons, Inc. in New

York City. The look at the pages of this book with dash-drawn coupler curves of four-bars having different dimensions was not exciting, but for Kurt Hain it was a key experience and meant the beginning of his development of design charts and thus helped the designer to find optimal mechanism solutions. Ten years before, Hermann Alt from Dresden had already shown the usefulness of the transmission angle as a criterion for the transmission of power from the input to the output link. Alt transformed his results to design chart curves for easy and overview use.

Hain's first design charts were for use with tractors and single axle trailers connected to them by one-point-couplings. Hain called the curves traversed by points of the tractor or the trailer "Schleppkurven" (tracking curves) (Hain, 1952b). Two years before, he had elaborated the (non-holonomic) theory of tracking curves on the base of two centrodes, the fixed and the moving one, rolling on each other (Hain, 1950). Ten years later Kurt Hain and his colleague and friend Johannes Volmer from Chemnitz (at that time Karl-Marx-Stadt in the former German Democratic Republic, DDR) followed the ideas of Hermann Alt and cooperated under the roof of the VDI in Düsseldorf to produce a series of design charts concerning adapted input/output motion for all kinds of four-bar linkages. The results were presented in several VDI guidelines for mechanical engineers. In the focus of interest is the transmission angle μ as the best value for characterizing the power transmission from the input to the output link.

A most interesting overview over design charts for four-bar linkages and their authors in Germany up to 1960 is given in Hain (1960a). Here, also some other respectable names occur, like Willibald Lichtenheldt, Kurt Luck, Walther Meyer zur Capellen, Helmut Rankers and Gerd Kiper, for example.

Again ten years later the digital computer with a corresponding plotter for digital data processing became an important tool for mechanism design in Germany (and worldwide). Hain and his young collaborator Michael Graef worked night and day to produce various design charts, especially concerning relative angles between links in four-bars. But in 1968, on the occasion of the 10th ASME Mechanisms Conference in Atlanta (GA), USA, Kurt Hain showed convincingly that his charts for simple four-bars could be combined in such a way that also six-bar linkages on the base of four-bars (of the Watt chain type, for example) were optimally designed for a given task (Hain, 1968). An example to demonstrate this is given in Figure 10. On the left side (A) we have the notions of links and angles in a four-bar linkage moving

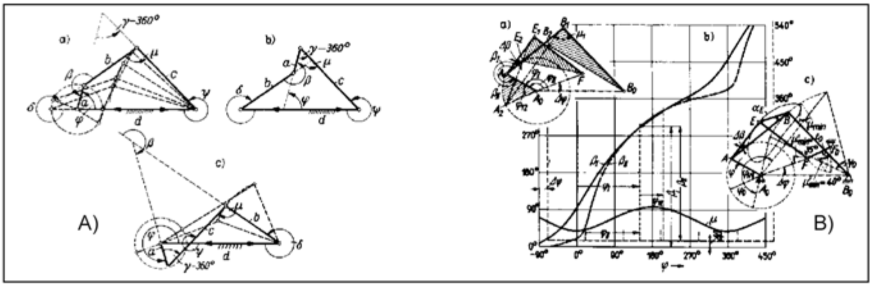

Fig. 10. Notions of links and angles in a four-bar (A), use of β-angle curves of two different four-bars to create a six-bar dwell mechanism (B).

as a crank-and-rocker (a). There are six different angles, namely φ (input), β, μ (transmission angle), ψ (output), δ and γ defining five relative motions. Arranging the same links in other sequences as in (b) and (c), we recognize that all angles between same links remain unchanged. On the right side (B) two four-bars $A_0A_1B_1B_0$ and $A_0A_1E_1F$ with the same crank A_0A and the coupler plane $A_1E_1B_1 = AEB$ can be moved as long as their relative angles β_I between links A_0A_1 and A_1B_1 and β_{II} between links A_0A_1 and A_1E_1, respectively, coincide, say, there is no relative angular motion between the corresponding links (a). This can be controlled by the proper β-curves of the two four-bars (b); these curves may be shifted, but the coordinate axes must be kept parallel one to another. The length $E_1F = EF$ is now chosen as the radius of curvature of the coupler curve of point E, and a double-lever EFF_0 is connected by rotary joints to point E. The result is a six-bar dwell mechanism (c). The duration of the dwell is given with φ_{12}, the angle of the coupler triangle AEB runs $\Delta\beta = \beta_{II} - \beta_I$, the angle of the base triangle $\Delta\varphi = \varphi_{II} - \varphi_I$. The quality of motion/force transmission is characterized by the minimum values $\mu_{\min}$ and $\mu'_{\min}$, as indicated (b).

Kurt Hain especially did pioneer work in the field of *planar mechanism systematics*. The famous Franz Reuleaux (1829–1905) was his example, and Hain wanted to follow Reuleaux's footsteps concerning the classification features of mechanisms and their elements. Hain's recipe to develop mechanisms and to present them in the form of catalogues may be summarized as follows:

1. One single kinematic chain can be the origin of a series of mechanisms by simply fixing one link to the base, one after another.
2. Turning pairs can be replaced by sliding pairs (one d.o.f.).

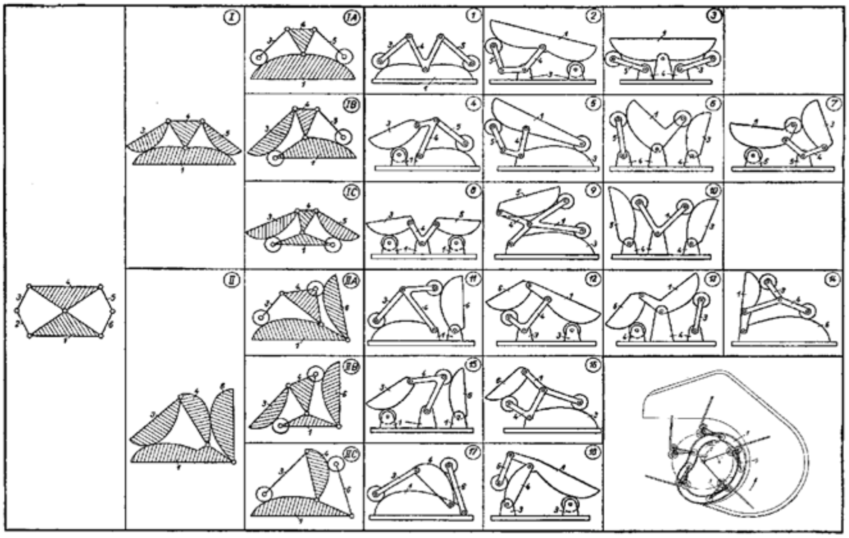

Fig. 11. Mechanisms with two cam joints derived from the Watt chain and application example (pick-up component of a hay press with fixed cam).

3. Single joint distances can be set to zero, thus double and triple joints are generated and the number of links is reduced.
4. A cam joint with two d.o.f. can be replaced by a binary link with two turning or sliding pairs (one d.o.f.) at its ends, and vice versa.

Based on these rules of design it becomes possible to create varieties of mechanisms that can be listed in mechanism catalogues. Sometimes it is necessary to pay attention to consequences, as for example with mobility restrictions.

In 1955 Kurt Hain published his first paper on systematics; it was focused on eight-link mechanisms and several examples, mainly for use in agricultural machines, were included (Hain, 1955). Wonderful catalogues about cam-linkage combinations are given in Hain (1960b) from which an example is presented in Figure 11: starting from the Watt chain, 18 mechanisms with 2 cam joints are derived; the mechanism no. ① is exploited for use as a pick-up component of a hay press with fixed cam and two roller paths (right side, below).

Also very interesting and unique at the time of origin is Hain's catalogue about four-, five- and six-link (pure) sliding-pair mechanisms (Hain, 1966b). An example: Figure 12 shows a series of sliding-pair mechanisms with four

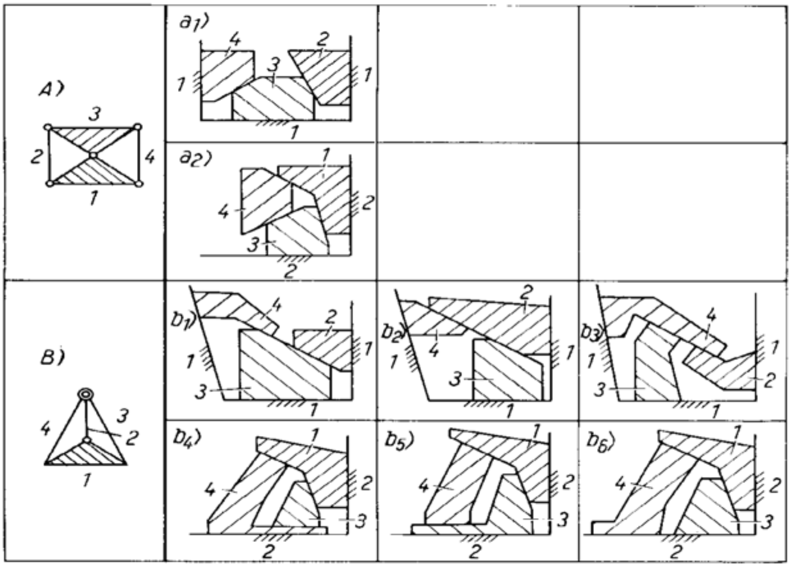

Fig. 12. A number of four-link sliding-pair mechanisms developed from two different kinematic chains A and B.

links and five joints. The normal one-joint versions are drawn above (A), the mechanisms with a double sliding pair below (B). General remarks about the transition from systematics to kinematics of mechanisms can be read in Hain (1976a).

A combination of design charts, catalogues and application examples is called "Getriebeatlas" (mechanism atlas) by Kurt Hain. It is his highest level of compact and summarized knowledge of a mechanism or a special group of mechanisms, a step being just before the construction of a mechanism model. Hain published three atlases; the first two dealt with four-bar linkages and with combinations of four-bar linkages, respectively (Hain, 1967a; 1972). The last one from 1973, the *Getriebebeispiel-Atlas* (mechanism example atlas) is the best (in the author's opinion) as regards the variety, types and number of application examples (Hain, 1973a). Hain finished this atlas just before his retirement. At the same time this atlas is an overview of almost all mechanism models that Kurt Hain designed since 1959.

According to the headings of the ten chapters the contents of the *Getriebebeispiel-Atlas* are as follows:

1. Mechanisms for oscillation-rotary and oscillation-linear motions

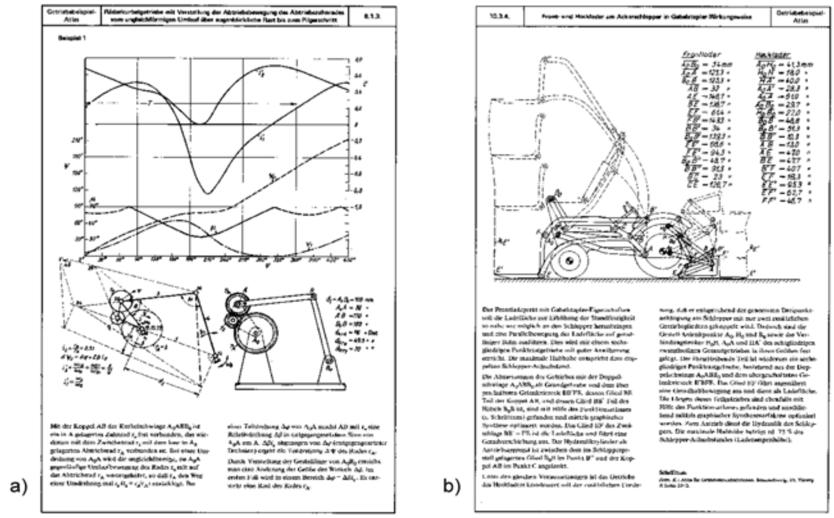

Fig. 13. Two pages from Kurt Hain's *Getriebebeispiel-Atlas* (1973): (a) Geared four-bar linkage, (b) tractor with front- and rear-forklift.

2. Mechanisms with revolving output link
3. Mechanisms representing coupler curves, cycloidal trajectories or polodes
4. Guidance mechanisms
5. Oscillating dwell mechanisms
6. Revolving dwell mechanisms (indexing mechanisms)
7. Spatial mechanisms
8. Adjustable mechanisms (with more than one d.o.f.)
9. Spring-loaded mechanisms
10. Examples of application

In Figure 13a the page no. 8.1.3 (Beispiel 1) of Hain's last atlas is shown, with a geared four-bar linkage A_0ABB_0. The rotation of the triangle crank (input shaft centre A_0) leads to a general motion of the coupler AB, but only the rotation of it is transmitted via gear wheels r_a and r_z to the output wheel r_A. So input and output link rotate around the same shaft centre A_0. By choosing proper dimensions it is possible to generate dwell or even pilgrim-step output motion. Above we see some characteristic functions of the linkage. Additionally, the dimensions of the drawn linkage are given, together with some important definitions.

In Figure 13b we look at page no. 10.3.4 of the atlas where a tractor with front- and rear-forklift is shown. The front-forklift is a six-link mechanism based on the double-rocker four-bar A_0ABB_0 and a second four-bar $BB'EF$ whose link BE is part of the coupler AB and whose link BB' is part of the rocker BB_0 of the first four-bar. By means of the functional curves in Hain (1972) and the *Roberts' theorem*, the dimensions of the two four-bars are chosen in such a way that points E and F move on an approximately straight line $k_E = k_F = k_{E'}$ from the ground to the maximal height of the front-forklift. A hydraulic input cylinder works between point B'' on the rocker BB_0 and point C on the coupler AB. The hydraulic cylinder extends the number of links from six to eight. Similar considerations and calculations are valid also for the rear-forklift with a so-called "triple suspension" of the forklift to the tractor via the points A_0, B_0 and H_0. The input link (crank) H_0H is now turned by the hydraulic drive unit of the tractor.

Mechanism Model Collection

In 1959, eleven years after starting his job in the Institute of Agro-Technical Basic Research of the FAL in Braunschweig, Kurt Hain began to establish a collection of mechanism models. These models were to close the gap between the scientific state of the art and the constructive practice in kinematics. Models, especially when driven manually, give an idea of how a mechanism runs and can even reveal or make directly felt the kinematic and dynamic weak points during operation. Almost without exception the links in Hain's models were made of plastic material. Dependent on the task definition of the link, the colour of the material was chosen; for example input links were red, output links light green or white. As for the joints, metal parts were used, and there was a unit construction system containing equal or similar elements to facilitate the assembly of the models. Until 1962 the Hain collection comprised around 130 models; at that time most of them were models of mechanisms for use in agricultural machines (Hain, 1962). Today, we have 176 genuine Hain models, but not all of them could be preserved for posterity. An impression of the variety of the Hain models is given in Figures 14 to 16.

Three Examples of Selected Highlights in Research Activities

Mechanisms with minus one d.o.f. Mechanisms have one fixed or reference link and at least one d.o.f. ($F = 1$), say, one drive is necessary and sufficient to make the mechanism work.

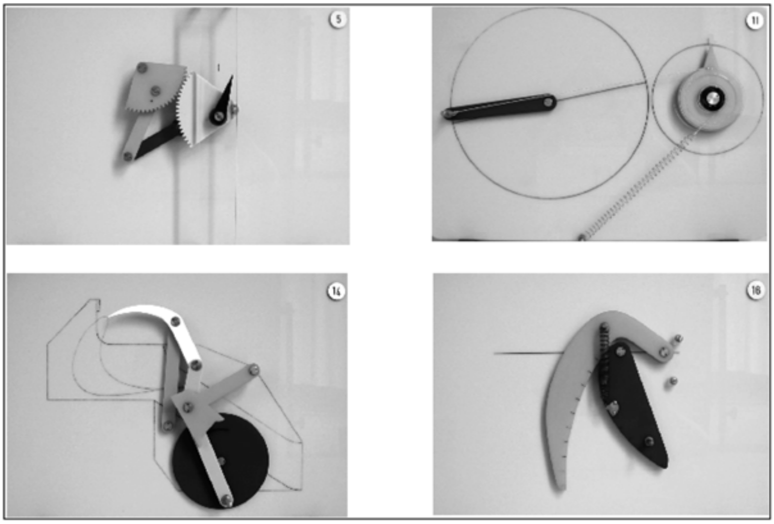

Fig. 14. Mechanism models nos. 5, 11, 14 and 16 of the Hain collection.

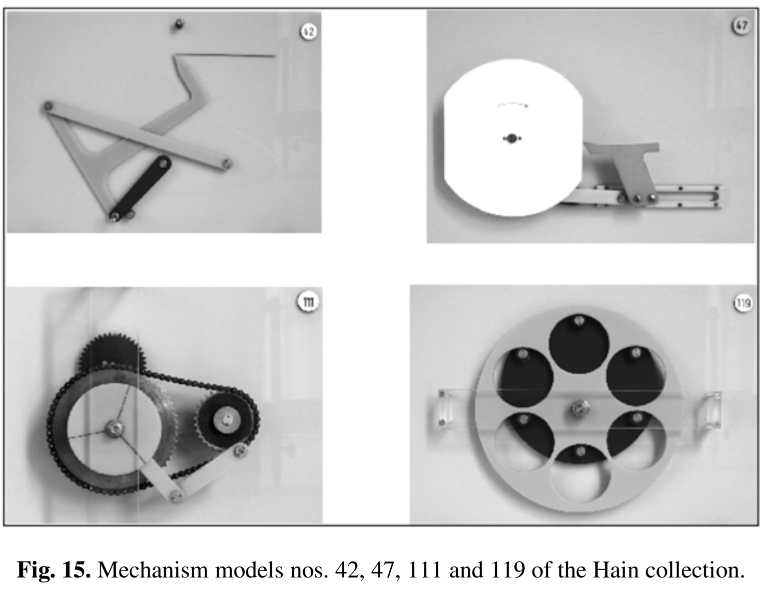

Fig. 15. Mechanism models nos. 42, 47, 111 and 119 of the Hain collection.

Normally, a planar mechanism is derived from a planar kinematic chain with one arbitrary link fixed and drawn with turning or sliding pairs only, and thus the Grübler formula

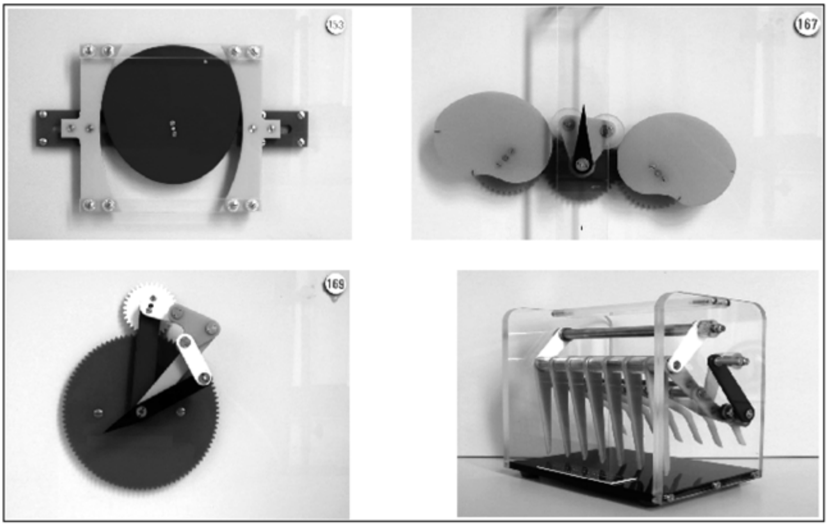

Fig. 16. Mechanism models nos. 153, 167, 169 and 148 of the Hain collection.

$$F = 3 \cdot (n - 1) - 2 \cdot g \tag{4}$$

(n: number of links, g: number of pairs) holds for the number F of d.o.f. of the mechanism. Mechanisms with $F = 0$ represent rigid frameworks, immovable, but nevertheless important intermediate structures for force analysis. But, who thinks of mechanisms with $F = -1$? Hain did, and developed clamping devices, for example, for workpieces on machine tools, with one or even more than one clamping contact location pairs. His idea is based on the replacement of binary links (bars) by input or output forces. In Figure 17, seven clamping devices are drawn ($n = 8$, $g = 11$) with one sliding pair between workpiece 3 and frame 1 and with the ability to centre the workpiece (rectangle between horizontal forces 6-6 or 7-7, respectively) (Hain, 1964b).

Planetary cam mechanism. In 1973 Kurt Hain invented a planetary cam mechanism (Figure 18 and model no. 167), very compact and stiff (Hain, 1973b).

Two conjugate cams are fixed to two planetary gear wheels each. Cams and gear wheels in centre points A and A' on the input bar rotate around A_0 with input angle φ. While the gear wheels mesh with the fixed sun wheel, the angle φ is transmitted with ratio 1:1 to the two gear wheels, i.e. $\beta = \beta' = \varphi$. Dependent on the contours of the cams the output triangle $F'F_0F$ rotates with

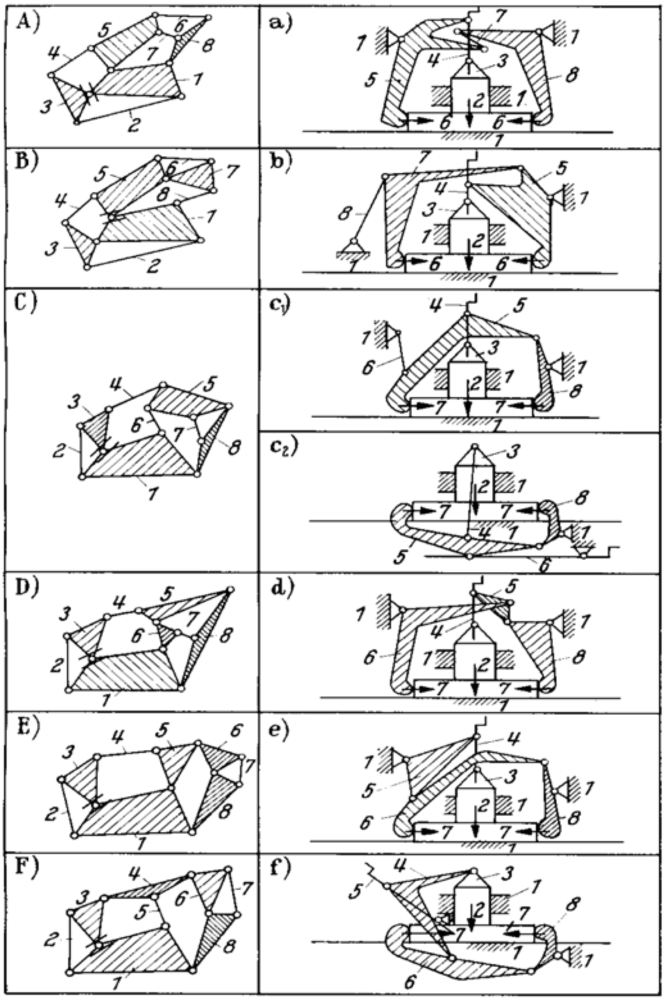

Fig. 17. Clamping devices derived from eight-link kinematic chains with −1 d.o.f. ($F = -1$).

angle ψ around $F_0 = A_0$, say, input and output axes coincide (Figure 18a). The motion law chosen is drawn in Figure 18b, it is a combination of an inclined sine curve with an intermediate straight line, thus leading to a dwell (standstill) of "length" $\varphi = 50°$ and a constant angular output velocity $\omega_{\text{out}} \equiv \mathrm{d}\psi/\mathrm{d}t$ of "length" $\varphi = 230°$ within the motion period of $\varphi = 360°$ ($\omega_a \equiv \omega_{\text{in}} \equiv \mathrm{d}\varphi/\mathrm{d}t$, $i = \omega_{\text{out}}/\omega_{\text{in}} = \mathrm{d}\psi/\mathrm{d}\varphi$, $\alpha_f/\omega_a^2 \equiv \mathrm{d}i/\mathrm{d}\varphi$).

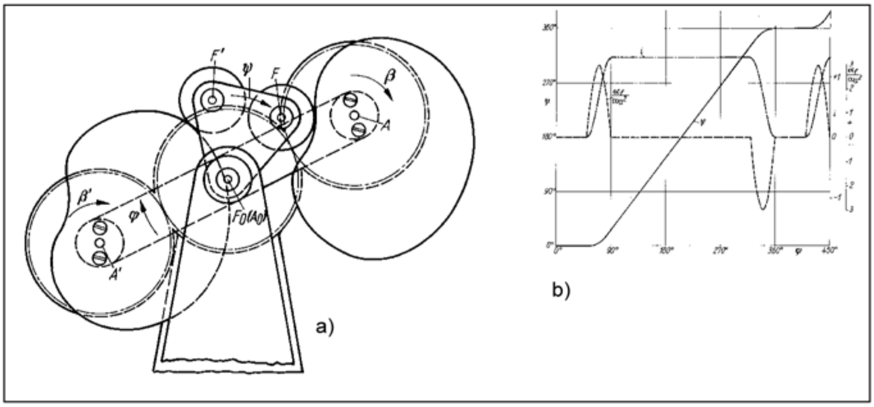

Fig. 18. Planetary cam mechanism (a) and assigned motion law with corresponding derivatives versus input angle φ (b).

Constraint handling devices. Planetary gears were also in the focus of attention when Kurt Hain presented some mechanisms for constraint handling devices, designed for the oriented guidance of workpieces in a plane. In combination with belts or chains his prototypes can move workpieces on circle segment paths and at the same time make them rotate by an angle dependent on the transmission ratio of the used gear wheels. Because of constraint motion condition, Hain's handling devices are fast machines in automated production processes, with exact positioning of the workpieces in a plane. Three examples of such handling devices are given in Figure 19 (Hain, 1976b); the sun wheels are always fixed in the frame and centred in A_0, while the input crank $a = A_0A$ with the belt-connected planet wheel and the workpiece holder fixed to the latter rotates around A_0. It was Hain's intention at the beginning of the industrial robot era to demonstrate that a lot of simple robotic trajectory tasks could be performed also by properly designed mechanisms.

Modern Interpretation of Main Contributions to Mechanism Design

Kurt Hain was Germany's leading practitioner of (planar) kinematics after World War II for at least forty years. He was educated in kinematics in a booming period of mechanization. The ideas and methods of some famous protagonists, for example Franz Reuleaux (1829–1905), Ludwig Burmester

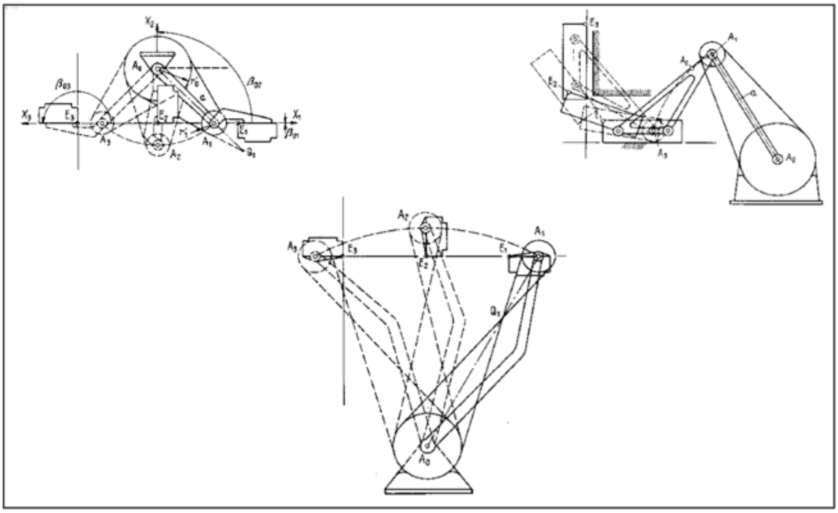

Fig. 19. Three examples of constraint handling devices for moving and orienting workpieces in a plane.

(1840–1927), Martin Grübler (1851–1935), Wilhelm Hartmann (1853–1922), Reinhold Müller (1857–1939), Rudolf Mehmke (1857–1944), Karl Kutzbach (1875–1942) and Hermann Alt (1889–1954), formed milestones and gave standards in mechanism theory. But, theory and practice are two completely different things. To be candid, not every professor named above was able to explain complicated theoretical coherences conceivable to the mechanical engineer who worked in an industrial company and tried to find a solution for his mechanical task. Kurt Hain, being somewhat of an autodidact, was always aware of the problem of transferring knowledge between the two worlds “theory” and “application”. He developed his own illustrative and comprehensible way of knowledge transfer: He normally started with a systematic approach, used mainly graphical methods or numerical methods strongly based on graphical methods, his *Zeichnungsfolge-Rechenmethode*, and finally proved the efficiency of his procedures by a number of application examples in different fields of mechanical engineering. Due to the strong dependence on graphical methods, Kurt Hain was especially most successful with planar kinematics.

The rough overview in Table 2 of the categories of Kurt Hain’s research activities comprises all the items in kinematics he dealt with in the course of his life. Systematics stands at the beginning, and Hain was a juggler of ideas in this field in developing new astonishing solutions with mechanisms from simple kinematic chains by variation of links and joints. For example, we owe him for insight into the usefulness of mechanisms with zero or less than zero d.o.f., with only swinging links, and for the world of equivalent mechanisms with simplified structures.

The *Zeichnungsfolge-Rechenmethode* created and cultivated by Kurt Hain is a mixture of Descriptive and Analytic Geometry. Kurt Hain alone was able to make the best of it. When following (and/or believing) only numerical results, it is often difficult to find correct line and vector orientations or to make the correct choice between two possible signs of an ambiguous mathematical expression. Close as he was to his drawings, Kurt Hain could avoid mistakes of this kind. It was his major tool when dealing with kinematic analysis for all types of mechanisms.

In the field of kinematic synthesis two Hain methods must be emphasized: the *point position reduction* and the generation of *point-dwell mechanisms*. At a time when digital computers were not yet known or in general use, Hain presented his solutions with a strongly reduced geometric-mathematical background. So, it was possible on the one hand to avoid the complex Burmester curves in many cases of application, and on the other hand to develop a new class of six-link mechanisms with a temporary standstill of one (output) link of a mechanism or linkage while the input link was running continuously.

The broadness of Kurt Hain's field of application examples is indeed overwhelming; on one side there are sensitive and precise computing mechanisms, on the other side there are big and robust mechanisms for use in transport devices and agricultural machines. Kurt Hain's retirement in 1973 signified by no means a transition into a more contemplative life: Free from professional constraints he could work all the more effectively and could occupy himself with the rising computer technology to study kinematic problems more basically and to solve them more precisely. During the last twenty years of his life Kurt Hain especially dealt also with mechanisms for handling devices and simple robots and their components, for example grippers (Hain, 1976b, 1982, 1984b, 1985, 1989).

Kurt Hain left a collection of 176 mechanism models to the posterity of kinematicians. Not all of the models could be saved, but it is still possible to rebuild them. He described most of the models in his publications, and his

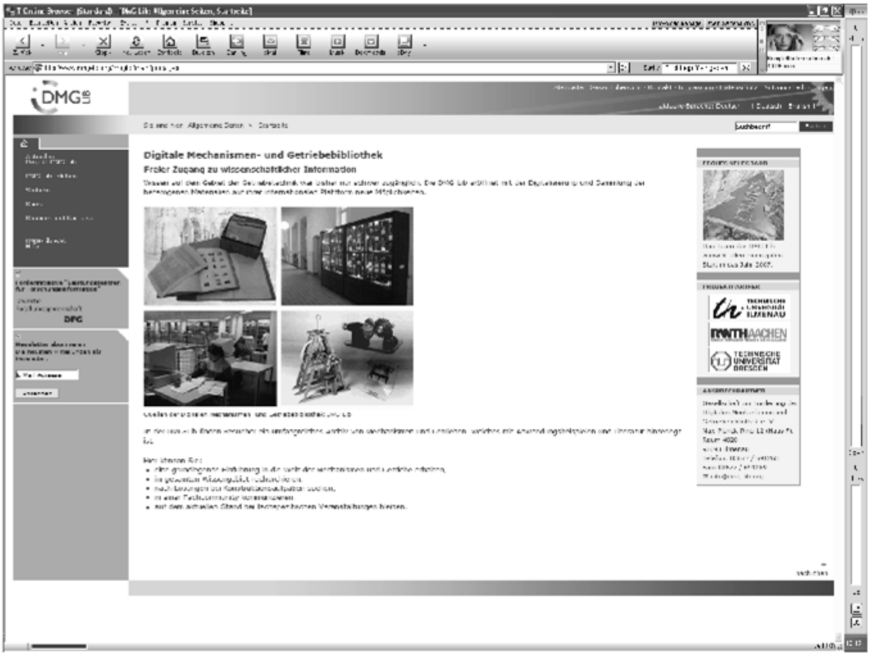

Fig. 20. Homepage of the DFG project DMG-Lib.

Getriebebeispiel-Atlas also contains the dimensions of the links, model by model and page by page.

The complete lifework of Kurt Hain with all his books, papers and models is, at this time, about to be digitized in order to allow access to students, experts and interested people all over the world via internet. There is a German national preserve activity named “Digitale Mechanismen- und Getriebebibliothek” (DMG-Lib) (Digital Mechanism and Gear Library). The project is financed by the “Deutsche Forschungsgemeinschaft” (DFG) (German Research Foundation) and is essentially performed by three German Technical Universities, the TU Ilmenau, the RWTH Aachen and the TU Dresden. The first German kinematician whose complete lifework will be gathered in digital form is Kurt Hain. Access to the homepage of the DMG-Lib is by the address http://www.dmg-lib.org (Figure 20).

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HERON OF ALEXANDRIA
(c. 10–85 AD)

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Abstract. Heron of Alexandria was a mathematician, physicist and engineer who lived around 10–85 AD. He taught at Alexandria's Musaeum and wrote many books on Mathematics, Geometry and Engineering, which were in use till the medieval times. His most important invention was the Aeolipile, the first steam turbine. Other inventions include automated machines for temples and theaters, surveying instruments, and military machines and weapons.

Introduction

The ancient Greek technology developed mostly in the period 300 BC to 150 AD and was in use for more than one thousand years. It had a profound impact both on Western and Muslim civilization. Notable inventions include cranes, screws, gears, organs, odometer, dial and pointer devices, wheelbarrows, diving bells, parchment, crossbows, torsion catapults, rutways, showers, roof tiles, breakwaters, and many more. Greek engineers were pioneers in three of the first four means of non-human propulsion known prior to the Industrial Age: watermills, windmills, and steam engines, although only water power was used extensively (Lahanas, Web). Among the Ancient Greek Engineers, the most prominent include Archimedes, Ktesibios, Heron, and Pappos.

Heron (or Hero) of Alexandria (in Greek ʹΗρων ο Αλεξανδρεύς), see Figure 1, was a mathematician, physicist and engineer who lived in the Hellenistic times in Alexandria, Egypt, at that time part of the Roman empire. He was made famous for documenting the first steam turbine, the *aelolipile*. He also invented many mechanisms for temples and theaters while he advanced or improved inventions by others, for example the *hydraulis*, originally in-

M. Ceccarelli (ed.), Distinguished Figures in Mechanism and Machine Science, 217–245.

Fig. 1. Heron of Alexandria (O'Connor, 1999).

vented by Ktesibius. Heron was also called *Michanikos* (Μηχανικός), the Greek word for Engineer.

It is important to stress that in the Ancient World, technology was not considered as very important for the growth of philosophy and science. The dominant motive in philosophy was understanding or wisdom, while the connection between science and technology was not as extensive as it is today. In this context, Heron, as well as the other engineers, were on the exception side (Lloyd, 1991).

Biographical Notes

The chronology of Heron's works is disputed and not absolutely certain to date. Many contradictory references on Heron exist, partly because the name was quite common. However, historians cite that he came after Apollonius, whom he quotes, and before Pappos, who cites him. This suggests that he must have lived between 150 BC and 250 AD (Thomas, 2005). In 1938, Neugebauer, based on a reference in Heron's *Dioptra* book of a moon eclipse, he found that this must have happened on March 13, 62 AD, (Neugebauer, 1938). Since the reference was made to readers who could easily remember the eclipse, this suggests that Heron flourished in the late first century AD. According to Lewis (2001), and assuming that *Cheirobalistra*, a powerful catapult, is genuinely his, Heron should have been alive at least till 84 AD, the year in which the *Cheirobalistra*, was introduced.

Because most of his writings appear as lecture notes for courses in mathematics, mechanics, physics and pneumatics, it is almost certain that Heron taught at the Musaeum of Alexandria, an institution for literary and scientific scholars supported by the Ptolemies, which included the famous Library of Alexandria. Many scholars believe that not only he taught at the Musaeum, but that in addition he served as its Director and that he developed it as the first Polytechnic School, or Technical Institute. He is the last recorded member of the School, and the best known (Lewis, 2001).

According to Drachmann (1963), Heron was a man who knew his business thoroughly, who was a skillful mathematician, astronomer, engineer and inventor of his time. Based on the content of the book *Pneumatica*, a number of researchers expressed doubts about his capabilities. However, this book appears to be an unfinished collection of notes and may have been altered through the years.

An important characteristic of Heron's work was clarity in expressing his ideas, something not common in ancient writings. As Drachmann (1963) states, "a man who is always able to present his subject in such a way that is readily understood, is a man who understands it himself, and he is certainly not a fool or a bungler."

Mahoney notes the following about Heron, "In the light of recent scholarship, he now appears as a well-educated and often ingenious applied mathematician, as well as a vital link in a continuous tradition of practical mathematics from the Babylonians, through the Arabs, to Renaissance Europe" (Drachmann and Mahoney, 1970). Furthermore, Heath writes that, "The practical utility of Heron's manuals being so great, it was natural that they should have great vogue, and equally natural that the most popular of them at any rate should be re-edited, altered and added to by later writers; this was inevitable with books which, like the *Elements* of Euclid, were in regular use in Greek, Byzantine, Roman, and Arabian education for centuries" (Heath, 1931).

In many of his works, Heron would start by reviewing past works. However, he would not always give credit to previous inventors and would tend to dismiss easily the work of others, before presenting his own solutions.

Heron recognized the value of experimental work. The example passage, taken from Lloyd (1973), attacks first those (like Aristotle) who denied absolutely that a void can exist, accusing them of following their faith as opposed to evidence:

> *Those then who assert generally that there is no vacuum are satisfied with inventing many arguments for this and perhaps seeming plausible with their theory in the absence of sensible proof. If, however, by*

> *referring to the appearances and to what is accessible to sensation, it is shown that there is a continuous vacuum, but only one produced contrary to nature; that there is a natural vacuum, but one scattered in tiny quantities; and that bodies fill up these scattered vacua by compression; then those who put forward plausible arguments on these matters will no longer have any loop-hole.*

Following this statement, Heron described an apparatus designed to show the existence of vacuum. This is basically a metal hollow sphere with a small hole and a thin tube of bronze attached to the hole. Heron argued that if one blows air into the sphere, then air enters it and therefore it must be compressible. This compressibility was attributed to the existence of small pockets of vacuum. He also continued his argument by saying that one can also draw air out of the sphere by inhaling air. Once this is done, then the sphere must contain more vacuum than before.

Although arguments of this sort may be commonplace today, they were not necessarily the norm two thousand years before, and therefore, this attitude towards experiments is considered to be very important.

List of Main Works

The main works of Heron are published in five volumes in the Teubner Series, (Heiberg, 1912; Schmidt, 1899). Among them, the most well-known books related to engineering include

- *Pneumatica* (Pneumatics), a treatise on the use of air, water, or steam, in Greek.
- *Automatopoietica* (Gr. Περί αυτοματοποιητικής, i.e. about making automatic devices), a description of automated machines using mechanical or pneumatical means, most for temples, in Greek.
- *Belopoeica* (from the Greek βέλος, meaning arrow, and ποιώ, meaning to make), on the constructions of machines of war, in Greek.
- *Mechanics*, which covers mechanisms and simple machines and has survived in Arabic, with a few fragments in Greek preserved by Pappos.
- *Barulkos* (Gr. Βαρούλκος from βαρύς, meaning heavy and έλκω, meaning to pull), that discusses methods of lifting heavy weights. Perhaps this is the same as *Mechanics*.

- *Dioptra*, which describes a theodolite-like instrument used in surveying and methods to measure length, in Greek and Arabic.
- *Catoptrica* (Catoptrics), on light propagation and reflection, and on the use of mirrors.
- *Cheirobalistra* (On Catapults), about catapults, in Greek.

Heron has also contributed to Mathematics and Geometry. Although some of them are of disputed authorship, his works in this area include

- *Metrica*, describes how to calculate surfaces and volumes of diverse objects, in Arabic.
- *Geometrica* (Geometria), a collection of equations based on the first chapter of Metrica, in Greek.
- *Stereometrica* (i. and ii.), examples of three-dimensional calculations based on the second chapter of Metrica, in Greek.
- *Geodaesia*, surveying analysis, in Greek.
- *Mensurae*, tools which can be used to conduct measurements based on *Stereometrica* and *Metrica* (disputed authorship), in Greek.
- *Definitiones* (Definitions), containing definitions of terms for geometry, in Greek, (disputed authorship).

Unlike other ancient works in Greek, the language in Heron's book is quite easy to be read by non-scholars, even today. Except the *Definitiones*, these books mostly consist of methods for obtaining the areas and volumes, of plane or solid figures. In these, Heron gave methods for computing very close approximations to the square roots of numbers, which are not complete squares and even cubic roots of numbers, which are not complete cubes. Heron also provided expressions for computing the areas of regular polygons of five to twelve sides in terms of the squares of the sides that lead to important trigonometrical ratio approximations. In general, it is believed that these books were based on Heron's works, but also that they were altered by people after him. Heron's most important work on geometry, *Metrica*, was missing until it was discovered in Constantinople in 1896 by R. Schöne. This work is the closest to its original form.

Review of Main Works on Mechanism Design

Mechanics

Heron wrote important books on mechanics that describe simple mechanical machines and methods for lifting weights. His *Mechanics* are divided in three books. The first is an introduction and describes the theory of motion, statics, balance, and how to construct three-dimensional shapes in proportion to a given shape (pantographs). The second contains an exposition of the theory of the five “powers”: the windlass, the lever, the pulley, the wedge, and the screw (incl. the worm gear) and examines methods of lifting heavy objects with their help. It also deals with finding of the center of mass of planar bodies. The third book presents applications of the five powers, i.e. methods of moving objects by means of sledges, cranes, etc. He also discusses wine presses. *Mechanics* is written for architects and contractors, and except for some chapters that appear to be out of place, the work is well arranged (Drachmann, 1963). The contents of this book have a lot of overlap with *Barulkos*, so many believe that these are not separate books.

Dioptra

Dioptra is a book on surveying and instruments for it. It begins with an introduction to “the science of dioptrics” and gives a description of the *dioptra* (Gr. διόπτρα), a combined theodolite and water-level. Heron here presents all previous works on the subjects and quickly dismisses them. Later, he gives instructions on how to construct a dioptra instrument, and how to use it. The book also contains a description of a *hodometer* (Gr. oδόμετρον), a device for measuring displacements. The book starts to degenerate after chapter 35, while the next chapter is missing. Chapter 37 describes the *barulkos*, a device designed for lifting weights, while the next one and proposes a hodometer for ships. The book ends with an appendix on other surveying methods (Drachmann, 1963; Lewis, 2001). The contents of *Dioptra* up to chapter 34 are listed in the Appendix and can give a good idea of the issues discussed in the book.

Automatopoietica

Automatopoietica is the oldest text that describes automatic machines and devices. Heron’s automatic devices were based on water, fire, and compressed air. Among these, the most well-known include an automatic system to open the doors of a Temple when a fire was started at the altar, a coin-operated machine that was providing water, and toy-like motions of puppets, such as bird automata. Another device, called the Hercules and the Dragon, has Hercules hitting the head of the dragon, while the dragon shoots water on his face. Heron is also credited with the construction of the first analog computer, a computing device based on gears and pins. Many of the "automata" of Heron's were constructed around 1589 by Giovanni Battista Aleoti.

Belopoeica and Cheirobalistra

The *Belopoeica* deals with the construction of war machines. It has some similarities with works written by Philon and Vitruvius, and perhaps was based on the work of Ktesibios.

The *Cheirobalistra* deals with catapults and serves as a lexicon (dictionary) of their parts. However, it is not certain that it was written by Heron.

Pneumatica

The *Pneumatica* is a controversial work in two books, with 43 chapters in the first and 37 in the second. The book starts with an analysis on fluid pressure, which at some parts is correct and elsewhere is not. It also describes mechanical toys, singing birds, sounding trumpets, etc. In total, more than one hundred machines and devices are described in its chapters. Although most of them work with steam or water, they all include mechanisms, either for transmitting power, or motion and signals.

The most famous device is the *aeolipile*, a steam turbine device, which is described in more detail later. The aeolipile was not used to produce mechanical work, perhaps because at that time, the need to use machines for producing mechanical work was not so crucial and, hence, it was not driving the process of building such devices.

This observation is more general. Researchers agree that most of these toys and devices were not designed to perform particular tasks, but rather to teach physics to students, i.e. Heron was demonstrating what can be done with physics, but not how to solve particular engineering problems.

Pneumatics stirred great interest among Renaissance scholars. The works were translated and published for the first time by Giovanni Battista Aleotti in 1589 under the title *Gli Artificiosi et Curiosi Moti Spiritali dit Herrone*,

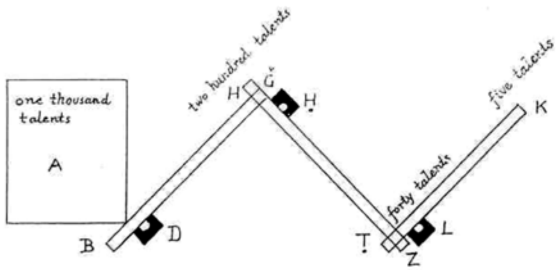


Fig. 2. Multi-link lever (Drachmann, 1963).

where the translator added some of his own ideas. Other translations were provided by Alessandro Giorgi da Urbino in 1592 and 1595 (Lahanas, Web).

The contents of *Pneumatica* are listed in the Appendix. They provide a good picture of the devices and methods discussed therein.

Modern Interpretation of Main Contributions to Mechanism Design

It is quite interesting to examine some of the machines and devices described by Heron from a modern standpoint. In particular, we will attempt to do this, focusing in a number of characteristic devices, which are of interest regarding the included mechanisms and/or their design. To this end, we will present and discuss (a) mechanisms such as *levers*, *gears*, *weight-lifting devices*, *presses*, *pantographs*, and *hodometers*, (b) automatic devices such as *automatic libations* and *automatic opening of temple doors*, (c) engines of war such as the *cheirobalistra* and the *palintonon*, and (d) important devices, such as the *dioptra*, the *aeolipile*, and the *hydraulis*.

(a) Mechanisms

Levers

In Heron's *Mechanics*, one can find many types of levers, some simple, some more complex. Figure 2 shows a multi-link lever system with many fulcra. The figure also shows the multiplication of force at various points. A force equal to five talents (one talent is about 26 kg), is multiplied by two hundred to become one thousand talents after fulcrum D. The original text, translated into English reads (Drachmann, 1963)

<25> As for the lever, the same weight is moved by the same power by this arrangement: Let the weight be at point A, and let the lever be BG, and let the stone that is under the lever be at point D, and let the moving of the weight by the lever take place while it is parallel to the ground, and let GD be five times DB; then the power that is at G, which balances the thousand talents, is two hundred talents. Let there now be another lever, which is HZ, and let point H be the point be the one at the head of the lever, engaging point G, so that G is moved by the moving of H, and let the stone that is under the lever be at point H, and let it be moved towards D, and let ZH be five times H′H; then the power that is at Z will be forty talents. Let there be another lever, which is T′K, and let us place the point T′ on the point Z, and let it be moved in the opposite direction of H. And let the stone that is under the lever be at the point L, and let it be moved the way in which the point H is not moved, and let KL be eight times LT′; then the power that is at K will be five talents, and it will balance the weight. And if we want the power to overcome the weight, we shall have to make KL more than eight times LT′; but if KL is eight times LT′ and ZD five times H′H and GD more than five times DB, the power will overcome the weight.

Reading the above passage, one finds that it is quite descriptive and quantitative and predicts the force (power) multiplication correctly. In the next paragraph, the text also recognizes that this force multiplication is done at the expense of a large delay (large displacement), which is of the same proportion as the force magnification.

One may note that the figure itself is drawn in a singular configuration in which actually the mechanism will not work as described. However, this particular configuration is not inferred from the text. Perhaps this was done either to save paper, or this figure was drafted by someone who was not aware of the exact workings of levers.

Gears

Heron used gears extensively. In general, unlike today's gears, these tend to be triangular in shape. The spur gears are more common, but Heron also used other types, like the worm gear in conjunction with a worm, see Figure 3.

Again, the text is quite descriptive and explains how to calculate the velocity (gear) ratio as follows (Drachmann, 1963):

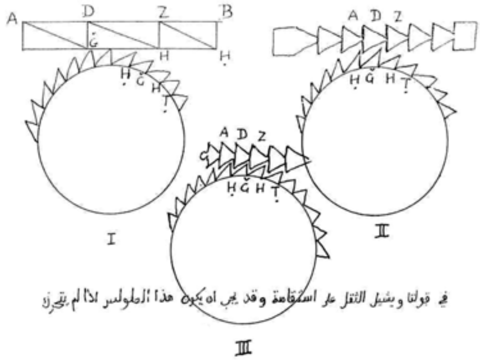


Fig. 3. Endless screw (worm gear) and driven gear (worm). Inscriptions in Arabic (Drachmann, 1963). The figure is used to calculate the velocity ratio of the gears.

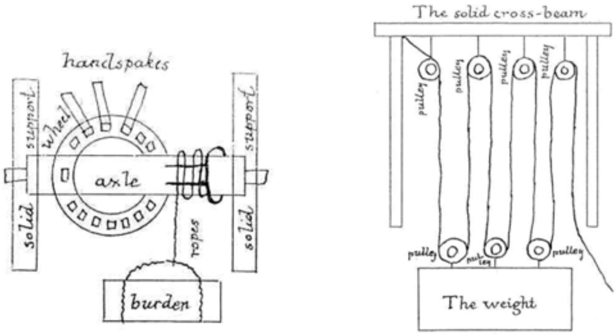
> *<18> When it is the case that a wheel with teeth engages the screw furrow, then for every one turn the screw is turned, it will move one tooth of the wheel. And we can prove this in the following way . . .*

It is worth noting that Heron used the worm gear and worm transmission in many of his mechanisms, possibly because this combination is not backdriveable, a characteristic very handy in the case of lifting ways. In such cases, the worm is connected to a handle and the operator of the mechanism can stop applying torque on it without having the weight fall.

Weight-lifting devices

Weight-lifting devices were very important in the ancient times. Traditionally, this was done with the use of enormous man-power, for example with many people pulling or lifting weights against their own. The mechanisms discussed by Heron allowed the lifting of big weights used in construction, by a single person.

Figure 4a shows an axle supported on its two ends and connected to a wheel with handles (handspakes). A rope is wound around the axle and lifts a weight. Again, this is a force multiplying device, and works best when the wheel has a large diameter R and the axle has a small one, r, since the multiplication of the force is proportional to R/r. As one can see in Figure 4a, someone who read the book with the figure was aware of this, and therefore designed the part of the axle around which the rope winds with a smaller



(a) (b)

Fig. 4. Lifting mechanisms. (a) Small force maltiplication. (b) Compound pulley mechanism (Drachmann, 1963).

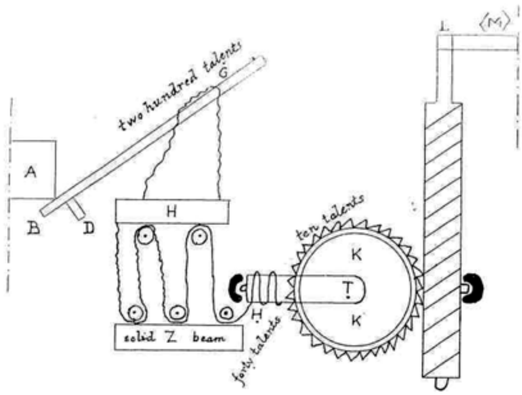


Fig. 5. Complex lifting mechanism showing force amplification (Drachmann, 1963).

diameter than that of the rest of the axle. Here, too, a "delay" is present, as many wheel rotations are needed to lift the weight.

Figure 4b shows a compound pulley system that is used to lift a weight. One end of the rope is attached to the solid cross-beam, while the other end is pulled by a person. Heron recognized that levers, lifting axles and pulleys, all work similarly in that they multiply forces by a factor and require at the same time displacements also multiplied by the same factor.

Figure 5 shows a complex lifting mechanism that contains a lever, a compound pulley system, an axle, and a worm gear with a worm. From its appearance, it seems that this mechanism is provided as a teaching example and not as the description of a particular lifting device. The figure also shows the force multiplication from the hand lever to the weight. It also shows the bearings of the axle, and the lever (wedge) fulcrum. The base of the entire mechanism is not shown in the figure. However, Heron mentioned that

> *<29> … And this support like a chest should be in a firm place, in a place strong in its foundation, solidly built. When the handspake is turned, the weight is lifted.*

From the above passage, one can notice that not only a strong supportive base is required, but also Heron recognizes the importance of estimating correctly the need for taking into account strength calculations.

Heron also provides the description of a weight lifting device, based entirely on gears, see Figure 6.

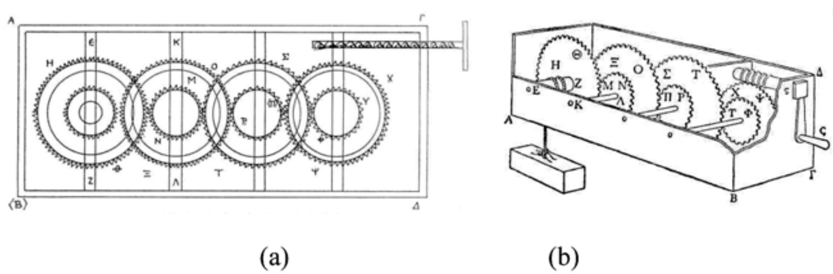


Fig. 6. Weight lifting compound gearbox. (a) Original drawing (Drachmann, 1963). (b) 3D drawing (Thomas, 2005).

The device is called the *barulkos*, and includes a non-backdriveable worm gear and a compound gear train with four parallel axes. It allows one to lift a large weight with a small effort, by turning the crank. The description of this device is found in Heron's *Dioptra*, chapter 37. In this, the author describes in detail the gearbox and recognizes that all axles must be able to rotate freely. This means of course that first the axles should be rotating, but also that they must not be subject to large frictional torques. This is important, as gear trains with many stages, like the *barulkos* were not very efficient.

Presses

In ancient times, presses were used to produce oil from olives or wine from grapes. In Heron's *Mechanics*, the author describes a two-screw press that is

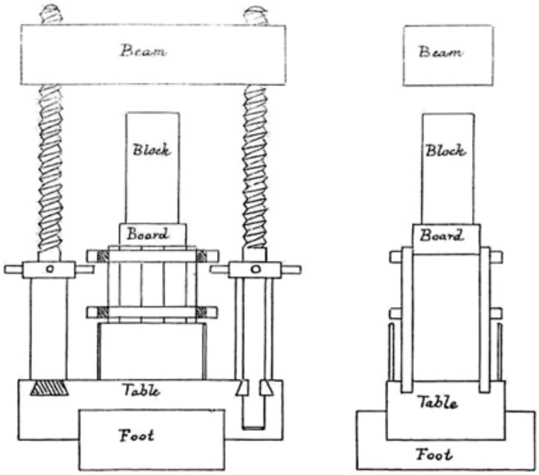


Fig. 7. Double-screw press (Drachmann, 1963).

used for pressing olives, see Figure 7. Unlike the single screw press, which may be backdriveable, the double screw press is not backdriveable, and therefore can hold its pressure without the need to apply force on the hand wheels. According to Drachmann (1963), Heron introduces the press as follows:

> *<19> These instruments, whose construction we shall now describe, serve for pressing of oil, and they are easy to work, they can be moved and put up in any place we want, and there is no need in them for a long, straight beam of a hard nature, nor for a very heavy stone, nor for strong ropes, and there is in them no hindrance from the stiffness of the ropes, but they are free from all that, and press with a strong pressure and the juices come out altogether. And their construction is what we now are going to describe.*

Passages like the above illustrate the clarity and compactness of Heron's writings. In this, he recognizes that the press works due to internal forces, and therefore, no long levers, pulleys, or mere weights are necessary to produce a large force (pressure). Despite the fact that large forces are produced, the press itself can be relatively lightweight and mobile; it does not even need a permanent base.

Figure 7 was drawn by Drachmann (1963), based on size information given in the original text. Since people at Heron's times had lathes, it seems that the screws were made with the help of one. However, it is Heron in

Dioptra, who presents the first extant description of a machine for cutting screws (Lloyd, 1973).

Pantograph

In his *Mechanics*, Heron described pantographs, i.e. mechanisms used for copying figures at a different size from the original. As explained in chapter 15, see (Drachmann, 1963):

> *<15> And let us now prove how to make a figure similar to the known plane figure in the given ratio by means of an instrument. We make two wheels on the same centre, fixed to it, and provided with teeth, moving on a single axle in the plane where is the figure we want to copy; and the ratio of one wheel to the other should be the given ratio* ...

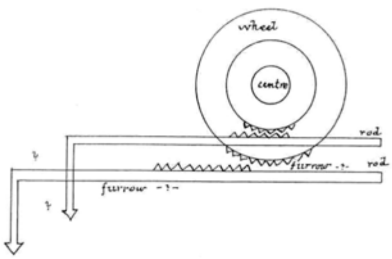
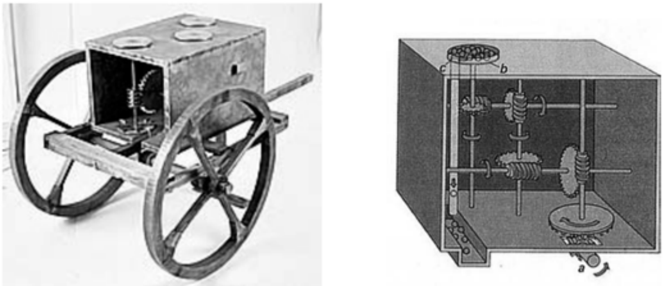


Fig. 8. Pantograph (Drachmann, 1963).

Figure 8 shows a pantograph taken from a copy of the book at the British Museum. It shows the two teethed wheels and the teethed copying rods, and mostly illustrates the principle and not the exact construction of the pantograph. The two wheels rotate at the same angular speed. When one of the rods is displaced in a given direction, the corresponding wheel rotates and since this is connected to the other one, this rotates and carries with it the other rod. Obviously, the two rods then have velocities and displacements proportional to their distance from the center of rotation, and therefore, their path is similar, too. If one of the rods is rotated, as opposed to be translated, then the other rod must remain parallel to the rotated one.

Hodometer

A *hodometer* (or odometer, Gr. οδόμετρον) is a device that indicates the dis-



(a) (b)

Fig. 9. (a) A reconstruction of Heron's hodometer, (b) mechanism geartrains (Lahanas, Web).

tance traveled by a vehicle. Hodometers have been described by others before Heron, including Vitruvius (c. 25 AD), and even Archimedes (c. 287 BC–c. 212 BC). However, it seems that none of these had ever worked or even that they were built. The same is probably true for his hodometer. Leonardo da Vinci tried to build it according to the given description, but did not succeed.

Heron describes the construction of his hodometer in chapter 34 of the *Dioptra* as a mechanism made of wheels and axles and housed in a small wooden box, see Figure 9. Heron observes that chariots with wheels of four feet diameter turn exactly four hundred times when the chariot covers one Roman mile, or about 1500 m.

Therefore he proposed a mechanism in which a pin on the chariots axle engages a fourhundred tooth cogwheel, that makes a complete rotation per mile. Then, a transmission of five axles and four worm gear and worm geartrains, drive an index showing the miles traveled. Alternatively, they engage a disk with holes along the circumference, where pebbles are located, that drop one by one into a box. In this case, the number of miles traveled is given simply by counting the number of pebbles (Lahanas, Web).

In chapter 38 of *Dioptra*, a naval hodometer is mentioned. Here, an external to a vessel paddlewheel is connected to a mechanism like that of the hodometer, which is located inside the vessel. The last wheel in the series made a full revolution every Roman mile. The naval log was replicated by K. N. Rados in wood and brass and exhibited at the International Exhibition in Bordeaux in 1907. The odometer was reconstructed recently by Dutch engi-

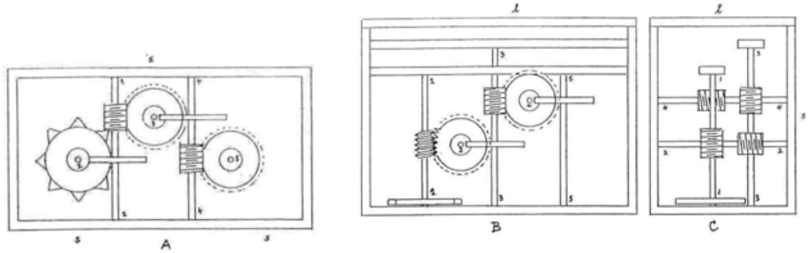


Fig. 10. Five axle hodometer described by Heron in *Dioptra*, chapter 34. (A) Aspect from above, (B) from the side, (C) from one end (Drachmann, 1963).

neer Andre Sleeswyk, who presented it at a special congress on technology held in Athens in 1987 (Thessaloniki Technology Museum, Web).

(b) Automatic Devices

Automatic libations
In *Pneumatics*, Heron describes several devices that operate on hot air or steam and are designed to produce astonishment and wonder (Lloyd, 1973). One such device is described in *Pneumatics I*, chapter 12, and is shown in Figure 11. The figures standing next to a hollow altar pour libations when a fire is lit on the altar.

The way this works is the following. When a fire is lit, the air in the hollow altar expands and drives out the liquid contained in the altarŠs pedestal. Then, the liquid passes through tubes in the figure bodies and appears to be poured by the figures.

Automatic opening of temple doors Another device designed by Heron, allows the doors of a temple to open when a fire is lit at the altar, see Figure 12. The doors are connected though a set of axles, pulleys, and ropes to a large bucket. Initially, the system is statically neutral and the weight of the bucket is balanced with counterweights. When a fire is lit, the air in the altar is heated and, as it expands, it enters a hollow sphere full of water. Due to the rising pressure in the sphere, some of the water is displaced into the bucket. As the bucket becomes more heavy, it is lowered, opening the doors of the temple (Figure 12). When the fire at the altar was put out, the pressure inside the altar would drop, and water would go back to the hollow sphere, pushed by

Fig. 11. Altar libations produced by fire (Lloyd, 1973).

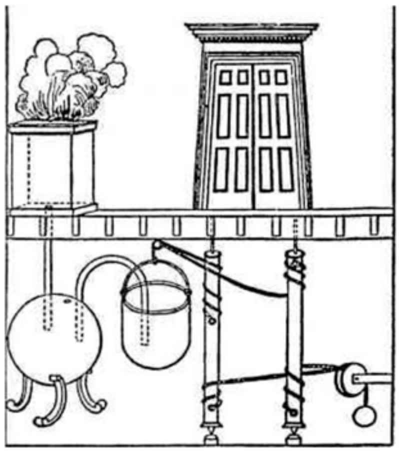


Fig. 12. The doors of the temple open automatically when a fire is started at the altar (Lloyd, 1973).

atmospheric pressure. Then, the counterweights would force the doors of the temple to close.

(c) Engines of War

Cheirobalistra

The *Cheirobalistra* (Gr. χειροβαλίστρα from χείρ which means hand and βάλλω which means to throw) is a device that hurls arrows over a large distance. In this device, the springs made of twisted hair or tendons, are stretched in two separate metal casings. A metal stud is attached at the top of

Fig. 13. The cheirobalistra as reconstructed by E. W. Marsden (Lahanas, Web).

each of the field frames, to hold them together. Another stud was attached to the bottom of the field frames and the base of the engine, to hold the spring casings in place (Marsden, 1971). A small handle wheel at the back of the base was used to load the springs. The cheirobalistra must have had a quick release mechanism for throwing the arrow. Heron's cheirobalistra was the most advanced two-armed torsion engine used by the Roman army (Lahanas, Web). The cheirobalistra was probably introduced around 84 AD and was definitely in service in 101–102 AD, as attested on Trajan's Column (Lewis, 2001).

Palintonon

A device similar to the cheirobalistra, but much bigger and powerful was the *Palintonon* (Gr. παλίτονον, from πάλιν meaning backwards, and τόνος/τείνω, meaning force, stress. It translates to a V-spring (see Figure 14).

This device is described in chapter 3 of *Belopoeica* and was made for throwing stones. It appears that it could fire an 8-pound stone over 300 yards. A similar but smaller device to throw arrows was called *Euthytonon* (Gr. ευθύτονον from ευθύς, meaning straight, and τόνος/τείνω, meaning force, stress. It translates to a straight-spring).

(d) Important Devices

Dioptra

A simple dioptra consists of a long rectangular rod with two sights. One of them has a pin-point aperture and is fixed on the rod, while the other one is

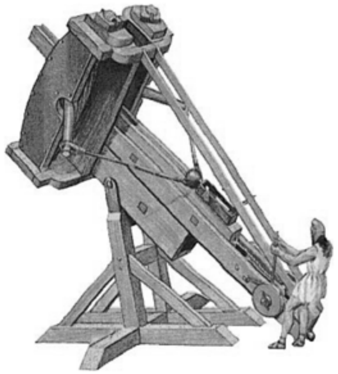


Fig. 14. Heron's palintonon (stone-thrower) (Lahanas, Web).

movable along the rod and is aligned with a target (see Figure 15a). Heron dismissed all previous dioptra designs as inadequate stating that they were all good for limited uses only. Some of them could be only mounted vertically, while others only horizontally.

Heron claimed to have constructed one that was able to perform all the tasks of his predecessors, and more (Lewis, 2001). Indeed, his dioptra could be used. It could be used as a level, a distance-measuring device, or an angle-measuring instrument. The dioptra, see Figure 15b, was probably based on a tripod and was designed such that the dioptra could attain any attitude. This was done with a 3R mechanism, with all three axes passing through the same point. This design is employed today in robot wrists, and one could describe easily its attitude with a Z-Y-Z type of Euler angles. The first two degrees of freedom were based on his favorite worm gear and worm transmission, driven by a small crank, while the last one was carrying the dioptra and was simply rotated by hand. The instrument could be leveled, but it was quite expensive to built and to many difficult to operate.

Aeolipile

Although there are indications that Archimedes and Philo made some simple use of steam, scholars agree that the discovery of the steam engine belongs to Heron. Heron's *aeolipile* is described in his *Pneumatics* 2.11. The name aeolipile is derived from Aeolos (Gr. Αίολος, the Greek god of the winds) and the Greek word pilos (Gr. πίλος, meaning sphere), and translates to "the sphere of Aeolos".

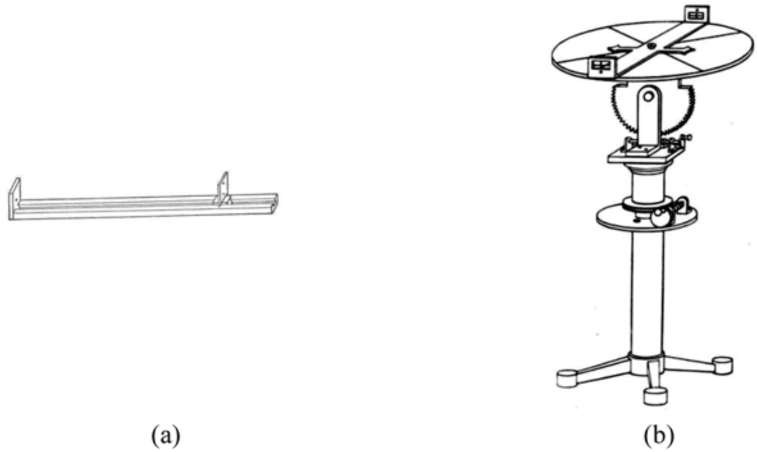


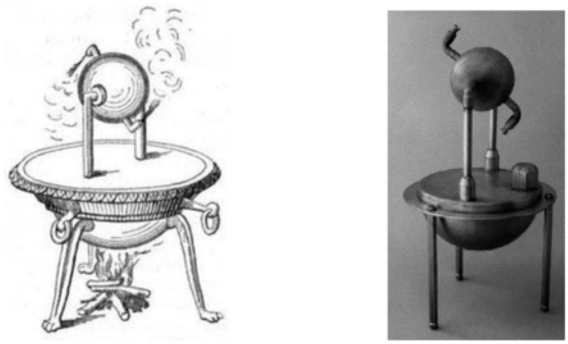
Fig. 15. (a) Simple dioptra, (b) Heron's dioptra (Drachmann, 1963).

The aeolipile (see Figure 16), is a device consisting of an air-tight sphere that receives steam through tubing along one of its major diameters. This piping also serves as an axis of rotation for the sphere. The steam is produced in a cauldron that also serves as the base of the device. The sphere is equipped with two L-shaped bent tubes, which allow the steam accumulated in the sphere to exit in such a way as to create a reaction torque around the axis of rotation of the device. This torque makes the sphere rotate at high speeds (1500 rpm).

The device can be described as a reaction turbine, since it makes use of the reaction force that appears due to the momentum change in the jet of steam which is applied to the bent pipe.

At the time the aeolipile was invented, the device was thought of as a toy. It was only much later that the device gained recognition and accumulated interest.

According to Landels (2000), who made a working reconstruction of the device, due to the device's high rotational speed, a high gear ratio would be needed to make it useful (i.e. develop a high torque at low speeds). The worst engineering problem of the device was the sleeve joint, where the pipe from the cauldron enters the sphere. If this joint is too loose, steam escapes and the device becomes inefficient; if it is too tight, there is a lot of friction increasing the power losses. Therefore, due to the technology level of the time, this jet engine, "to do the work of one man, it would have required the



(a) (b)

Fig. 16. (a) Heron's aeolipile, and (b) a modern replica photographed by Katie Crisalli (Wikipedia).

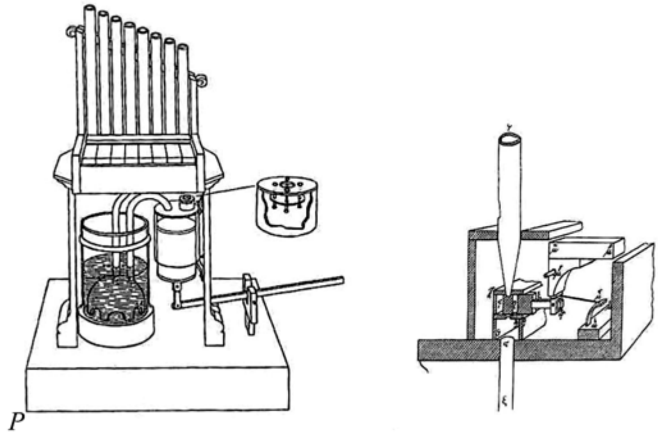
input of several men. In other words, it would have been a labor-using device rather than a labor-saving one" (Landels, 2000).

Hydraulis

Hydraulis (Gr. ύδραυλις from ύδωρ, meaning water, and αυλός, meaning pipe), was invented by Ktesibios of Alexandria (285–222 BC) and was a water organ with keyboard, see Figure 17a. It is generally considered to be the precursor of the modern pipe organ. Hydraulis works by forcing air coming from an air pump to reach a large copper chamber with water, which also contains a hemispherical or funnel-like copper "wind chest". The air is in the wind chest and its pressure is kept constant by water rising in it. The compressed air is driven continuously at constant pressure upwards, to blow the organ pipes.

To improve keyboard air valve of the organ, Heron designed a spool-type mechanism that would slide to open the air stream when a keyboard key was depressed, and would be restored to it original position by a spring force, see Figure 17b.

Heron describes the organ in *Pneumatics*, without referring to Ktesibios. However, it is believed that this was due to the fact that the inventor was well known. Since the organ requires an operator to drive the air pump, Heron designed a wind turbine (see Figure 18), and a mechanism for converting rotary motion to a periodic motion lifting the piston of the air pump. There is no other mention of wind power in Ancient works.



(a) (b)

Fig. 17. (a) Hydraulis was initially designed by Ktesibios and improved by Heron. The detail shows the air pump valve. (b) The organ air valve mechanism (Schmidt, 1899).

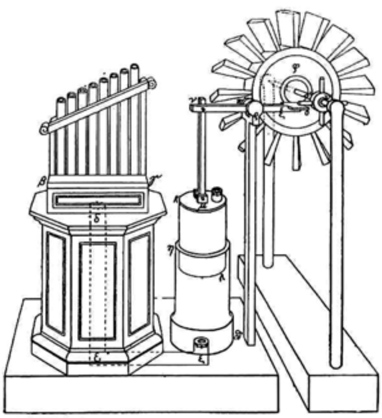


Fig. 18. Hydraulis connected to a wind turbine through a cam-like mechanism (Lazos, 1999).

As seen in this figure, the wind turbine axle included had radial rods, acting as primitive cams, that were forcing a lever connected to the air piston to move downwards, pushing the piston up. When the cam-like rod had rotated away from the lever, the lever was returning to its original position due to the weight of the piston. One could say that this is similar to the cam-driven automobile valve mechanism, where of course the weight restoring force is replaced by a spring force.

Professor Pantermalis of the Aristotelian University of Thessaloniki, recreated the organ recently which played during the Athens Olympics in 2004.

Conclusions

In this chapter, we presented a short introduction to Heron of Alexandria and his works. As many of the ancient scientists, Heron was a mathematician, a physicist and an engineer who wrote many books on Mathematics, Geometry and Engineering, in use till the medieval times. His devices were powered by single humans, water, steam or the wind, and contained many simple mechanisms. His major inventions include the Aeolipile, the first steam turbine, automated machines for temples and theaters, surveying instruments, and military machines and weapons.

Appendix

In detail, the first thirty-four *Dioptra chapters* include (Lahanas, Web):

1. & 2. Introduction to "the science of dioptrics".
3. & 4. Instructions on how to construct a dioptra instrument.
5. Instructions on how to produce a stave for measurement.
6. To observe the difference in height between two points or if their height is the same.
7. To draw a straight line by dioptra from a given point to another invisible point, whatever the distance between them.
8. To find the horizontal (pros diabeten) interval between two given points, one near us, the other distant, without approaching the distant one.
9. To find the minimum width of a river while staying on the same bank.
10. To find the horizontal interval between two visible but distant points, and their direction.
11. To find a line at right angles at the end of a given line, without approaching either the line or its end.
12. To find the perpendicular height of a visible point above the horizontal plane drawn through our position, without approaching the point.

13. (a) To find the perpendicular height of one visible point above another, without approaching either point. (b) To find the direction of a line connecting two points, without approaching them.
14. To find the depth of a ditch, that is the perpendicular height from its floor to the horizontal plane either through our position or through any other point.
15. To tunnel through a hill in a straight line, where the mouths of the tunnel are given.
16. To sink shafts for a tunnel under a hill, perpendicular to the tunnel.
17. To lay out a harbour wall on a given segment of a circle between given ends.
18. To mound up the ground in a given segment of a spherical surface.
19. To grade the ground at a given angle, so that on a level site with the shape of an equal-sided parallelogram its gradient slopes to a single point.
20. To find a point on the surface above a tunnel so that a auxiliary shaft can be sunk.
21. To lay out with the dioptra a given distance in a given direction from us.
22. To lay out with the dioptra a given distance from another point, parallel to a given line, without approaching the point having the line on which to lay it out.
23. –30. The first five chapters refer to the dioptra setting out irregular shaped plots of land, while the remaining three explain how to determine the areas from those figures.
31. To measure the discharge or outflow of a spring.
32. & 33. Describes how to utilize the dioptra in a vertical mode for the purposes of astronomical observations.
34. This chapter informs the reader about the usage of another measuring instrument called the odometer, which has a device fitted to the wheels of a carriage such that the horizontal distance is evaluated in a very similar fashion in which a modern-day perambulator gives distance.

A good idea of the particular contents of the *Pneumatica* can be taken by listing the chapters of the books, as translated by Woodcroft (1851):

A Treatise on Pneumatics

1. The bent siphon.
2. Concentric or enclosed siphon.
3. Uniform discharge siphon.

4. Siphon which is capable of discharging a greater or less quantity of liquid with uniformity.
5. A vessel for withdrawing air from a siphon.
6. A vessel for retaining or discharging a liquid at pleasure.
7. A vessel for discharging liquids of different temperatures at pleasure.
8. A vessel for discharging liquids in varying proportions.
9. A water jet produced by mechanically compressed air.
10. A valve for a pump.
11. Libations on an altar produced by fire.
12. A vessel from which the contents flow when filled to a certain height.
13. Two vessels from which the contents flow, by a liquid being poured into one only.
14. A bird made to whistle by flowing water.
15. Birds made to sing and be silent alternately by flowing water.
16. Trumpets sounded by flowing water.
17. Sounds produced on the opening of a temple door.
18. Drinking horn from which either wine or water will flow.
19. A vessel containing a liquid of uniform height, although a stream flows from it.
20. A vessel which remains full, although vater be drawn from it.
21. Sacrificial vessel which flows only when money is introduced.
22. A vessel from which a variety of liquids may be made to flow through one pipe.
23. A flow of wine from one vessel, produced by water being poured into another.
24. A pipe from which flows wine and water in varying proportions.
25. A vessel from which wine flows in proportion as water is withdrawn.
26. A vessel from which wine flows in proportion as water is poured into another.
27. The fire-engine.
28. An automaton which drinks at certain times only, on a liquid being presented to it.
29. An automaton which may be made to drink at any time, on a liquid being presented to it.
30. An automaton which will drink any quantity that may he presented to it.
31. A wheel in a temple, which, on being turned liberates purifying water.

32. A vessel containing different wines, any one of which may be liberated by placing a certain weight in a cup.
33. A self-trimming lamp.
34. A vessel from which liquid may be made to flow, on any portion of water being poured into it.
35. A vessel which will hold a certain quantity of liquid when the supply is continuous, will only receive a portion of such liquid if the supply is intermittent.
36. A satyr pouring water from a wine-skin into a full washing-basin, without making the contents overflow.
37. Temple doors opened by fire on an altar.
38. Other intermediate means of opening temple doors by bire on an altar.
39. Wine flowing from a vessel may be arrested on the introduction of water, but, when the supply of water ceases, the wine flows again.
40. On an apple being lifted, Hercules shoots a dragon which then hisses.
41. A vessel from which uniform quantities only of liquid can be poured.
42. A water-jet actuated by compressed air from the lungs.
43. Notes from a bird produced at intervals by an intermittent stream of water.
44. Notes produced from several birds in succession, by a stream of water.
45. A jet of steam supporting a sphere.
46. The world represented in the centre of the universe.
47. A fountain which trickles by the action of the Sun's rays.
48. A thyrsus made to whistle by being submerged in water.
49. A trumpet, in the hands of an automaton, sounded by compressed air.
50. The steam-engine.
51. A vessel from which flowing water may be stopped at pleasure.
52. A drinking-horn in which a peculiarly formed siphon is fixed.
53. A vessel in which water and air ascend and descend alternately.
54. Water driven from the mouth of a wine-skin in the hands of a satyr, by means of compressed air.
55. A vessel, out of which water flows as it is poured in but if the supply is withheld, water will not flow again, until the vessel is half filled and on the supply. Being stopped again, it will then not flow until the vessel is filled.
56. A cupping-glass, to which is attached, an air exhausted compartment.
57. Description of a syringe.

58. A vessel from which a flow of wine can be stopped, by pouring into it a small measure of water.
59. A vessel from which wine or water may be made to flow, separately or mixed.
60. Libations poured on an altar, and a serpent made to hiss, by the action of fire.
61. Water flowing from a siphon ceases on surrounding the end of its longer side with water.
62. A vessel which emits a sound when a liquor is poured from it.
63. A water-clock, made to govern the quantities of liquid flowing from a vessel.
64. A drinking-horn from which a mixture of wine and water, or pure water may be made to flow alternately or together, at pleasure.
65. A vessel from which wine or water may be made to flow separately or mixed.
66. Wine discharged into a cup in any required quantity.
67. A goblet into which as much wine flows as is taken out.
68. A shrine over which a bird may be made to revolve and sing by worshippers turning a wheel.
69. A siphon fixed in a vessel from which the discharge shall cease at will.
70. Figures made to dance by fire on an altar.
71. A lamp in which the oil can be raised by water contained within its stand.
72. A lamp in which the oil is raised by blowing air into it.
73. A lamp in which the oil is raised by water as required.
74. A steam-boiler from which a hot-air blast, or hot-air mixed with steam is blown into the fire, and from which hot water flows on the introduction of cold.
75. A steam-boiler from which either a hot blast may be driven into the fire, a blackbird made to sing, or a triton to blow a horn.
76. An altar organ blown by manual labour.
77. An altar organ blown by the agency of a wind-mill.
78. An automaton, the head of which continues attached to the body, after a knife has entered the neck at one side, passed completely through it, and out at the other; which animal will drink immediately after the operation. Appendix.

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WILLIBALD LICHTENHELDT (1901–1980)

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Abstract. Professor Lichtenheldt was the Nestor of Mechanisms and Machine Theory after the 2nd World War at TU Dresden, Germany. He became an honorary member of IFToMM in Zakopane 1969. He was an excellent teacher and educated a new generation of students in mechanism design. He also wrote several textbooks and numerous reports and papers on his research.

Biographical Notes

Willibald Lichtenheldt, Figure 1, was born on 30 October 1901, the son of a painter in Werdau (Saxony), where he received his early education and training.

Fig. 1. Willibald Lichtenheldt (1901–1980).

M. Ceccarelli (ed.), Distinguished Figures in Mechanism and Machine Science, 247–265.

He completed his pre-college schooling and Abitur in 1918 at the Realschule in the city of Werdau. His continuation school training as probationer in fine-mechanics was done at MASS-Industrie GmbH Werdau (MASSI). Then he studied at the Ingenieurschule Zwickau for three years. One of his teachers was Dr. Curt Beyer, who later became famous at the Oskar v. Miller Polytechnikum Munich. Lichtenheldt earned his Ing. Degree in 1922 and went directly to textile industry for practical work. For several years he worked as a designer of weaving machines in the Maschinenfabrik Oscar Moeschler, Meerane (Saxony).

The topic "weaving machine" encouraged him to study the analysis and synthesis of mechanisms more deeply. He came into contact with the scientific work of Burmester [1], Krause [2] and Alt and transferred the results directly to practical applications. His first papers [3–5] in this field were published in connection with the VDI-Mechanism Conferences, which were held in Germany from 1926 every two years. He became also a member of the VDI in 1931 and was continuously active in VDI-Conferences [6–8].

It is remarkable that Lichtenheldt after ten years practical work in industry started in 1934 his study at TU Dresden to improve his scientific foundations. He attended basic lectures by Lagally in mathematics, by Kutzbach in machine elements, by Alt in mechanisms theory, etc. After achieving the Vordiplom he concentrated especially on analysis and synthesis of mechanisms and textile machines. He graduated in 1938 as Diplom-Ingenieur at TU Dresden (Figure 2), and was appointed a scientific assistant at TU Berlin-Charlottenburg by his former teacher Professor Hermann Alt.

His active scientific work in Berlin culminated in his doctor dissertation "Einfache Konstruktionsverfahren zur Ermittlung der Abmessungen von Kurbelgetrieben" in 1940 [9]. This paper is still important today for finding good approximate solutions which can be improved with modern computer technology. Lichtenheldt was very active in teaching and research work at TU Berlin. His second Dissertation (Habilitation) on the topic "Kinematische und dynamische Untersuchung der Webladen-antriebe" was finished in 1943 [10] and he was awarded at TU Berlin the academic degree Dr.-Ing. habil.

In 1943 he became Professor at TU Dresden in the theory of field mechanisms and started his lectures in the difficult atmosphere of wartime. On 13 February 1945 the TU Dresden was almost destroyed by intensive bombing. Lichtenheldt was one of the active Professors who worked very hard to re-

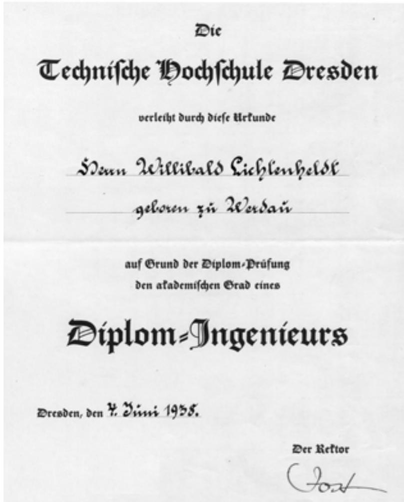

Fig. 2. Diplom-Certificate of W. Lichtenheldt.

construct the campus. During the following five years Lichtenheldt was once more active in industry, ending this period of his career at Carl Zeiss Jena. On 4 August 1950 he was appointed once more as full Professor at TU Dresden for mechanisms theory and fine mechanics and as director of the same institute in the framework of the faculty of mechanical engineering.

His lectures included the following topics:

- Projektionslehre und Kinematik (Technical drawing and kinematics)
- Getriebelehre I, II, III (Mechanisms theory, three courses I, II, III)
- Feinmechanik (Fine mechanics)
- Textilmaschinenkonstruktion (Design and construction of textile machines).

He encouraged his students with his excellent pedagogical talent; his drawings at the blackboard trained the eye to look for the details in design. The students enjoyed watching the development of his sketches and appreciated the clarity of his presentation of the engineering principles. On the basis of these lectures he wrote his textbook *Konstruktionslehre der Getriebe*, which has been published in five editions [20] (Figure 3). The first edition was translated into the Russian language. The fifth edition has been extended by Professor Kurt Luck. Moreover he wrote many outstanding scientific papers

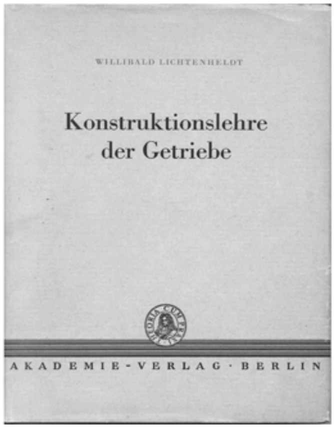

Fig. 3. Textbook of Willibald Lichtenheldt.

[11–19] and excellent material for self-study corresponding to his lectures [21, 22, 23].

He was elected as Dean of the Faculty of Mechanical Engineering for the period 1952–1955. In his welcome address to the new students, matriculated in 1954, he characterized the important work of the designer with the following words:

> The designer is very important for realizing the technical progress at all. The efficiency of the industrial production is determined by the technical standard of the used machines as well as by the standard of the produced machines. The basis for realizing this fact is the creativity of design engineers. The standard of life in our country is based upon the efficiency of such experts in engineering sciences.

For improving effectiveness in the design process he supported the creation of diagrams for mechanism design. Such diagrams are very useful in finding optimal solutions for realizing special transfer functions by mechanisms. The scientific work of his team was presented at a number of mechanism conferences in Germany, especially at TU Dresden in 1953, 1956, 1958, 1960, 1962 and 1966. The last event was the 25th mechanism conference in Germany with around 500 participants and represented the coronation of his career. Many scientists took part in this important conference, among others Academician Professor I.I. Artobolevsky from Moscow, Professor M. Konstantinov from Sofia, Professor I. Salyi from Hungary, Professor J. Oderfeld

Fig. 4. Mechanism Conference in Germany at TU Dresden in 1966; 1st row, left to right: B. Dizioglu, M. Konstantinov, K. Hain, W. Rehwald, W. Lichtenheldt; 2nd row, left to right Kohler, Nina & Professor I.I. Artobolevsky, Moscow, W. Rössner, A. Bock.

and Professor A. Morecki from Warsaw, Professor B. Dizioglu and Dr. K. Hain from Braunschweig, Professor W. Rehwald from Stuttgart (Figure 4).

His teaching as Ordinarius in the field of mechanism theory was excellent and particularly significant after the 2nd World War to the new generation of engineers at TU Dresden. During his lectures in the Auditorium Maximum (which could hold 500 students) it was absolutely silent, no noise, you could hear a pin falling. Many students remember these years with great fondness.

The results of the scientific work inspired by Lichtenheldt in mechanisms theory, produced by undergraduate students in joint projects and diplom-theses and by postgraduate students in doctor-theses, found direct applications in several branches of industry, e.g. in textile machinery, food machinery, packing machinery, printing machinery, earthmoving machinery, agricultural machinery, precision machinery.

For his meritorious work Lichtenheldt was honoured in 1961 by the "Vaterländischen Verdienstorden in Silber", in 1962 by the "Nationalpreis III. Klasse", and in 1978 by "Hervorragender Wissenschaftler des Volkes". He was appointed as a member of the Berlin Academy of Sciences in 1959. The TU Magdeburg granted to him the "Doctor Degree Ehrenhalber, Dr.-Ing.

Fig. 5. II. IFToMM Congress 1969 in Zakopane (Poland), organized by Professor. J. Oderfeld, left to right Professor W. Lichtenheldt, Nina & Professor I.I. Artobolevsky, Moscow, Professor Muster, USA.

Eh." in 1976. During the Inaugural Assembly of the II. IFToMM Congress in 1969 in Zakopane (Figure 5), Lichtenheldt was recommended as a member of the Honorary Roll of the International Federation for the Theory of Machines and Mechanisms.

Today "Mechanism and Machine Science" is an important part in the education-curriculum of students in the field of mechanical engineering. The work and life of Willibald Lichtenheldt was always directed to this aim.

List of (Main) Works

- Zur Getriebetechnik der Webstühle (Analysis of mechanisms in weaving machines), *Zeitschrift VDI* **75** (1931) 904–908.
- Koppelgetriebe für den Webstuhl (Mechanism design for a weaving loom), *Zeitschrift VDI* **78** (1934) 352–354.

- Zur Synthese der Webstulgetriebe (Synthesis of mechanisms in a weaving loom), *Meliand Textilberichte* **16** (1935) 477–480.
- Über den Schlagmechanismus des mechanischen Webstuhls (Remarks on the impact-mechanism of a weaving loom), *Masch.-Bau Betrieb* **15** (1936) 405–407.
- Die Koppelkurvenfräsmaschine (The coupler curve-cutting machine), *Masch.-Bau Betrieb* **17** (1938) 145–147.
- Reguliergetriebe für Wasserturbinen (Adjustable mechanism for water-turbines), *Masch.-Bau Betrieb* **17** (1938) 596–597.
- Einfache Konstruktionsverfahren zur Ermittlung der Abmessungen von Kurbelgetrieben (Simple drawing methods for determination of the geometrical dimensions of linkages), *VDI Forschungsheft* Nr. 408, VDI-Verlag, Berlin, 1941.
- Kinematische und dynamische Untersuchung der Webladenantriebe (Kinematical and dynamical investigation of mechanisms in weaving machines), *Faserforschung und Textiltechnik* **2** (1951) 89–104 and 141–153.
- Zur Konstrukion von Gelenkgetrieben (General remarks on the design of mechanisms), *Wiss. Zeit. der TH Dresden* **1** (1951/1952) 71–76.
- Die Bedeutung der Konstruktionslehre für die Feinmechanik (The importance of mechanisms theory to fine-mechanics), *Wiss. Zeitschrift der TH Dresden* **3** (1953/1954) 211–214.
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- Zur Geometrie des Wippkranes (Remarks on the geometry of the luffing crane), *Wiss. Zeitschrift der TH Dresden* **3** (1953/1954) 555–558.
- Lenkergeradführungen im Feingerätebau (Straightline-mechanisms in precision mechanics), *Feingerätetechnik* **4** (1955) 447–450.
- Die Methode der Partialsynthese (The partial-synthesis – A new method in mechanism synthesis), *Wiss. Zeitschrift der TH Dresden* **5** (1955/1956) 79–82.
- Konstruktionstafeln für Geradführungsmechanismen (Diagrams for the design of straightline-mechanisms), *Maschinenbautechnik* **7** (1958) 609–611.
- Die Anwendung der Geometrie bei Getriebekonstruktionen (The importance of Geometry for linkage-design), *Wiss. Zeitschrift der TH Dresden* **8** (1958/1959) 341–346.

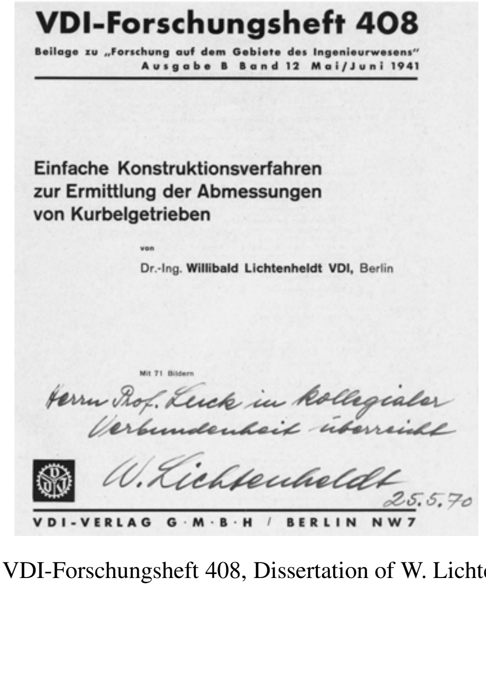
- Zur Krümmung der Bahnen von Koppelpunkten (Remarks on the curvature of coupler curves), *Wiss. Zeitschrift der TU Dresden* **10** (1961) 1411–1416.
- *Konstruktionslehre der Getriebe* (Theory and Design of Mechanisms and Linkages), 1st edition, Akademie-Verlag, Berlin (1961), 2nd edition (1965), 3rd edition (1967), 4th edition (1970), 5th extended edition (1977).

Review of Main Works on Mechanism Design

Several main works of Professor W. Lichtenheldt will be discussed below.

(1) His doctor dissertation "Einfache Konstruktionsverfahren zur Ermittlung der Abmessungen von Kurbelgetrieben" (Simple drawing methods for determination of the geometrical dimensions of linkages), which was published in VDI Forschungsheft Nr. 408, VDI-Verlag, Berlin 1941 (Figure 6).

He deepened the investigation of the Burmester theory, especially the useful application of special cases of Burmester curves, e.g. circle and straight-line, equilateral-hyperbola and infinite-straight-line, two perpendicular-



VDI-Forschungsheft 408

Beilage zu „Forschung auf dem Gebiete des Ingenieurwesens"
Ausgabe B Band 12 Mai/Juni 1941

Einfache Konstruktionsverfahren
zur Ermittlung der Abmessungen
von Kurbelgetrieben

von

Dr.-Ing. **Willibald Lichtenheldt VDI,** Berlin

Mit 71 Bildern

Herrn Prof. Luck in kollegialer
Verbundenheit überreicht
W. Lichtenheldt
25.5.70

VDI-VERLAG G·M·B·H / BERLIN NW7

Fig. 6. VDI-Forschungsheft 408, Dissertation of W. Lichtenheldt.

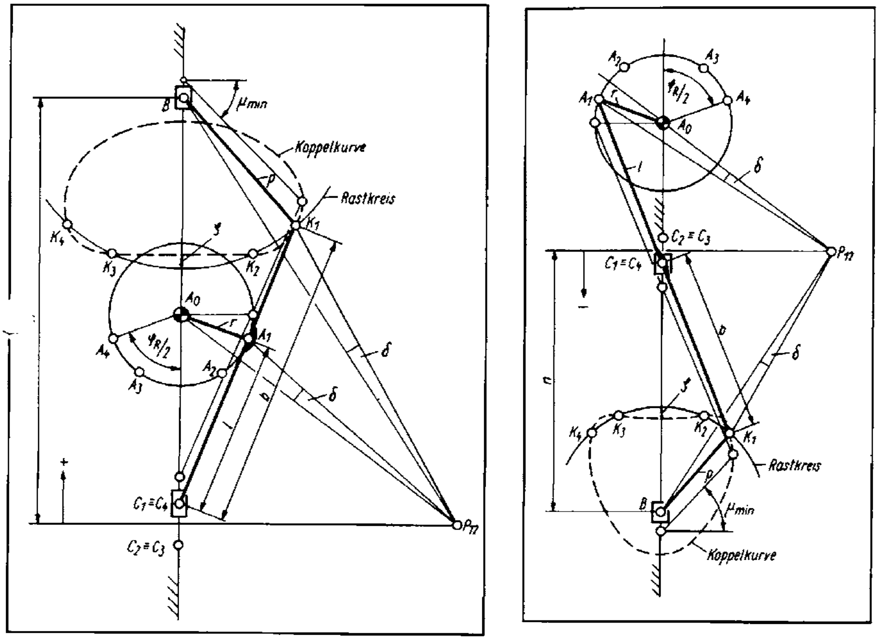


Fig. 7. (a) Linkage Type I, (b) Linkage Type II.

straight-lines and infinite-straight-line. Both types of Burmester curves, centre-point curves and circle-point curves, are taken into consideration to find simple graphical methods for the design of mechanisms with respect to special transfer functions.

On the basis of these graphical methods in the 50th and 60th decades of the 20th century many diagrams were developed for rationalization of the designing process.

Example: Graphical method to design a six-bar linkage for a good resting period of the output, correspondent to a given input angle [20, 24, 25]. Figure 7 demonstrates the graphical construction.

The construction of linkage type I is the following: Starting from the external dead-centre position of the slider crank mechanism the angle $\varphi_R/2$, drawn symmetrically to the line, in the points A_1 and A_4 on the circle with radius r around A_0. The quarter of the resting-angle φ_R results in the points A_2 and A_3 The coupler-length $l = 2.5 \cdot r$ delivers $C_1 \equiv C_4$ and $C_2 \equiv C_3$. The middle-perpendicular lines through A_1A_2 and C_1C_2 result in the pole P_{12}; the angle $\angle A_0P_{12}A_1 = \delta$ put at BP_{12} in P_{12} in the same sense results in K_1 on

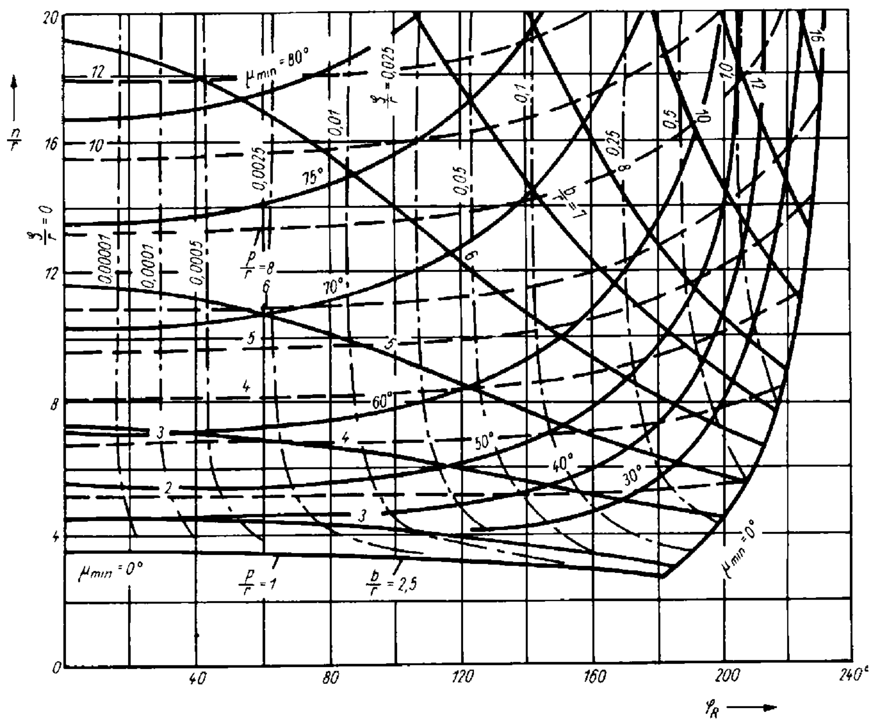


Fig. 8. Diagram for linkage type I.

the line C_1A_1. Point B is chosen on the sliding line according to the distance "n" in the positive direction. A circle with radius $p = BK_1$ approximates the coupler curve of point K_1 corresponding to the resting-angle φ_R, therefore point B is nearly resting in this moving branch. The slider in B can be compensated by an oscillating link with given swinging angle. The slider in C also can be compensated by a long link (length $> 6r$). The corresponding diagram is demonstrated in Figure 8.

This diagram has been created for the basic slider crank mechanism with $r : l = 1 : 2.5$; it is constant for all six-bar linkages of type I; and the radius is $r = 1$. The diagram includes the following parameters; all parameters for length are connected to the radius r.

- $r = A_0A_1 \rightarrow$ radius of the input link,
- $n \rightarrow$ distance of the resting point B from the middle perpendicular line m', rectangular to C_1C_2,
- $b \rightarrow$ distance between the coupler point K and the sliding point C,
- $p \rightarrow$ link-length between the points K and B,
- $\varphi_R \rightarrow$ resting angle,
- $\mu_{\min} \rightarrow$ transmission angle, see Figures 7a and 7b,
- $\varsigma \rightarrow$ criterion for the quality of resting period.

Practical example. Given parameters: $\varphi_R = 60°$, $r = 10$ mm, $l = 2.5 \cdot r = AC = 25$ mm,

$$n/r = 10 \rightarrow n = 10 \cdot r = 100 \text{ mm.}$$

Solution: According to the diagram in Figure 8, we get the following parameters:

$$p/r = 5.3 \rightarrow p = 53 \text{ mm}, \quad b/r = 4.7 \rightarrow b = 47 \text{ mm},$$

$$\varsigma/r = 0.002 \rightarrow \varsigma = 0.002 \text{ mm}, \quad \mu_{\min} = 67°.$$

Analogous to linkage type I we can find the parameters for linkage type II by using the diagram in Figure 9.

(2) His textbook *Konstruktionslehre der Getriebe* (Theory and Design of Mechanisms and Linkages) was published in five editions from 1961 through 1977 [20]. It includes the basics of analysis and synthesis of mechanisms. The special topics are listed in the following chapters:

- Type synthesis
- Scientific foundations of kinematics
- Synthesis of plane mechanisms – Burmester theory
- Design methods for exact synthesis of four-bar linkages and its numerical solution
- Cam mechanisms, e.g. type-synthesis, transfer functions, design
- Stepping mechanisms, e.g. geneva drive mechanisms, geared linkages
- Force analysis in linkages and cam mechanisms
- Spatial linkages, e.g. type-synthesis, analysis and synthesis
- Diagrams for rationalization of the design work.

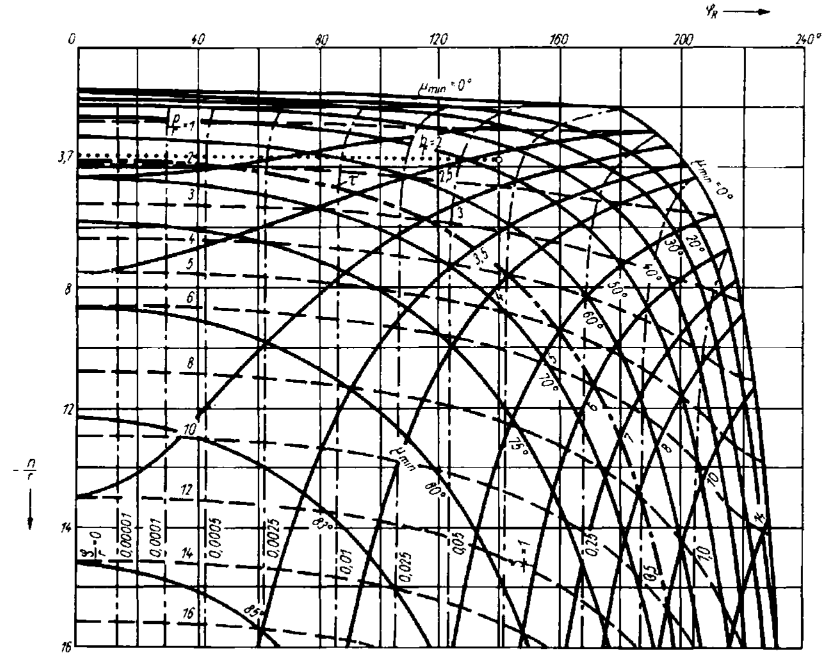


Fig. 9. Diagram for linkage type II.

A special topic of the last chapter is the so-called dead-position construction of the crank-rocker mechanism according to Professor Alt (Alt'sche Totlagen-Konstruktion). This graphical method gives a possibility of finding good solutions by respecting special restrictions, e.g. input-output angle, transmission angle etc. This dead-position construction of Professor Alt is demonstrated in Figure 10.

The angles φ_0 and ψ_0 are the corresponding input-output angles of the crank and rocker mechanism. A dead centre position is characterized by two infinitely separated positions of the crank A_0A_1, which are correspondent to two identical positions of the output link B_0B_1. In this position the output link changes its direction. The oscillating angle ψ_0 is limited by two dead positions of the output. On the basis of the dead-position construction of Professor Alt, the diagram in Figure 10 has been created.

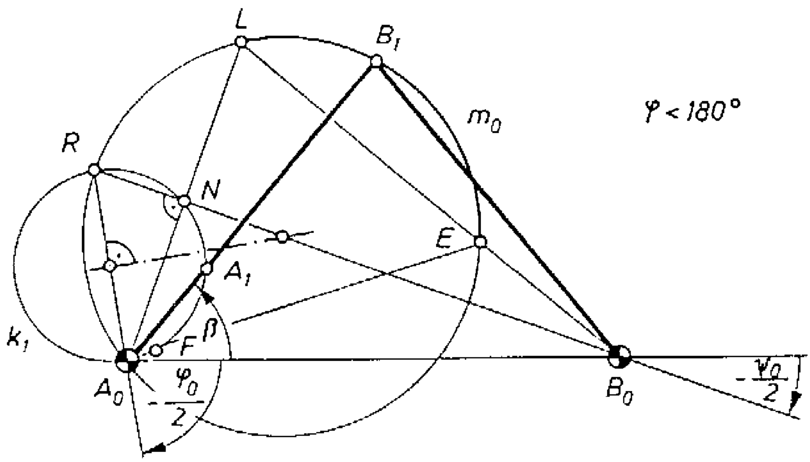


Fig. 10. Alt'sche Totlagenkonstruktion (Dead-position construction of Alt).

The diagram (Figure 10) includes the following parameters:

- φ_0 and ψ_0 input-output angles of the crank-rocker mechanism
- $< \beta =< B_0A_0B_1$, see Figure 10
- max $\mu_{\min}$, maximal transmission angle.

Given parameters: input angle $\varphi_0 = 160°$, output angle $\psi_0 = 40°$.

Solution: According to the diagram in Figure 11, we get the following parameters: $< \beta = 50°$, max $\mu_{\min} = 33°$.

Angle β crosses the circles k_1 and m_0 in A_1 and B_1 according to Figure 10. In this graphical way the length of crank A_0A_1, coupler A_1B_1 and output link B_0B_1 are determined. A chosen length of the frame $A_0B_0 = 150$ mm results in the following length of crank $A_0A_1 = 38$ mm, coupler $A_1B_1 = 80$ mm, and oscillating link $B_0B_1 = 117$ mm.

Modern Interpretation of Main Contributions to Mechanism Design

The most important achievements of Professor Willibald Lichtenheldt exist in research work and education of the new generation (after the 2nd World War) in the field of mechanism theory and design. He was a master of interpretation of geometrical connections for the design process by linkages. On

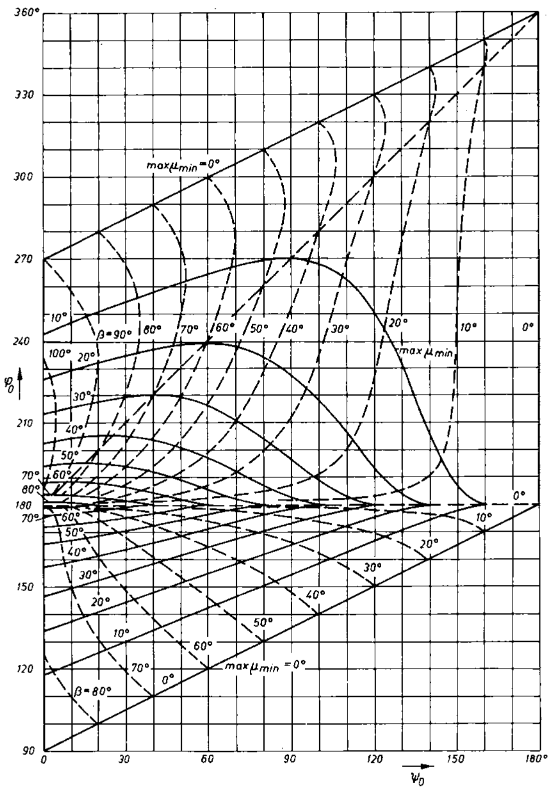


Fig. 11. Diagram for crank-rocker mechanisms with input-output angles φ_0-ψ_0 and optimal transmission angle max $\mu_{\min}$.

the basis of Burmester's theory [1] he investigated simple graphical methods to improve the design process, to raise the level of efficiency, analogous to the modern CAD – Computer Aided Design. The TU Magdeburg granted to him the Doctor Degree Ehrenhalber; during the ceremony on behalf of this event he gave an excellent speech on the topic "Integration of CAD – Computer-Aided-Design in Mechanisms Theory".

The last decades starting from 1970 have exhibited great progress in the field of machine and mechanism science by using modern CAD equipment. The diagrams for mechanism design have been improved by numerical calculations by using CAD computer techniques.

Most mechanisms of the modern day, as well as those developed in the last century, have become increasingly complex. Common examples are automobiles, tractors, bulldozers, carpet sweepers and concrete mixers. Computer Aided Analysis is now considered essential in the design of such complex mechanical systems as well as to respond to the market trends by developing new products in a shorter time. Coupled with dynamic analysis, the field of mechanisms has reached a high perfection and reliability. Mechanisms is the foundation for ultimate machine design. During the last 15 years Rehwald and Luck investigated the topic “Computer Aided Linkage Simulation” [26–32].

Several software packages are available on the market today for analysis and simulation of mechanisms, which makes these analyses and incorporation of multi-system concepts much easier. It must, however, be emphasized that such software is no substitute for human ingenuity and imagination.

A variety of machinery used in critical operations such as packaging, conveying and off-shore handling, still consists of several types of two-dimensional kinematic chains and their respective inversions. Most software currently available on the market does not address the analysis of two-dimensional kinematic chains using mathematical and graphical techniques, such as coupler curves and hodographs, developed over the past decades, but instead encourage the designer to work from a clean slate in three-dimensional space. The software KOSIM developed by Rehwald and adapted by Luck to various applications differs in this aspect.

The KOSIM software [30] focuses on the design and analysis of planar mechanisms. The simplicity and elegance of the software and the short time required for mastering it, make it attractive for designers of several types of machinery. The authors of KOSIM have conceived an ingenious nomenclature for designating planar mechanisms and the constituent kinematic chains based on terminology evolved by the International Federation for the Promotion of Mechanism and Machine Science (IFToMM).

The book *KOSIM – Computer Aided Linkage Simulation*, which is based on the original version in the German language by the authors, is an excellent description of the novel methodology used in the KOSIM software [31]. The book also describes in vivid detail the theory of kinematic and dynamic analysis of mechanisms. Numerous practical examples and graphical illustrations make the book one of the most useful texts for designers, practicing engineers, software developers, academicians and students. Figures 12–14 demonstrate some applications of this modern software.

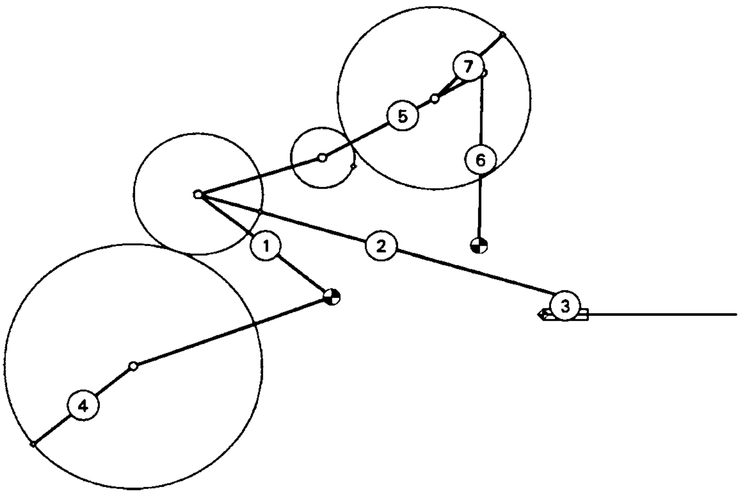


Fig. 12. Animation of an eight-bar linkage.

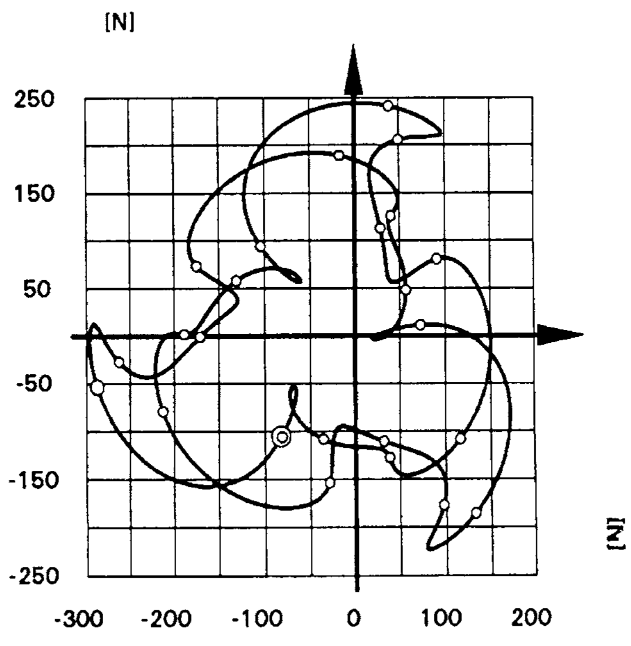


Fig. 13. Hodograph of the force-vector of an eight-bar linkage.

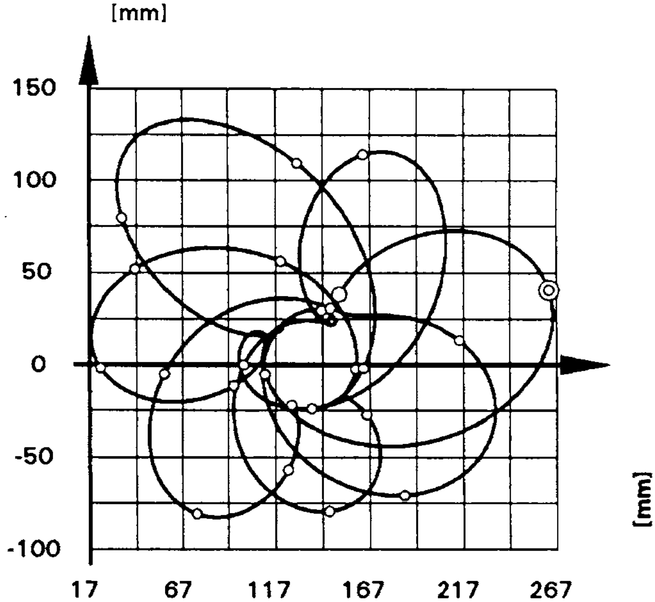


Fig. 14. Coupler-curve of another eight-bar linkage.

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XIAN-ZHOU LIU
(1890–1975)

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Abstract. Liu Xian-Zhou was a pioneer Chinese scientist who studied the history of ancient Chinese mechanisms and the first to write, in Chinese, systematic descriptions of the machines and their functions. He was truly China's leading pioneer in mechanical engineering. In addition to his scholarly publications, he was the first author to write textbooks in Chinese for students entering the fields of mechanisms and machines.

Biographical Notes

Liu Xian-Zhou was born on January 27, 1890 at Wan County in He-Bei Province, China (Figure 1). He passed away in 1975 at the age of 86.

Fig. 1. Liu Xian-Zhou (1890–1975) (Dong and Li, 1990).

M. Ceccarelli (ed.), Distinguished Figures in Mechanism and Machine Science, 267–278.

When Liu was a child, he directly experienced the hard labor of agriculture by working with his father on the family farm. When Liu was 8 to 16 years old (1897–1905), he studied in an old-style private school for traditional Chinese and classical literature. With such a basic training, he had a solid background to study ancient Chinese historical records. Between 17 to 23 years old (1906–1912), Liu was a middle-school student in He-Bei Province, graduating with an excellent academic grade of 96.4. In 1914, Liu enrolled in the Department of Engineering at Hong Kong University, at government expense. He graduated in 1918 with a B.S. degree and received the "First Class Honor".

During 1918 to 1921, Liu was a teacher at a vocational school in He-Bei Province. He emphasized that theory and experiment should be equally important, and set up a practical training factory for the school. At age 34 (1924), Liu accepted an invitation to become President of the oldest Chinese engineering university, Bei Yang University, now Tianjin University. He set out to upgrade the academic excellence of the University to become the "Massachusetts Institute of Technology of China".

During 1928 to 1931, he was Chairman of the Department of Engineering at Northeastern University and offered courses such as Theory of Machines and Thermodynamics. In 1932, he joined the Department of Mechanical Engineering at Tsinghua University. He studied numerous ancient Chinese books and literature relating to mechanical technology and engineering, and he proposed standard Chinese terminology in mechanical engineering. During 1938 to 1946, Liu was a member of the faculty of the Department of Engineering at South-West Associated University and a Vice-Chairman in the Chinese Society of Mechanical Engineers.

During 1946 to 1947, Liu visited the USA to investigate and study agriculture machinery. After returning to China, he started to write teaching materials in agriculture machinery and taught the subject at Tsinghua University. After 1949, Liu was a Vice-President at Tsinghua University, and the Chairman of the Chinese Society of Mechanical Engineers and the Chinese Society of Agriculture Machinery.

Liu devoted his life to the development of a mechanical engineering curriculum, especially mechanism and machine science. He supervised many students and played an important role in early Chinese engineering education in the first half of the 20th century. Liu Xian-Zhou had two major academic achievements: writing Chinese teaching materials in mechanical engineering and studying the history and inventions of ancient Chinese mechanisms and machines. During 1918 to 1948, he wrote fifteen books. He was the first professor to write college textbooks in modern science and technology in Chinese. Furthermore, Liu was the first Chinese to study and present the inventions of ancient Chinese mechanisms and machines systematically. He wrote eighteen papers and two books regarding ancient Chinese mechanisms and machines.

Works on Mechanism and Machine Theory

Contributions to Textbooks and Education in Chinese

In the early 20th century, almost all teachers in higher education in China adapted foreign materials and conducted classes in foreign languages. Liu insisted on using Chinese teaching materials for engineering and technology, and he wrote Chinese textbooks by himself. From 1918 to 1948, he completed a series of Chinese textbooks relating to mechanical engineering, such as Engineering Drawings, Principles of Machinery, Dynamics of Machinery, Agricultural Machinery, Thermodynamics, Heat Engines, Steam Engines, Internal Combustion Engines, Gas and Valve Sensors, and Physics. Thus Liu became the first scholar to edit foreign teaching materials in engineering and technology and translate them into Chinese, which was a truly important contribution to the development of the curriculum of modern mechanical engineering in China.

From the end of the 19th century to the beginning of the 20th century, the Chinese translation of terminology in mechanical engineering was not unified and was confusing. Liu accepted an assignment from the Chinese Institute of Engineers to compile “English-Chinese Terminology of Mechanical Engineering”. He extensively reviewed relative books and literature relating to engineering and technology since the Ming Dynasty (1368–1644). Then, he identified suitable words as the Chinese translation according to the following four points: should be feasible, should be simple, should be general, and should be familiar. This book was published in 1934 with around 11,000 words. Two later versions were published in 1936 and 1945, respectively, with about 20,000 words.

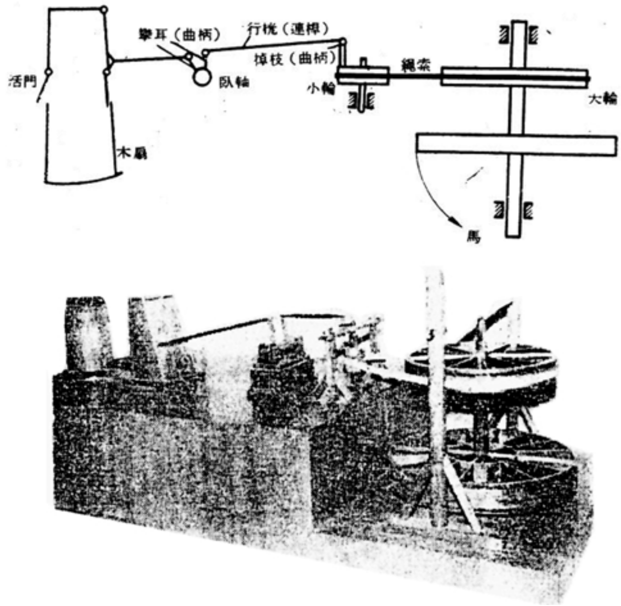

Fig. 2. Animal and water-driven mechanisms of piston bellows (Liu, 1962).

Achievements in Agricultural Machinery

In 1920, Liu designed two types of waterwheels to improve the efficiency of water supply for farmers. One type was designed for manpower input and another was for use of animal power. Not only was the manufacturing of the two designs simple, but also the efficiency of the water supply process was improved. During 1937–1945, Liu also studied and modified other machines such as the plow, waterwheel, and water pump. And, he published a paper entitled “Problems Regarding the Improvement of Chinese Agricultural Machinery”.

In 1946, Liu went to the USA and for one and a half years investigated and studied American agricultural machinery. He collected over 500 related books and documents. After the trip, Liu realized that agriculture was the foundation of every development in China. Since the major powers of agricultural machinery at the time were manpower and animal power, he suggested a temporary hold on importing heavy machines from abroad. Instead, he focused on the modification of existing manpower and animal power machines.

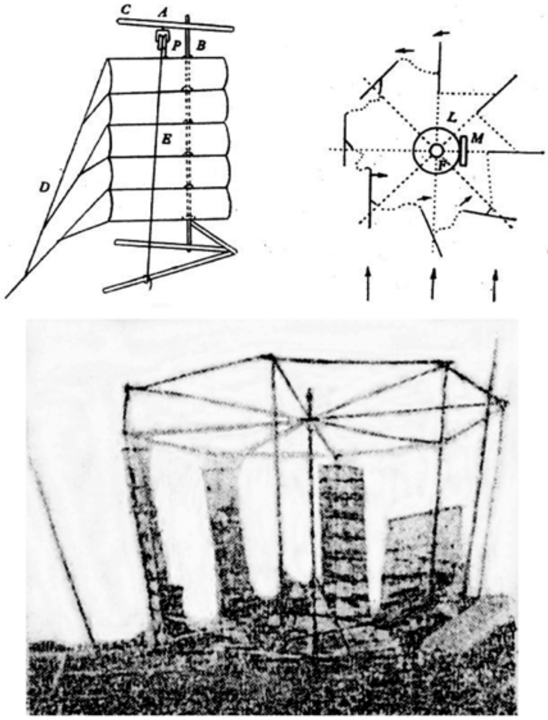

Fig. 3. A windmill (Liu, 1962).

And, he suggested a long-term plan for the development of agricultural machines with other types of power.

Liu's improved designs of waterwheels were soon adopted by many farmers and this key technology of agricultural machinery continued to be developed. A schedule for applying electrical power was also planned. Truly this was the time of the birth of technological development in Chinese agriculture.

In 1963, Liu authored a book entitled *History of Inventions of Ancient Chinese Agricultural Machinery*. In the book, Liu provided his comments and introduced many designs in agricultural machinery over the past several thousand years in China, including the following areas of agriculture machines: soil preparation, seeding, weeding, irrigation, harvesting and threshing, manufacture, and transportation. Reconstructions of the water-driven mechanism of piston bellows (Figure 2) and a windmill (Figure 3) were demonstrated and displayed at the National Museum of China in Beijing.

Research in Ancient Chinese Mechanisms and Machines

Liu studied extensive literature regarding the inventions of Ancient Chinese mechanisms and machines. In 1933, he wrote a book entitled *Introduction to Ancient Chinese Engineering Books*. In 1935, he presented a book entitled *Historical Records of Mechanical Engineering in China* including hand tools, transportation machines, agriculture machines, irrigating machines, weaving machines, an armory, potential fuels, printing on hard materials, a timekeeper, and foreign-import machinery. Liu studied over 2,000 ancient books and articles, and he produced about 16,000 files on cards. Furthermore, based on historical records and archaeological excavation, he thoroughly analyzed the development of ancient Chinese mechanical technology and commented extensively on them.

In 1962, Liu completed a classical book entitled *Chronology of Inventions of Ancient Chinese Mechanical Engineering, Volume 1*. In the book, the development and inventions of ancient Chinese mechanisms and machines was systematically introduced, including simple labor-saving devices and various types of mechanisms and machines. This book became an important publication regarding the history of ancient Chinese technology and deeply influenced later researchers in this topic. Many inventions introduced in this book, such as a water-driven device for astronomical observation, the south-pointing chariot, the hodometer, and five-wheel sand-clocks, were reconstructed and displayed at the National Museum of China in Beijing.

As to the device for water-driven astronomical observation, Liu studied and designed a gear mechanism and he reconstructed a prototype (Figure 4). Regarding the south-pointing chariot, Liu studied the designs by Wu De-Ren

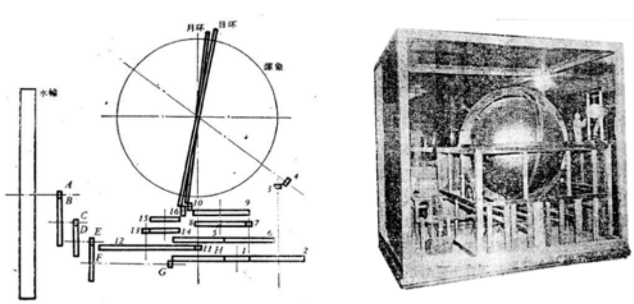

Fig. 4. Reconstruction of the gear mechanism of the device for water-driven astronomical observation (Liu, 1962).

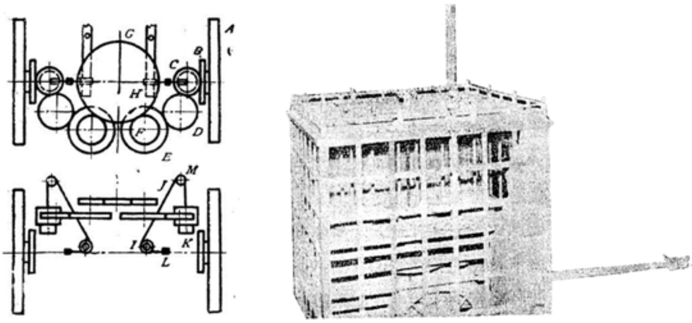

Fig. 5. Reconstruction of the mechanism of the south- pointing chariot (Liu, 1962).

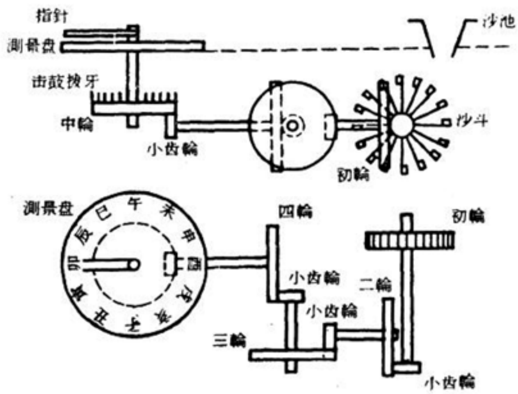

Fig. 6. Mechanism of the sand-driven wheel-clocks (Liu, 1962).

and George Lanchester. Furthermore, he cooperated with Wang Zhen-Duo to reconstruct a prototype (Figure 5). Regarding the five-wheel sand-clock, Liu reconstructed the mechanism of one of the sand-driven wheel-clocks (Figure 6). The five wheels are the driving-wheel (with scoops) below the feed, three large gear-wheels, and one middle wheel fitted with audible signal trip-lugs and borne on the shaft of the pointer (which made the rounds of the dial-face). On this design, the markings for the twelve double-hours can be seen. And, four small gear-wheels were connected below the main gears to transmit the motion.

Based on long-term study of literature regarding the development of ancient Chinese mechanisms and machines, Liu identified many unfamiliar designs to the western world, in addition to the four well-known inventions: paper, print, compass, and gunpowder. With regard to wind power, Liu pointed out that Chinese invented the sail 3,000 years ago, and the windmill was invented 900 years ago. As for water power, the water-driven mill was invented 2,000 years ago. The water-driven wind box was invented 1,900 years ago. The water-driven large spinner was invented 600 years ago. With regard to heat power, the prototype of a steam engine was invented 1,200 years ago. A rocket driven by gunpowder was invented 700 years ago. The prototype of a rocket was invented in the 14th century and the prototype of a two-stage rocket was invented in the beginning of the 17th century. With regard to gear transmissions, the hodometer and the south-pointing chariot, transmitted by gear trains, were invented at least 1,000 years ago. In the Eastern Hang Dynasty (25–220 AD), Zhang Heng invented the armillary sphere, which automatically indicated the date, 1,800 years ago. Monk Yi-Xin and Liang Lin-Chan invented a complicated astronomical and time-counting instrument 1,200 years ago. Zhan Xi-Yuan invented a timekeeper similar to the modern western clocks about 600 years ago.

Liu included these materials as part of his class notes in mechanical engineering. This greatly motivated young Chinese students' interest in learning and studying ancient inventions. His efforts also attracted scholars and students to study history of ancient Chinese mechanisms and machines in addition to popular areas such as histories of physics, chemistry, architecture, agriculture, and medicine.

These great echoes of history created an unforgettable image for students who had previously been educated in the ideas of Euclid, the ancient Egypt mathematician and his geometry, and Archimedes with his rules. They had no knowledge about ancient Chinese scientists and inventors in engineering and technology (Figure 7).

Liu's research efforts on ancient Chinese mechanisms and machines also attracted the attention of western scholars. He was invited to attend the Eighth International Conference on History of Science held in Florence in 1965 (Figure 8). He addressed the topic entitled "On Chinese Inventions of Time-Keeping Apparatus" (Liu, 1965). And, he indicated that the earliest time-counting device was designed on Zhang Heng's armillary sphere, driven by water power and transmitted by gear trains around 130 AD in China.

Fig. 7. Liu taught his student at museum (You, 2006).

Fig. 8. Group photo in the 8th International Conference on History of Science in 1965, X.Z. Liu (second from the left) and J. Needham (third from the left) (You, 2006).

Methodology on Studying Ancient Chinese Machines

Liu devoted most of his career to studying the development of ancient Chinese mechanical inventions. His methodology was scientific and effective.

In the process of research and writing, Liu studied as many original records and reports as possible. He compared documents and archaeological excavations to prove historical facts and to reveal the pattern of development. He also paid attention to new discoveries in archaeology, and he always tried his best to visit the sites. If he could not visit the excavation, he always asked for photos or copies of reports as part of the records.

An additional dimension to Liu's research was his comparison of ancient Chinese machines with Western designs and his studies of the social systems that affected the development of ancient science and technology.

Liu also devoted his efforts to the practical aspects of reconstruction of ancient machines. In 1959, he presented a paper entitled "Intensive Applications of Gear Trains in Ancient China". And, he built a facsimile of the transmission mechanism of Zhang Heng's armillary sphere. Liu further co-operated with Wang Zhen-Duo at the National Museum of China, and they reconstructed a wooden prototype to display to the public.

Modern Interpretation of Main Contributions to Mechanism Design

Liu was a pioneer in developing Chinese terminology in mechanical engineering. This work required familiarity with modern English and traditional Chinese, even local culture, including different Chinese spoken languages regarding mechanical terminology in the factories. Liu's efforts received a positive response in academia and industry. The "English-Chinese Terminology of Mechanical Engineering" published by the Chinese Academy of Sciences was based on the results of Liu's study. Furthermore, the Chinese terminology of mechanism and machine science was based on this publication.

In Liu's three books, he analyzed, organized and summarized historical literature, mechanical inventions, and agricultural machines in ancient China. This work needed tremendous manpower and long hours to review large numbers of documents from ancient literature. Liu never stopped his work of literature survey during his lifetime. He studied numerous ancient records and books, and he filed various ancient inventions on cards which became a very valuable database for other researchers to follow up in the future.

Liu's achievements in ancient Chinese mechanisms and machines also influenced the field of history of technology in the world. At the time, people thought that the astronomical clocks driven by mechanical gear transmissions were invented by the Europeans in the 14th century. After Liu's intensive study for about twenty years, he pointed out that there was a mechanical timekeeper in Zhang Heng's water-driven armillary sphere in the East Hang Dynasty around 130 AD. According to Liu's paper, the applications of gear transmissions in ancient China were already highly developed in the second century and the timekeeper by Zhang Heng was designed with gear transmissions.

Although Liu did not complete his work, many scholars followed his research approach and spirit. During the thirty years after Liu died, four scholars at Tsinghua University, Zhang Chun-Hui, You Zhan-Hong, Wu Zong-Ze, and Liu Yuan-Liang, continued and completed Liu's unfinished work. They edited a book entitled *Chronology of Inventions of Ancient Chinese Mechanical Engineering – Vol. 2* in 2004. They led the study in history of ancient Chinese mechanisms and machines to another landmark. The study of ancient mechanisms and machines is the key that links ancient history and modern history of mechanical engineering. With today's ever developing technology and requirements for better and newer products, reflection on and understanding of the past is crucial to drawing on that knowledge and generating new ideas. The birth of any new academic subject is difficult and arduous, but it requires the work of many followers of the pioneers in the field. Liu Xian-Zhou was such a pioneer.

Acknowledgement

The authors are grateful to Gao Xuan, vice-chairman of the Institute for History of Science and Technology & Ancient Texts at Tsinghua University, Beijing, for providing the literature regarding Liu Xian-Zhou.

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GIULIO MOZZI
(1730–1813)

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Abstract. Giulio Mozzi was the first to attack the study of the general helicoidal motion of a rigid body in a completely rigorous way. He outlined a Screw Theory with a mathematical formulation in a Treatise that was published in 1763 but had a limited circulation.

Biographical Notes

Giulio Giuseppe Mozzi del Garbo (known as Giulio Mozzi), see Figure 1, was born in Florence on February 23, 1730 to an aristocratic family.

He received a humanistic education that motivated him to become a poet. In 1756 he wrote the short poems "Inno al Sole" and "Ode alla Noia".

Later he was attracted to "mathematical studies" and he joined Paolo Frisi at the University of Pisa. Frisi was quite famous for his works on Astronomy and Mathematics, and he was a member of several European Academies. According to Marcolongo (1906), Frisi is believed to have first formulated correctly the composition of instantaneous rotations, as pointed out also in Ceccarelli (2000a).

During a period of home studies, he wrote the "Discorso matematico sopra il rotamento momentaneo dei corpi" (which can be translated as "A Mathematical Treatise on the Instantaneous Rotation of Bodies"), Figure 2, that was published in 1763 (Mozzi, 1763). Then, Mozzi travelled around Europe to learn about new developments in the scientific and political world.

Curiously, Giulio Mozzi wrote his book during an isolated period of sickness, to keep himself occupied. He thought that the Treatise would be of very limited interest or could be completely unnoticed (he mentioned this remark in introductory notes). In fact, he did not continue to study and work on the

M. Ceccarelli (ed.), Distinguished Figures in Mechanism and Machine Science, 279–293.

Fig. 1. A marble bust of Giulio Mozzi (1730–1813) from the archive of the Bartolini Salimbeni family in Florence.[1]

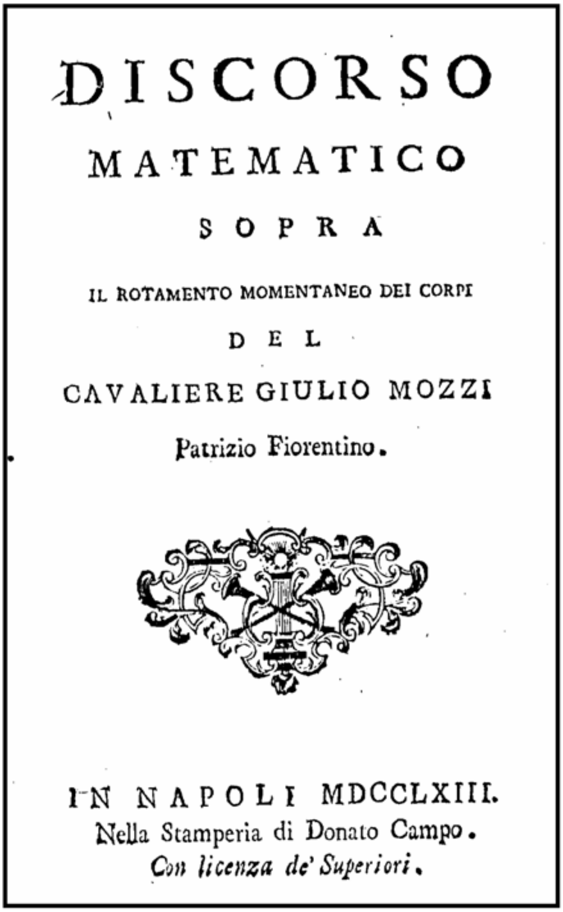
subject of his book, although he was recognised to be "a gentleman with delightful mind and nice knowledge" as indicated in De Tipaldo (1837).

After his return to Florence, he married Luisa Bartolini Salimbeni, who gave him two sons.

Giulio Mozzi was elected Academician in the "Accademia della Crusca" on September 7, 1754. He was so active that he was also elected in 1784 and again in 1808 as President of the new "Accademia Fiorentina" which had merged with the "Accademia della Crusca" in 1783.

Although during the political changes in that time he suffered some misfortune, his character was publicly recognised and he was elected senator in 1785. In 1801 he was appointed as Minister for Foreign Relations of Etruria during the reign of Ludovico I di Borbone. Subsequently Maria Teresa d'Austria confirmed him in this position until 1807. He was also appointed

[1] Marchesa Clementina Bartolini Salimbeni is gratefully acknowledged for the picture (Figure 1) she provided to the author. The picture was identified also by Dr. Arch. Lorenzo Bartolini Salimbeni. Unfortunately the marble statue has been stolen in the 1990s and only a picture remained in the archive of family Bartolini Salimbeni in Florence. No other images of Giulio Mozzi have been found although the research has been extended to several historical archives in Italy and in particular in Florence. In particular Academia della Crusca, Galleria degli Uffizi, Cimitero Monumentale di Firenze, Archivio di Stato per i Beni Culturali in Florence are gratefully acknowledged.



DISCORSO
MATEMATICO
SOPRA
IL ROTAMENTO MOMENTANEO DEI CORPI
DEL
CAVALIERE GIULIO MOZZI
Patrizio Fiorentino.

IN NAPOLI MDCCLXIII.
Nella Stamperia di Donato Campo.
Con licenza de' Superiori.

Fig. 2. Title page of the Treatise by Giulio Mozzi published in 1763.

by Napoleon in 1812 as a member of the re- established the "Accademia della Crusca" and he received the "Gran Croce della Riunione".

Giulio Mozzi died in Florence on April 16, 1813.

These biographical notes are outlined in agreement with an anonymous publication (1813), De Tipaldo (1837), Zobi (1850) and Marcolongo (1905), and are meant to portray a significant memory of him in the Italian tradition.

A Review of the Treatise by Giulio Mozzi

The book by Giulio Mozzi (Figure 2) can be considered of great importance not only for its fundamental contribution to kinematics, but also from a general Mechanics viewpoint.

In fact, the Treatise approaches Statics and Dynamics of rigid bodies with respect to impulsive forces and consequent motions. Indeed, the Treatise deals with several cases of determining the helicoidal motion for given forces and vice versa.

The screw axis was introduced by Giulio Mozzi in his Treatise on page 5 in Corollary IV:

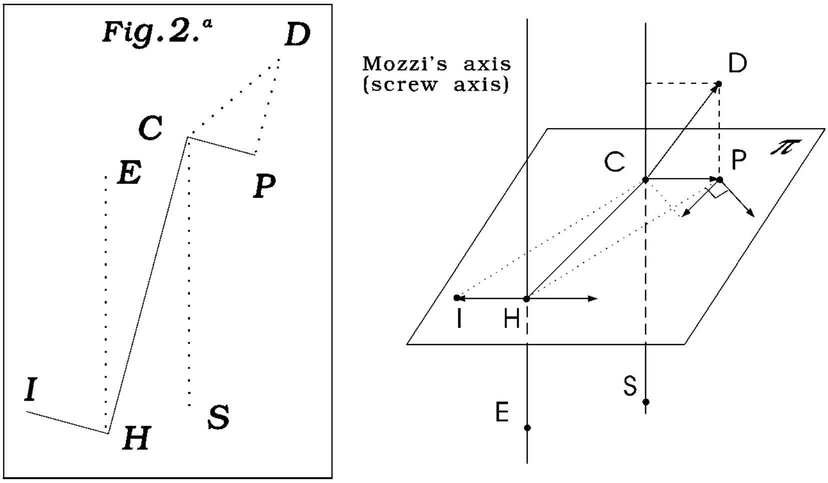
> Quindi ancora si potrà dedurre, che i suddetti due movimenti si riducono a due altri, uno de' quali sarà rettilineo e comune a tutte le parti del corpo, e parallelo all'asse di rotazione, che passa per il centro di gravità, e l'altro pure di rotamento, che avrà un asse di rotazione parallelo all'asse mentovato.

This Corollary can be translated as:

> Therefore you can deduce that the above mentioned movements become two others. A first one is linear and common to all the points of a body; it is parallel to the axis of rotation which crosses the center of gravity. The latter is a rotation motion whose axis of rotation is parallel to the above mentioned axis.

At the end of the demonstration the screw axis is defined on page 6 as "asse spontaneo di rotazione" (spontaneous axis of rotation) "Accademia della Crusca". Both the definition and proof refer to figure 2.a of table 1 in the Treatise, which has been reproduced in Figure 3a. A modern interpretation has been reported in the drawing in Figure 3b, as proposed in Ceccarelli (2000b).

The proof by Giulio Mozzi is as follows, referring to Figures 3a and 3b. Let us consider point C as the center of gravity of a rigid body, CS as the line of a rotation axis through C and CD as a displacement of C. Then CD can be solved into a component PD along a line parallel to CS axis and a component CP lying on a plane π orthogonal to CS. The displacement can be considered common for all the points of the rigid body and particularly the component CP for all points of the plane π, since it is considered referring to the center of gravity. In addition, assuming a rotation about the line CS, if we consider a line CH orthogonal to CP, we may determine a point H



a) b)

Fig. 3. A schematic drawing to prove the existence of a screw axis: (a) by Giulio Mozzi as HE line; (b) a modern view of Mozzi's description.

for which the rotational displacement HI with respect to C can be equal but opposite to CP, so that point H does not rotate in the plane π. Thus point C will have CP as a rotational displacement about a line HE, that is parallel to CS; point P will have a displacement which can be also considered as a vector sum of CP and a rotational displacement about CS again. This will result in a rotational displacement, which is orthogonal to HP and proportional to HP, which is the case because HE is a rotation axis. Consequently, point H individuates a screw axis, about and along which a general helicoidal motion can be determined through a translation component given by DP and a rotational component given by CP.

The demonstration proves the existence of a screw axis by means of a descriptive geometric reasoning based on Figure 3, and it was considered "chiara ed esatta" (clear and correct) in 1830 by Giorgini (1836) and in 1905 by Marcolongo (1905). Therefore, the screw axis has been also called Mozzi's axis because of this geometrical definition and following the Italian tradition, as pointed out in Levi-Civita and Amaldi (1950).

In addition, a very first concept of a force couple can be defined and has been used throughout the Treatise, starting with Lemmas II and III on pages 6 and 10, and Corollary VII on pages 10 and 11. In the next corollaries Mozzi analyzed several different cases to give computational results for compositions of forces and couples of forces as based on geometrical interpretation of physical situations.

Moreover, Mozzi approached the problem of solving a given system of forces into an equivalent set of two forces, one orthogonal to a given plane and the latter parallel to the plane itself.

Then, a general problem of determining the instantaneous helicoidal motion of a rigid body for a given impulsive force is formulated in Problem 1 on page 22. Thus, dynamics equations are formulated analytically in page 30 (Figure 4), in the form of the Theorem of Impulsive Forces for a given force acting on the body, and the consequent instantaneous helicoidal motion is determined. In Figure 4 the dynamic equations are reported in the original form that Mozzi had formulated by geometrical expressions following the custom of the time. In particular, main terms are indicated in the figure as PRp, which is the screw axis passing through an unknown point; G, which is the center point of the mass M of the body; MU, which is the applied force FL in the screw axis direction; GE and $M\Delta$ which are the resultants of inertial translational forces. By using geometrical reasoning, Mozzi deduced the expressions in Figure 4 to compute the dynamic equilibrium as a function of six unknowns that will represent the screw motion by identifying the screw axis and the rotational and translational velocity components of the motion due to the action of an external force FL. Therefore, the equations in Figure 4 can be recognized as an early expression of dynamic equations for a general screw motion.

Mozzi discussed the obtained formulas with some examples, also with the aim to better illustrate the kinematic and dynamical feasibility of the concept of instantaneous helicoidal motion. He also compared his results with some criticism with respect to previous uncompleted results by Bernoulli (1742). He mentioned as fundamental the works cited in Euler (1736) and D'Alembert (1749, 1796) with the aim to review the recent interest on the topic and to acknowledge advances due to Frisi (1765) and Perelli (the works of Perelli on this subject are lost).

In Problems II to V, Mozzi proposed a generalisation of the obtained results for the case with two acting forces to the case with a system of several acting forces by using an equivalence to two suitable forces that are normal to each other. The result was verified in the Treatise by computing the instanta-

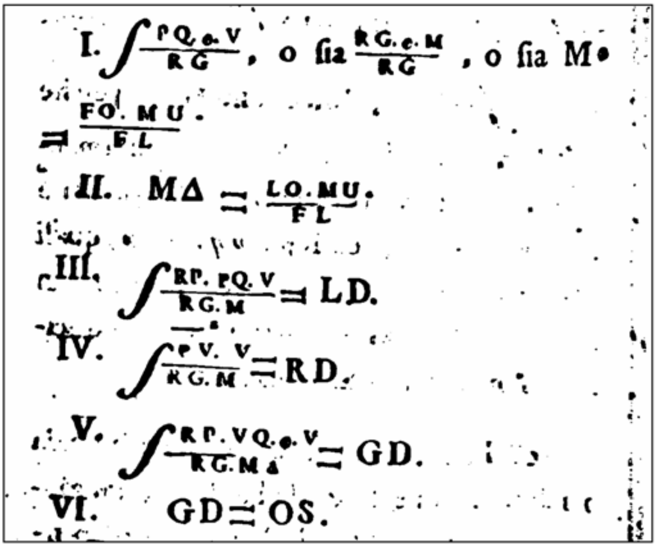


Fig. 4. Dynamic equations for a general helicoidal motion on page 30 of the Treatise by Giulio Mozzi.

neous motion for each force and then resolving them into a unique helicoidal motion.

Then, a reverse problem was formulated in Problems VI and VIII and corresponding corollaries, from pages 50 to 59, to compute a force that must act to give a prescribed motion or to change it from a given motion to another one.

Mozzi approached also the problem for the case of constrained rigid bodies. Particularly, the cases with a fixed point or a fixed axis of the body, or a fixed plane in contact with the body, are discussed in the last part of the Treatise. An interesting note is a first application of the D'Alembert Principle in terms of instantaneous forces, in Lemma VII on page 64. Mozzi used it to determine the instantaneous motion in Problems IX to XI for a free or constrained body, and when it occurs also as a helicoidal motion. Finally he approached the case of anelastic collision in Problems XII to XIV for a single contact and multiple contacts.

On the Circulation of the Treatise

In 1763, Mozzi's Treatise approached geometrically and analytically the problem of general motion for rigid bodies, a long-time famous problem that had been widely studied at the beginning of the 19th century. The definition of "screw axis" can be attributed to Mozzi as an important contribution to kinematics, and in the Italian tradition called "Mozzi's Axis". Unfortunately, the book by Mozzi passed almost unnoticed (as he had feared) and nowadays is sometimes forgotten.

One reason for the unsuccessful circulation of "Discorso matematico sopra il rotamento dei corpi" was surely the great changes during the Period of the French Revolution, but perhaps the most telling lay in the fact that Mozzi, who was not a University professor, did not continue to work and publish on the topic.

In his Treatise, Mozzi referenced works on astronomical studies (Bernoulli, 1742; Euler, 1736; D'Alembert, 1749; Frisi, 1765) and by D'Alembert, Bernoulli, Euler, Frisi, and Perelli (1704–1783) (whose works on the topics are lost, respectively, to give a fuller account of previous studies on the problem of general motion.

In 1836 Gaetano Giorgini (1795–1874) referenced the definition by Mozzi in a postscript appendix (written in 1832) of his paper (Giorgini 1836), in which he claimed to be contemporaneous first with Michel Chasles (1793–1880), who wrote the famous short note (Chasles, 1830), in formulating a theorem for general helicoidal motion of a rigid body, as pointed out in Ceccarelli (2000b). Although he recognised the validity of Mozzi's theorem, he did not appreciate the necessary rigor in Mozzi's Treatise and he claimed an analytical proof of the theorem for himself.

However, in that time also Cauchy (1789–1857), Poisson (1781–1834), Poinsot (1777–1859), and Rodrigues (1794–1851) approached the problem in Cauchy (1827), Poisson (1834), Poinsot (1834), and Rodrigues (1840), respectively. Also they did not refer to Mozzi's book. Indeed, neither did Poinsot or Poisson reference Chasles' paper or Giorgini's paper. The lack of reference to early studies seems to have recurred over time.

Once the general theorem was discovered and proved, it was thought to be obvious, as it is today. For example, Poisson himself did even not reference his own work (Poisson, 1834) in his treatise on Mechanics (Poisson, 1838). In Italy, in 1870 G. Battaglini wrote a paper with an analytical formulation for infinitesimal motion of a rigid body in which he declared and analytically proved once again the general theorem (Battaglini, 1870). However, he did not reference his paper in his Treatise on Mechanics in the section dealing with general motion (Battaglini, 1873). At the same time in France, De Saint Venant wrote a specific text on Kinematics (De Saint Venant, 1850), in which he discussed the general motion of a rigid body and he cited neither Mozzi nor Chasles.

Frisi also reported a first version of the theorem in his textbook (Frisi, 1777), without referencing his friend and pupil Mozzi. Again, Cayley (1821–1895) in a list of interesting papers on the basic kinematics of rigid body seems not to be aware of all the works on the topic (Cayley, 1891), and even he did not reference any Italian author. Indeed, the difficulties for Italian works can be clearly understood when it is observed that Giorgini's work was ignored also in his country, as for example was the work by Chelini (1862), who referred to general motion as Chasles' theorem, although he cited Giorgini too. Frisi was a famous Professor and, by acknowledging Mozzi's work, he helped the circulation of the main results, one of which he recognised to be the definition of a general helicoidal motion. As an illustrative example we can cite the reference (Frisi, 1768) in which Frisi clearly mentioned Mozzi's Treatise in the introduction.

Moreover, it is probable that Giulio Mozzi brought the book with him whilst travelling around Europe and discussed the topic with other scientists. His position as President of the Accademia Fiorentina should have contributed to a distribution of his work. Probably, the repute of his personality also helped circulation of the Treatise.

Therefore, it is quite curious that, although the book by Mozzi obtained some distribution and success in the academic world, not only in Italy, nevertheless it was forgotten quite early or even was not noticed. Some letters between Paolo Frisi and Roger Boscovich (1711–1787) of Milano University, reported in Costa (1967), illustrate the intention of Frisi to help the circulation of Mozzi's work in a formal way. The letters report also an appreciation of the study by Boscovich. Moreover, Frisi was in contact with D'Alembert and indeed they were good friends, as pointed out in Frisi (1786) and Grimsley (1963). They had frequent correspondence, with exchange also of works by friends and pupils of each other, as indicated in Grimsley (1963). Therefore, it seems likely that Frisi sent a copy of Mozzi's Treatise to D'Alembert. But today there is no evidence of a previous presence of Mozzi's book in academic libraries in Paris.

The fact that Mozzi's Treatise was soon ignored is probably (as mentioned above) due both to the turbulence in the French Revolution period and to the theoretical approach with neither practical nor astronomical applications. The great theoretical aspect of the subject can be appreciated when it is observed that for example in the Encyclopédie (D'Alembert and Diderot, 1785), in the item "mouvement", spatial motion was not even mentioned.

Indeed, general motion had no technical applications as the work by L.M.N. Carnot (1753-1823) illustrates (Carnot, 1803), since it deals with Mechanics applied to machines by using a theoretical approach but without mentioning spatial motion.

Symptomatic of failure to cite sources is the Treatise (Francouer, 1807) published in 1807 by L.B. Francouer, professor at the Ecole Polytechnique, who stated the theorem of helicoidal motion on page 376 with no reference to previous works, treating it as if it was an obvious and well-known assertion.

In addition, it is notable that most scientific works in the 18th century were written in Latin, not in Italian. Therefore the Treatise by Mozzi could be not well accepted in the scientific community or at least it could not be easily understood because of the language barrier.

In addition, the previously mentioned turbulence of events during the French Revolution combined with consequent changes in Italian politics, may have contributed to the obscuring of scientific achievements and particularly to impeding circulation of Mozzi's Treatise. Indeed, the fragmentation of Italy into many small kingdoms could have contributed, due to his political positions in several circumstances as outlined in the biographical notes.

Modern Interpretation of Main Contributions to Mechanism Design

A modern interpretation of Mozzi's Treatise cannot be identified directly, since the work has been forgotten for a long time and even when rediscovered it has not been considered as a background for developing the today so-called Screw Theory. In fact, the main contribution of Mozzi's work can be recognized in the definition and use of the screw axis as a useful means to describe and deal with the general motion of rigid bodies. Indeed, the developments by Mozzi are not directed to any practical purposes, including mechanism design, but the Treatise can be considered a theoretical work that is fundamental for an elaboration of a mathematization that is, in turn, useful for applied me-

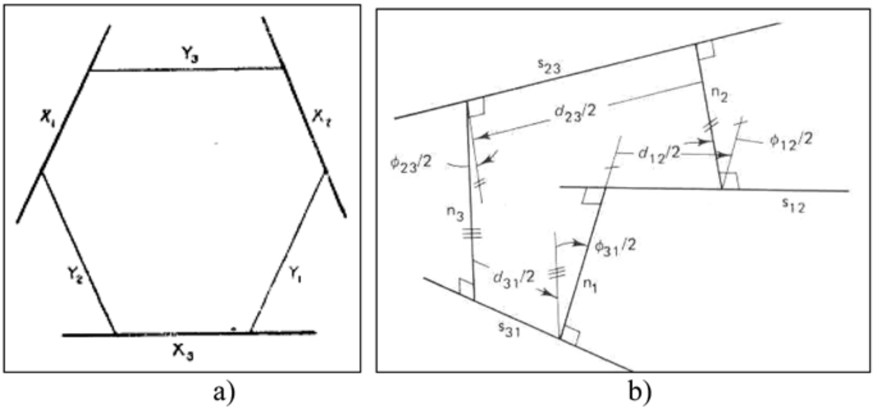


Fig. 5. A model of Screw Triangle for synthesis of spatial mechanisms: (a) by Bricard in (1927); (b) by Bottema and Roth in (1990).

chanics even for mechanism design as it developed in the 18-th century and later.

However, the modernity of Mozzi's approach can be appreciated by looking at the historical development of the Screw Theory from the first successful work (Ball, 1876) to a very recent book (Davidson and Hunt, 2004). In addition, today there is a wide use of the Screw Theory for studying mainly spatial mechanisms and particular robotic manipulators, attested to by a massive production of publications both in Journals and conference proceedings.

A few examples are outlined in this section with the main aim to show the modernity of Mozzi's work in terms of conceptual contributions, since his formalism in the Treatise has been already determined to be not suitable for a current practical implementation (see for example Figure 4).

In the introduction to the 1900 edition of his Treatise, Ball emphasized the character of the book as a comprehensive account and summary of his previous works and developments by others and as dealing with a general theory of the study of the motion of rigid bodies. Thus, the Treatise is not directed to any particular application and it is rather theoretical in the field of Rational Mechanics (likewise Mozzi's Treatise) but with an advanced modern formalism.

In 1927 in the books by Bricard, Screw Theory is considered as a part of Kinematics that can find application in practical problems for mechanical systems. In addition, formulation of screws is outlined as useful for graphical

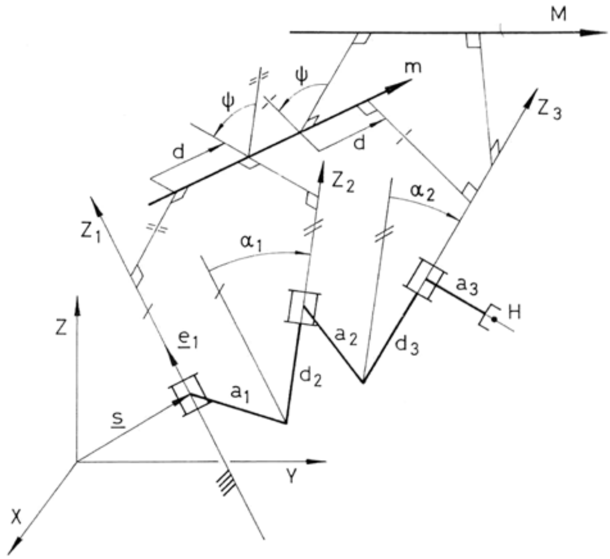


Fig. 6. A scheme of Screw Triangle for Three-revolute manipulators.

representations, such as in the example of Figure 5a). In Figure 5 a successful use of the screw axis (Mozzi's Axis) is reported in two examples for a study of a sequence of motions through the so-called Screw Triangle. Figure 5b is a modern representation as proposed by Bottema and Roth (1990) emphasizing the geometrical parameters. Indeed a modern systematization of the Screw Theory has been presented in the work by Bottema and Roth (1990), which is specifically dedicated to Kinematics with an approach that, although not directed to mechanism design, can be a source for it.

Such an orientation of the Screw Theory to mechanism design has been evolving since the 1960s and today is widely used mainly for robotic manipulators. Recently, the Screw Theory has been developed specifically for applications to robots, for example the book by Davidson and Hunt (2004). In fact, specific models are elaborated with great details and accurate formulation both for analysis and design, such as in the example of Figure 6 which refers to a general three-revolute open chain manipulator.

Indeed, today the Screw Theory is even considered as a fundamental approach for the study of three-dimensional design and operation of mechanisms.

Summarizing, the identification and formulation of the screw axis (Mozzi's Axis) are fundamental bases for achievements both in the theory and practice of mechanical systems. Although Mozzi's work seems not have been adequately recognized, his ideas have been successfully developed, even independently.

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THÉODORE OLIVIER
(1793–1853)

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Abstract. Théodore Olivier is mainly known as being, in 1829, one of the four founders of Ecole Centrale des Arts et Manufactures also named today Ecole Centrale Paris. He was a former student of Gaspard Monge and he taught descriptive geometry. He highly contributed to the science of ruled surfaces and to the theory of gearing. He designed many self-explanatory models; most of them are movable. In rigid-body kinematics the locus of the instantaneous axes of any time-dependent motion is a ruled surface. That way, Olivier pioneered Julius Plücker's work about straight-line geometry and, consequently, disclosed basic tools for the "screw theory" devised by Robert Ball. Moreover, with his book about the general skew arrangement of two gear wheels together with his models of gears, Olivier is one of the scientific ancestors of Jack Phillips with his book issued in 2003 on *General Spatial Involute Gearing*.

Biographical Notes

Théodore Olivier was born at Lyon (France) on the 21st of January 1793. He was admitted at Ecole Polytechnique in 1810, and because of his weak health, he spent four years in the famous school. There, he was a disciple of Gaspard Monge who is regarded as the inventor of descriptive geometry. This subject deals with projection, perspective and cross section but, in particular, it is concerned with the representation of three-dimensional objects on a two-dimensional plane. This problem is of interest not only to mathematicians and engineers but also to artists. In fact, the pioneer of descriptive geometry is the great German artist Albrecht Dürer (1471–1528).

Several historians have noticed that Olivier looks much like the Emperor Napoléon 1rst even though nobody could prove that Olivier was secretly a son of Napoléon Bonaparte. However, amazingly, Olivier suffered from a severe prostration in 1821 when the Emperor died at Sainte-Hélène. After that, he

M. Ceccarelli (ed.), Distinguished Figures in Mechanism and Machine Science, 295–318.

Fig. 1. Olivier portrait (Courtesy of Ecole Centrale Paris, France).

went to Sweden where the former Napoleonian Maréchal Bernadotte was still the king Charles XIV of Sweden. There, he taught descriptive geometry at the royal school of Morienberg. He became private tutor of the Prince; he organized schools for the engineer corps and artillery, too. He returned to France at the end of 1828.

In 1829, Olivier helped to found the Ecole Centrale des Arts et Manufactures. The scientific discoveries around that time were having a major impact on French industry. However, to fully benefit, a new type of engineer had to be trained with a broad knowledge of science and mathematics. Alphonse Lavallée, who was a lawyer and a businessman from Nantes, put most of his capital into the foundation of the Ecole Centrale des Arts et Manufactures. He obtained the help of three top scientists, one being Théodore Olivier, the other two being Jean-Baptiste Dumas and Eugène Péclet. They set up the school with the stated aim of training the doctors of factories and mills. Olivier became a professor at the Ecole Centrale des Arts et Manufactures when it opened in the Hôtel de Juigné in the Marais district of Paris in 1829. The two other scientists, Dumas and Péclet, who had joined with Lavallée, also became professors. In his role as professor, Olivier lectured on descriptive geometry and mechanics. From 1838, he also lectured on these topics at the Conservatoire National des Arts et Métiers in Paris. Moreover, during the period (1830–1844), Olivier was a lecturer at Ecole Polytechnique, too.

It is worth mentioning that, today, the Ecole Centrale des Arts et Manufactures is located at Chatenay-Malabry in the south suburb of Paris and is generally called Ecole Centrale Paris. The Hôtel de Juigné, which lodged the famous school from 1929 till 1884 still exists and is now devoted to the exhibition of important art works of Pablo Picasso. Some mementos of the former school are shown in one of the rooms of the present-day Picasso museum. The ancient monument was built in 1656 for a lord collecting the former tax on the salt, and, therefore, it is named also Hôtel Salé that means salty hotel.

At about the time of the foundation of the Ecole Centrale, Olivier became interested in making models of ruled surfaces. He made also models of various types of gearing. They are what we know today as the "Olivier Models". Olivier wrote also textbooks but his fame, however, is mainly the result of these models, which he created to assist in his teaching of geometry and mechanics. In fact, Olivier and his wife earned quite a good income from selling these models, particularly in the United States.

Almost all Olivier models are movable. One can differentiate two main categories of models, namely models of ruled surfaces with the only purpose of illustrating geometric properties and models of technical systems of gears for transmitting motion. Some of the models of surfaces with moving parts are done to show to students how the ruled surfaces are generated. Others were designed to show the curves of intersection of certain surfaces.

The models with gears bring forth by their material achievements the various kinds of gear coupling; moreover, Olivier proposed machines for manufacturing gears. Actually, Olivier is a great pioneer in the science of gearing.

The ruled surfaces exemplified by the Olivier models have attracted attention because of their beauty rather than for their scientific interest. They can be considered as "abstract sculpture". Nevertheless, the locus of the screw axes in any time-dependent rigid-body motion is a ruled surface. Hence, the Olivier models of ruled surfaces have some interest for kinematics.

Olivier was awarded by the French honor of Chevalier de la Légion d'Honneur and by Swedish fellowship of the Royal Order of the Swedish Pole-Star.

When Olivier died in 1853, his wife kept his personal collection of models numbering nearly fifty. Some years before his death, Olivier had given to the Conservatoire National des Arts et Métiers a collection of models. This set was almost a complete duplicate of his own. Olivier's widow sold most of the models that she had to US professors. That way, the US Military Academy of

West Point has about 24 models, Union College in Schenectady has 41 models, and also Columbia University has some. Maybe some Olivier models are possessed by Harvard University and also by other unidentified institutions.

List of Works

Main Papers and Textbooks

The Olivier publications are listed below.

[1] Olivier, Th., Mémoire de Géométrie descriptive sur la construction des tangentes en un point multiple d'une courbe plane ou à double courbure et dont l'équation n'est pas connue, *Journal de l'Ecole Polytechnique*, **13**, 1832, pp. 303–305.

[2] Olivier, Th., Rapport sur la balance de précision de M. Bockholtz. *Excerpt from Bulletin de la Société d'encouragement pour l'industrie nationale*, 3 pp., 1833 [in the file on Olivier at Ecole Centrale Paris].

[3] Olivier, Th., Construction des centres de courbure des épicycloïdes planes et sphériques, *Journal de l'Ecole Polytechnique*, **14**, 1934, pp. 85–152.

[4] Olivier, Th., De la courbure et de la flexion d'une courbe à double courbure, *Journal de l'Ecole Polytechnique*, **15**, 61–91, 1835, pp. 252–263.

[5] Olivier Th., Mémoire sur le système des courbes à petits rayons des chemins de fer de M. Laignel, 64 pp.: pl., errata, Paris: impr. de Mme Huzard, 1836.

[6] Olivier, Th., Des indications des divers ordres de contact entre deux surfaces, et des conditions géométriques auxquelles doivent satisfaire deux surfaces ayant un point de contact pour qu'elles aient un contact du nième ordre autour de ce point, *Journal de l'Ecole Polytechnique*, **15**, 1837, pp. 123–150.

[7] Olivier, Th., Sur les paraboloïdes osculateurs, *Journal de l'Ecole Polytechnique*, **15**, 1837, pp. 230–243.

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[13] Olivier, Th., Des propriétés osculatrices de deux surfaces en contact par un point, *Journal Math. Pures et Appliquées, Sér. I*, **6**, 1841, pp. 297–308.
[14] Olivier, Th., *Théorie géométrique des engrenages destinés à transmettre le mouvement de rotation entre deux axes situés dans un même plan*, Bachelier, Paris, 1842, 126 pp.
[15] Olivier, Th., *Théorie géométrique des engrenages destinés à transmettre le mouvement de rotation entre deux axes situés ou non dans un même plan*, Bachelier, Paris; Michelsen, Leipzig; Dulau, London, 1842, 118 pp.
[16] Olivier, Th., *Développements de géométrie descriptive [atlas]*, Carilian-Goeury et V. Dalmont, Paris, 2 vols., pl., 1843.
[17] Olivier, Th., *Rapport sur un abaque ou compteur universel par Léon Lalanne*, Mme Vve Bouchard-Huzard, C., Paris, 1846, ECP 3483, 12 pp.
[18] Olivier, Th., *De la cause du déraillement des wagons sur les courbes des chemins de fer*, L. Mathias, Paris, 1846, 92 pp.
[19] Olivier, Th., *Histoire de la fondation de la Société d'encouragement pour l'industrie nationale*, B. Huzard, Paris, 1850.
[20] Olivier, Th., *Mémoire de géométrie descriptive théorique et appliquée [atlas]*, Carilian, Goeury et V. Dalmont, Paris, 2 vols., pl., 1851.

Models

The Olivier models can be regarded as being a kind of publication. Unfortunately, some of the Olivier models were lost.

THÉORIE GÉOMÉTRIQUE

DES

ENGRENAGES

DESTINÉS A TRANSMETTRE

LE MOUVEMENT DE ROTATION ENTRE DEUX AXES SITUÉS OU NON SITUÉS DANS UN MÊME PLAN;

Par THÉODORE OLIVIER,

Ancien Élève de l'École Polytechnique et ancien Officier d'artillerie; Docteur-ès-sciences de la Faculté de Paris; Professeur de Géométrie descriptive au Conservatoire royal des Arts et Métiers, et Professeur-Fondateur de l'École centrale des Arts et Manufactures; Répétiteur à l'École Polytechnique; Membre de la Société Philomatique de Paris; Membre étranger des deux Académies royales des Sciences et des Sciences militaires de Stockholm; Membre du Comité des Arts mécaniques de la Société d'Encouragement pour l'Industrie nationale; des Académies de Metz, Dijon et Lyon; Chevalier de la Légion d'honneur et de l'Ordre royal de l'Étoile polaire (de Suède).

PARIS,

BACHELIER, IMPRIMEUR-LIBRAIRE

DE L'ÉCOLE POLYTECHNIQUE, DU BUREAU DES LONGITUDES, ETC.,

QUAI DES AUGUSTINS, N° 55.

A LEIPZIG, CHEZ MICHELSEN. ‖ *A LONDRES*, CHEZ DULAU ET C^ie^.

1842

Fig. 2. Front page of Olivier textbook [15].

An important collection of Olivier models is exhibited in the French "Musée des Arts et Métiers", 60, rue Réaumur, 75003 Paris; http://www.arts-et-metiers.net/. These models are listed below with their inventory numbers

(Inv.) at the museum. Their precise dates of construction are not always known, but, because of the Olivier death in 1853, one can guess that the models were made between 1929 and 1853. As far as the author is aware, the Olivier models of gears, which are indicated in the list by the sign ⋆ are not exhibited in another museum.

[AM1] Conoïde à cône directeur; Inv. 04452- Date of construction: 1830-
[AM2] Conoïde à plan directeur; Inv. 04451- Date of construction : 1830-
[AM3] Conoïde et son paraboloïde hyperbolique tangent; Inv. 04464- Date of construction 1830
[AM4] ⋆ Crémaillère circulaire à fuseaux; Inv. 04426-
[AM5] ⋆ Crémaillère circulaire, la roue est une lanterne; Inv. 04427-
[AM6] ⋆ Crémaillère rectiline; Inv. 05464-
[AM7] ⋆ Crémaillère rectiligne à chevrons; Inv. 05454-
[AM8] ⋆ Crémaillère rectiligne, dent carrée; Inv. 05448-
[AM9] ⋆ Crémaillère rectiligne, les dents sont des prismes rectangulaires obliques; Inv. 05452-
[AM10] ⋆ Crémaillère rectiligne, les dents sont des prismes rectangulaires obliques; Inv. 05451-
[AM11] Cylindre et plan se transformant, par un mouvement de rotation, l'un en hyperboloïde . . . ; Inv. 04471-
[AM12] Cylindres primitifs d'une engrenage gauche; Inv. 05463-
[AM13] ⋆ Dent hélicoïdale; Inv. 05462-
[AM14] ⋆ Dessins de mécanique: engrenages et crémaillères; Inv. 36270- Date of construction: 1840-
[AM15] Deux cercles égaux situés dans des plans parallèles sont divisés en un même nombre de parties égales; Inv. 04448- Date of construction: 1830-
[AM16] Deux cônes se coupant suivant une courbe plane; Inv. 04537-
[AM17] Deux cylindres quelconques se transformant, par un mouvement de rotation, en deux cônes; Inv. 04472-
[AM18] Deux cylindres se transformant par un mouvement de rotation en deux hyperboloïdes; Inv. 04469- Date of construction: 1830-
[AM19] Deux plans se transformant, par un mouvement de rotation, d'abord en deux paraboloïdes . . . ; Inv. 04470-
[AM20] ⋆ Elément de crémaillère rectiligne; Inv. 05468-
[AM21] ⋆ Elément de l'engrenage No. 3431; Inv. 04583-

[AM22] ⋆ Elément lisse d'engrenage cylindrique hélicoïdal à contact extérieur; Inv. 05466-
[AM23] ⋆ Elément lisse de crémaillère rectiligne; Inv. 05469-
[AM24] ⋆ Engrenage; Inv. 05459-
[AM25] ⋆ Engrenage à crémaillère; Inv. 05449-
[AM26] ⋆ Engrenage à cremaillère: les dents sont à profil carré; Inv. 05456-
[AM27] ⋆ Engrenage à crémaillère à lanterne, les dents de la crémaillère sont des fuseaux; Inv. 04431-
[AM28] ⋆ Engrenage à crémaillère et pignon à lanterne, les dents du pignon sont des fuseaux; Inv. 04432-
[AM29] ⋆ Engrenage à crémaillère, dents à profil en épicycloïde; Inv. 04430-
[AM30] ⋆ Engrenage à crémaillère, dents à profil triangulaire; Inv. 05457-
[AM31] ⋆ Engrenage à vis creuse dite vis tangente; Inv. 04437-
[AM32] ⋆ Engrenage conique à crémaillère; Inv. 04425-
[AM33] ⋆ Engrenage conique droit extérieur, angle aigu (profil de la dent: épicycloïde sphérique); Inv. 04421-
[AM34] ⋆ Engrenage conique droit extérieur à lanterne: les dents du pignon sont des fuseaux coniques; Inv. 04424-
[AM35] ⋆ Engrenage conique, angle aigu; Inv. 04422-
[AM36] ⋆ Engrenage cylindrique droit à contact extérieur, Profil des dents en développante de cercle; Inv. 04428-
[AM37] ⋆ Engrenage cylindrique droit à contact extérieur; Inv. 04429-
[AM38] ⋆ Engrenage cylindrique extérieur, à épicycloïde et à flanc . . . ; Inv. 04433-
[AM39] ⋆ Engrenage cylindrique extérieur, à épicycloïde et à flanc; Inv. 04434-
[AM40] ⋆ Engrenage extérieur; Inv. 04435-
[AM41] ⋆ Engrenage extérieur; Inv. 05467-
[AM42] ⋆ Engrenage extérieur, le pignon est une lanterne; Inv. 04423-
[AM43] ⋆ Engrenage gauche extérieur; Inv. 05455-
[AM44] ⋆ Engrenage hélicoïdal à cremaillère; Inv. 05453-
[AM45] ⋆ Engrenage intérieur; Inv. 05465-
[AM46] ⋆ Engrenage intérieur, la roue est une lanterne; Inv. 05450-
[AM47] ⋆ Engrenage intérieur, la surface de la dent de la roue conduite étant convexe; Inv. 05458-
[AM48] ⋆ Engrenage intérieur, le pignon est une lanterne; Inv. 04436-0000-

[AM49] ⋆ Equipage composé d'une roue et de trois pignons satellites; Inv. 02669-0000-
[AM50] Hyperboloïde à une nappe; Inv. 04443- Date of construction: 1830-
[AM51] Hyperboloïde à une nappe et son cône asymptote; Inv. 04444- Date of construction: 1830-
[AM52] Hyperboloïde à une nappe et son cône asymptote: le plan tangent au cone est l'un des deux plans: . . . ; Inv. 04560- Date of construction: 1830-
[AM53] Hyperboloïde à une nappe et son cône asymptote, plan tangent au cône . . . ; Inv. 04559- Date of construction: 1830-
[AM54] Hyperboloïde à une nappe et son paraboloïde tangent; Inv. 04882- Date of construction: 1830-
[AM55] Hyperboloïde à une nappe, ou à deux cercles, situés dans des plans parallèles et divisés; Inv. 04442- Date of construction: 1830-
[AM56] Intersection de deux cônes qui ont deux plans tangents communs; Inv. 04445-
[AM57] Intersection de deux surfaces du genre de la douelle de la vis Saint Gilles; Inv. 04473- Date of construction: 1830-
[AM58] ⋆ Machine de Théodore Olivier pour tailler les engrenages; Inv. 02668-
[AM59] ⋆ Machine de Théodore Olivier pour tailler les engrenages (Complementing Inv. 2668-0001): onze accessoires; Inv. 02668-0002-
[AM60] Matrice pour le centimètre et le millimètre; Inv. 05308-
[AM61] ⋆ Modèle d'engrenage constitué par un système de conoïdes; Inv. 04447-0002- Date of construction: 1830-
[AM62] Modèle montrant les arêtes de douelle en hélices des voûtes biaises (English apparatus); Inv. 04454-
[AM63] Modèle offrant deux systèmes de conoïdes; Inv. 04447-0001- Date of construction: 1830-
[AM64] Modèle servant à démontrer les propriétés de la surface règlée (douelle de la vis Saint-Gilles carrée); Inv. 04460- Date of construction: 1850-
[AM65] Modèle servant à transformer deux cylindres en deux conoïdes ou en deux surfaces gauches; Inv. 04462-
[AM66] Modèle servant à transformer les génératrices droites d'un cylindre de révolution en hélices rampantes; Inv. 04455-

[AM67] Modèle servant à transformer un conoïde en un cylindre ...; Inv. 04461-
[AM68] Modèle servant à transformer un cylindre en conoïde et vice versa; Inv. 04446- Date of construction: 1830-
[AM69] Paraboloïde hyperbolique; Inv. 04459- Date of construction: 1830-
[AM70] Paraboloïde hyperbolique; Inv. 04440- Date of construction: 1830-
[AM71] Paraboloïde hyperbolique; Inv. 04450- Date of construction: 1830-
[AM72] Paraboloïde hyperbolique; Inv. 04465- Date of construction: 1830-
[AM73] Paraboloïde hyperbolique Génération rectiligne double; Inv. 04439- Date of construction: 1830-
[AM74] Paraboloïde hyperbolique Génération rectiligne simple; Inv. 04463- Date of construction: 1830-
[AM75] Paraboloïde hyperbolique ou quadrilatère gauche dont les quate cotés sont égaux et ...; Inv. 04441-
[AM76] Paraboloïde hyperbolique ou quadrilatère gauche ayant ses quate cotés égaux projetés en un lozange; Inv. 04457-
[AM77] Paraboloïde hyperbolique, génération rectiligne simple; Inv. 04438- Date of construction: 1830-
[AM78] ⋆ Parallélipipède capable de contenir la dent hélicoïdale; Inv. 05461-
[AM79] ⋆ Parallélipipède capable de contenir la dent hélicoïdale; Inv. 05460-
[AM80] Pénétration de deux cylindres Courbe d'arrachement; Inv. 04536- Date of construction: 1830-
[AM81] Pénétration de deux cylindres. Courbe à points multiples; Inv. 04535- Date of construction: 1830-
[AM82] Pénétration de deux cylindres. Courbes d'entrée et de sortie; Inv. 04534- of construction: 1830-
[AM83] Petit modèle de charrue avec attelage; Inv. 05129-0003-
[AM84] Petit modèle de charrue avec attelage; Inv. 05129-0004-
[AM85] Surface de biais passé; Inv. 04555-
[AM86] Surface de douelle de la voûte d'arête; Inv. 04456-
[AM87] Surface gauche se transformant en une autre surface gauche; Inv. 04556- Date of construction: 1830-
[AM88] Surface hélicoïdale développable; Inv. 04466- Date of construction: 1830-

[AM89] Surface hélicoïde gauche, problème à résoudre; Inv. 04468- Date of construction: 1830-
[AM90] Surface hélicoïde gauche, spirale d'Archimède; Inv. 04467-
[AM91] Surface réglée, douelle de la vis Saint Gilles carrée; Inv. 04453-
[AM92] Surfaces réglées passant par deux cercles égaux et parallèles coupées par un plan parallèle au cercle; Inv. 04449- Date of construction: 1830-
[AM93] Transformation d'un conoïde en cylindre, du paraboloïde tangent au conoïde en plan tangent; Inv. 04562-
[AM94] Transformation d'un conoïde et d'un cylindre du paraboloïde tangent au conoïde . . . ; Inv. 04561-

Apparently, the Olivier models that are detained in the US seem to be replicates done by Olivier himself for his own interest of some of the models that Olivier gave to the former galleries of the Conservatoire Royal des Arts et Métiers.

Review of Main Works on Mechanism Design

Many Olivier models are devoted to the description of ruled surfaces.

For instance, the Olivier model of Figure 3 shows two ruled surfaces that are tangent along a common straight line. Both geometric surfaces are formed by sets of stretched strings. The strings are held in place by lead weights that are concealed by the wooden box of the fixed base. The metal piece at the bottom (or at the top) is the director curve of the ruled surfaces. In this example of model, the director curve includes a circle rigidly connected to a segment of a tangent straight line. The warping of the two ruled surfaces can be adjusted by the feasible displacements of the metal parts. In a special position of the metal parts, the two ruled surfaces are a cylinder and a plane as shown in Figure 4. In Figure 3, the surfaces are a hyperboloid of one sheet (also called *regulus*) and a hyperbolic paraboloid.

A conoid is a warped surface, which is generated by a straight line moving in such a manner as to touch a fixed straight line and a fixed director curve, and remaining parallel to a fixed plane. In the model of Figure 5, the plane of the two conoids is the vertical rear wall. The two straight lines or axes of the conoids are horizontal and parallel; the conoids share the same director curve that is an ellipse. Both are right conoids; as a matter of fact, the vertical plane is perpendicular to the two horizontal axes.

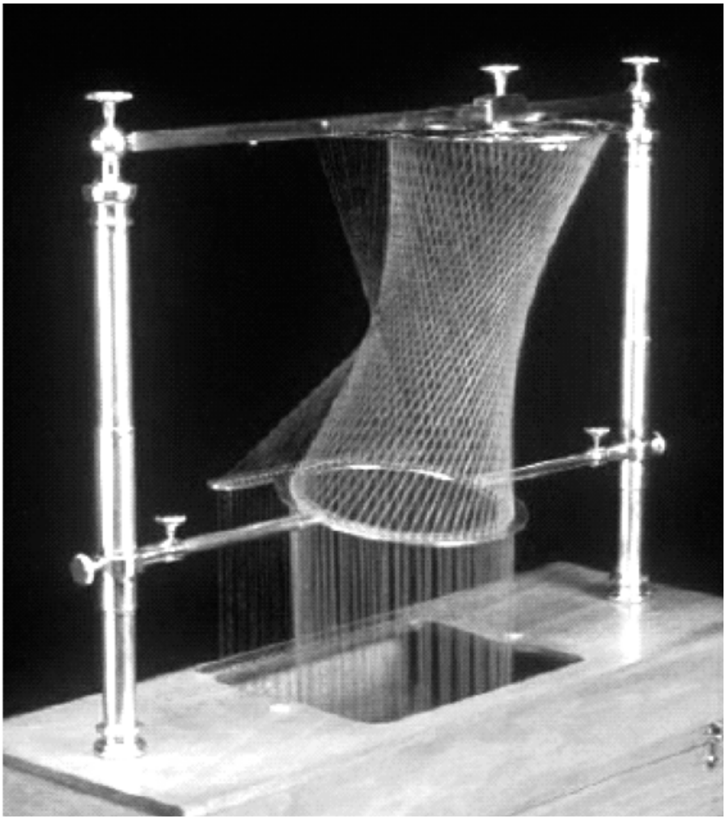


Fig. 3. Hyperboloid of one sheet and a tangent hyperbolic paraboloid (Museum of Arts et Métiers, No. 04882-).

Through his models, Olivier introduced various types of conoids. It is possible that he built a special right conoid playing a key role in kinematics; however, this is not certain. The conoid of Plücker was named *cylindroid* by Robert Ball in *A Treatise on the Theory of Screws* [A2], 1900, reprinted 1998, and it is sometimes called a skew arch. Ball mentioned in a footnote of his textbook the prior disclosure of his cylindroid by Julius Plücker in [A1], 1869.

Some surfaces among the Olivier models are various types of ruled helicoids that can be used for constituting screw joints. The more interesting helical surface is the developable helicoid also named involute (or convolute) helicoid. Such a surface is generated by the tangent lines to an helix drawn on a revolute cylinder. Moreover, pieces of involute helicoids can be used as

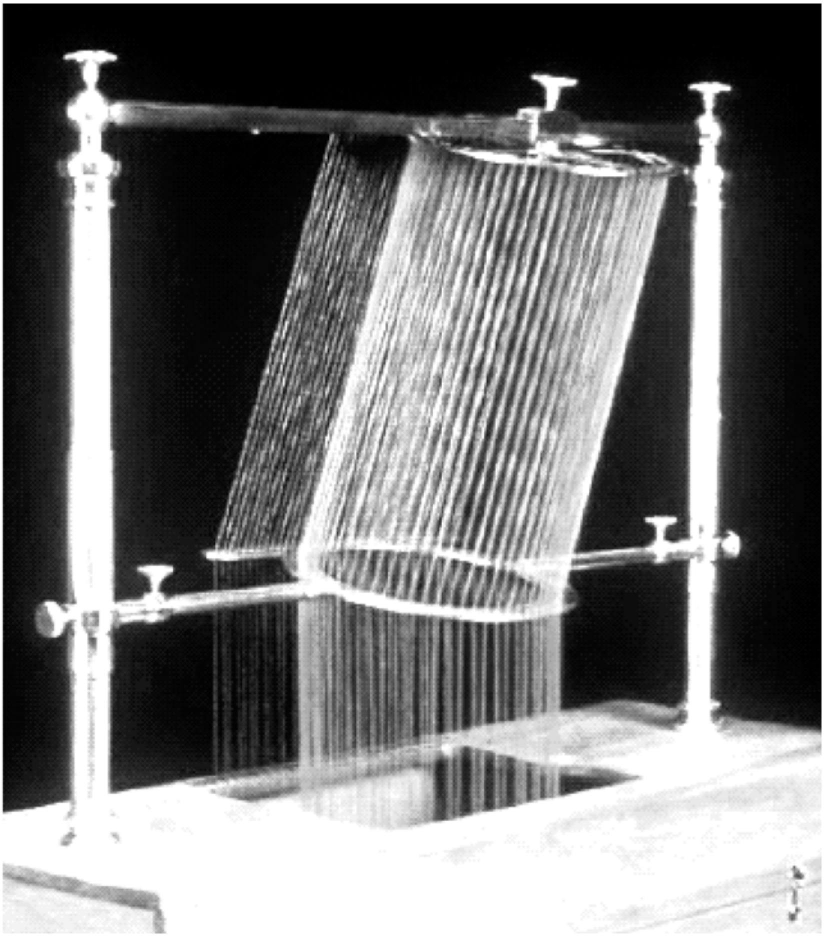


Fig. 4. The one-sheet hyperboloid and its tangent hyperbolic paraboloid are transformed into a cylinder and a tangent plane.

flanks of gear teeth. This fact might have been demonstrated by Olivier himself or it was not according to further work on the general spatial involute gearing by Jack Phillips in 2003 [A5].

Gears are one of man's oldest mechanical devices. The gear has been a basic element of machinery throughout all time from the earliest beginnings of machinery. The earliest known relic of gearing from ancient times is the "South Pointing Chariot" (China, about 2600 BC). However, the mathematical determination of the shapes of gear teeth is much more recent. The great

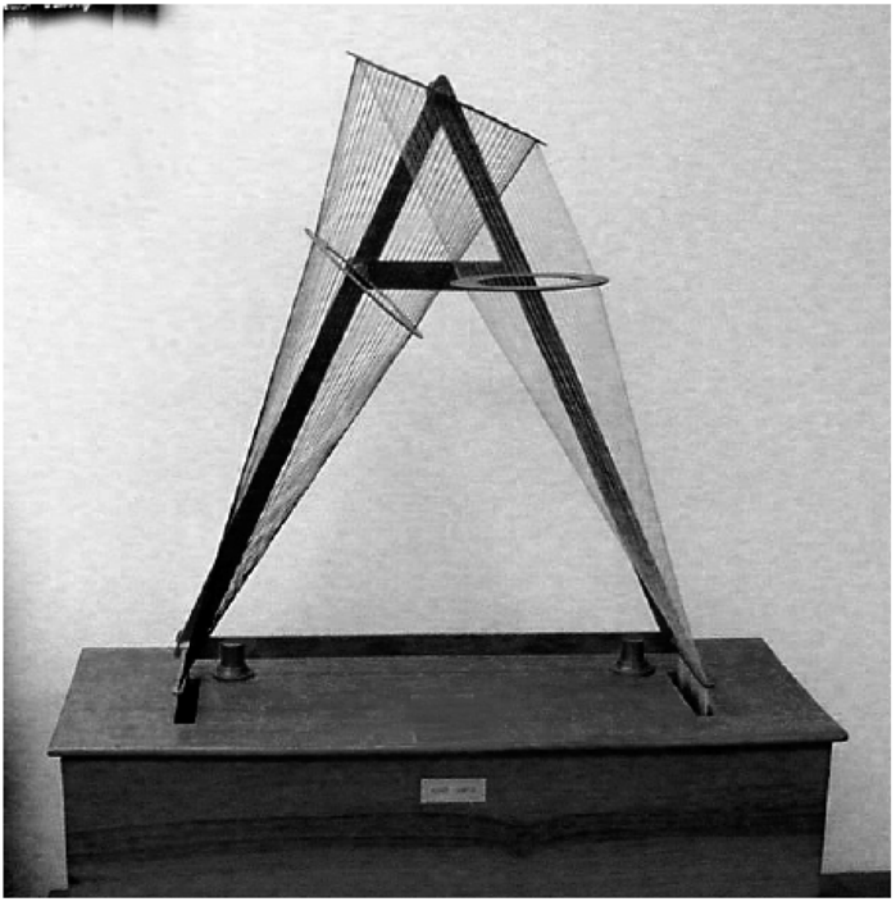


Fig. 5. Two right conoids intersecting along an elliptic curve (US Military Academy of West Point).

mathematician Leonard Euler in 1754 worked out rules for conjugate action of teeth in spur gears. Some consider him "the father of involute gearing". Nevertheless, Olivier is the first who explored in depth the profiles of gear teeth for practical application.

At the beginning of his career, Olivier was interested in a special kind of spur or bevel gear that was claimed in 1810, by an engineer named White whose biography is unknown, to be frictionless. Such an outcome seems to contradict Euler's work on tooth profiles. According to Olivier, who oddly

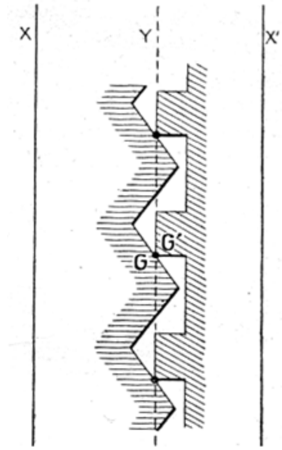


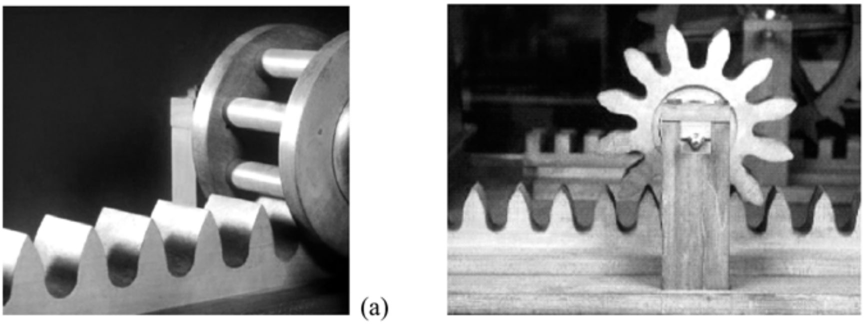
Fig. 6. Sketch of a White gearing as depicted in a Bricard book [A3].

misspelt *White* as *With* in [12] 1839, White was not a geometer and consequently was unable to prove the astonishing property of this gearing.

It may be worth recalling what is a White gearing. In Figure 6 that is excerpted from a Bricard book, 1927 [A3], one wheel has the fixed axis X and the other wheel has the fixed axis X'. Two virtual revolute cylinders with axes X and X' keep in touch along the straight line Y. The rotation around X can be transmitted to the X' axis through a relative rolling motion of the virtual cylinders along Y. Hence, one can synthesize conjugate gear teeth that are in point touch on the line Y. In the example of Figure 6, one wheel is a screw with a triangular filet and the other one is a screw with a square filet. The tooth contact is achieved at the point $G = G'$ belonging to the line Y. The trajectory of G with respect to the wheel of axis X is a helix drawn on the cylinder of axis X as well as the trajectory of G' with respect to the wheel of axis X'. One has to notice that, in White gearing, the tooth contact is the common point of two curves lying on the rolling virtual cylinders.

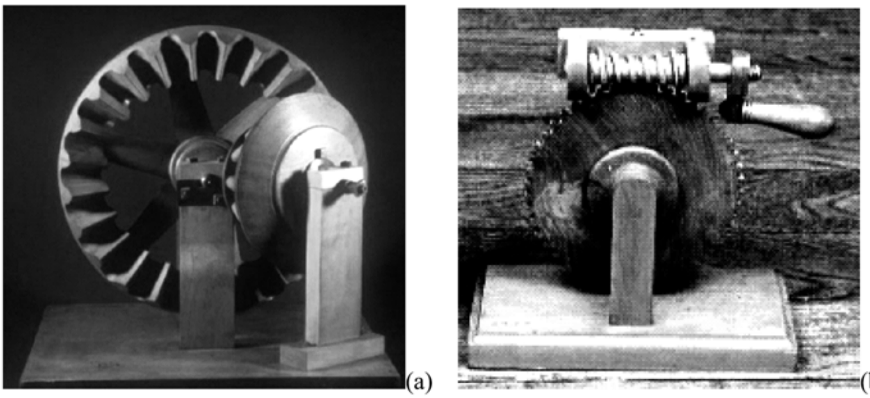
In his papers [11, 12], Olivier showed by geometry that, in White gear, there is no sliding motion at the point of contact but there is a relative rotation between the teeth around this point. Therefore, the White gearing is not deprived of friction and wear. Actually this gearing cannot transmit high power. However, it has been fruitfully implemented in mechanical clockworks.

Olivier wrote two treatises on gearing. His first book deals with wheel axes lying in the same plane [14]. Olivier described all the practical cases of tooth



(a) (b)

Fig. 7. Examples of spur gears among the numerous Olivier models (Museum of "Arts et Métiers", Nos 04432 and 04430).



(a) (b)

Fig. 8. (a) Model of bevel gears; (b) particular spatial gearing: worm screw gear (Museum of "Arts et Métiers", Nos 04424 and 04437).

shapes that are conjugate in both important situations, namely the two fixed shaft axes are parallel and the axes intersect. Moreover the three-dimensional embodiment of what Olivier found out by means of planar drawings and geometric reasoning was illustrated by numerous models made of wood. Alas, some of the models that are mentioned in Olivier books are lost. Fortunately, in the "Musée des Arts et Métiers" in Paris, an important collection of these gears remain safe behind glass. For instance, one can still admire a (squirrel-cage) lantern pinion with its conjugate rack (Figure 7a), the use of cycloidal teeth (Figure 7b), and the planar involute gearing (fathered by Euler).

In addition to the spur gears, various types of bevel gears were also studied by Olivier. A bevel gear with a conical lantern pinion is shown in Figure 8a.

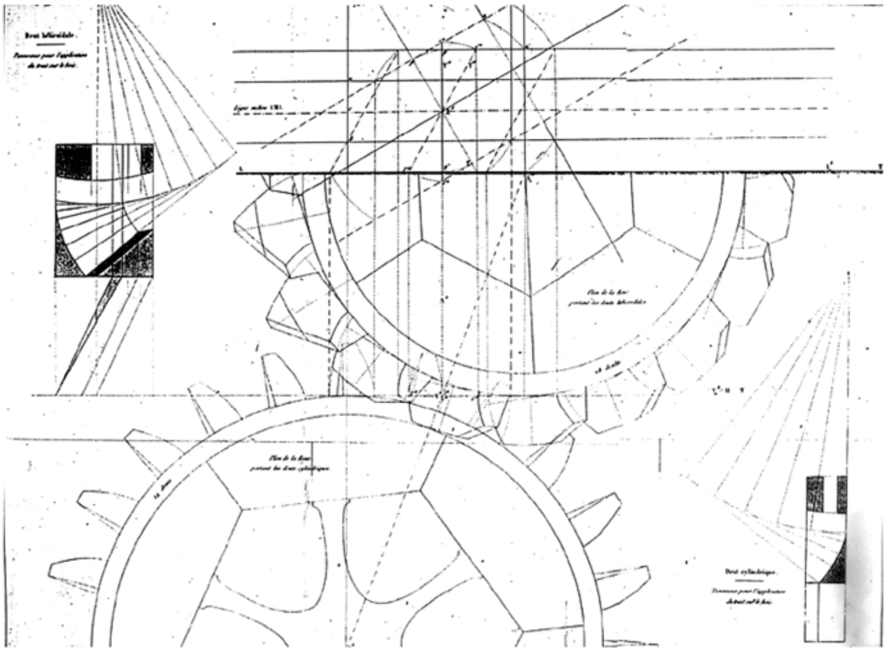


Fig. 9. Lower sized copy of Plate IV in Olivier book [15].

The special arrangement in space of a cogged wheel with a tangent worm screw is also illustrated by an Olivier model (Figure 8b).

In a second treatise on gears, 1842, [15], Olivier explored the general skew arrangement of wheels. Two wheel axes do not lie in that same plane and are not perpendicular. For instance, with Plate IV of [15] (Figure 9), Olivier showed how to obtain by geometric means, conjugate profiles for the teeth of a skew system, which can transmit rotation between two axes with an angle of 30°. One wheel has 18 teeth and the other one has 24 teeth.

The effectiveness of the design by descriptive geometry is confirmed by the actual construction of working models of gears. Alas, some of the models that are mentioned in [15] are lost. However, fortunately, some Olivier models of screw gearing were saved and they are still exhibited at the Museum of "Arts et Métiers" at Paris. Figures 10 and 11 show noteworthy models of an external skew gearing and an internal skew gearing, respectively.

In his book [15] on skew spatial gearing, one may wonder if Olivier found out general tooth profiles or only special possible surfaces. Despite

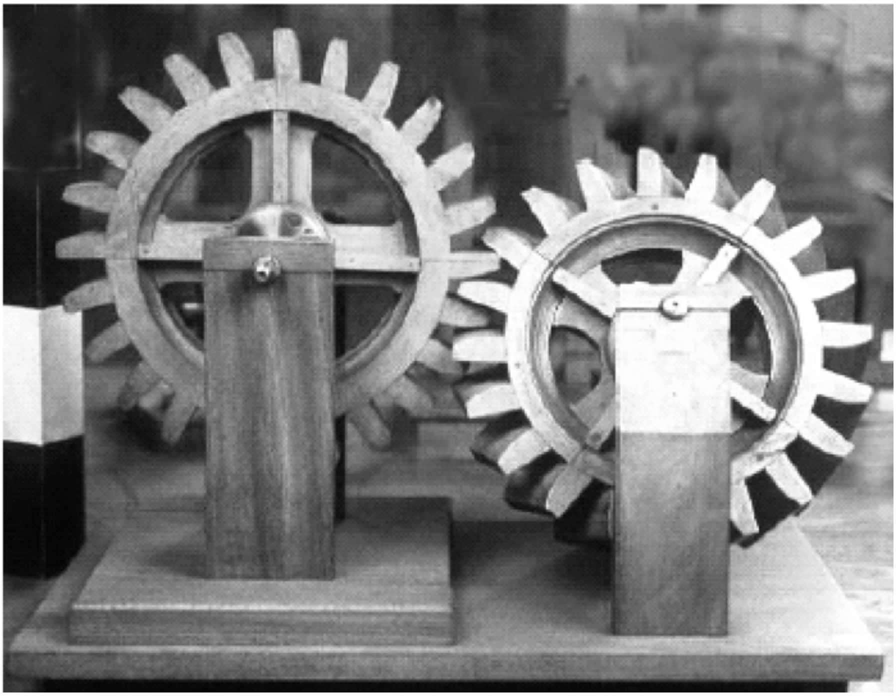


Fig. 10. Two gear wheels with a skew layout of their axes of rotation; external gearing (Musée des Arts et Métiers, No. 5455-).

the important historical contribution of Olivier, one of the two wheels has cylindrical teeth while the other wheel has helical teeth as written in table IV of [15]. This is confirmed by inspection of the corresponding spatial models (Figure 10). Hence, the conjugation of two helical surfaces was not disclosed in Olivier's work, and, therefore, the novelty of the "general spatial involute gearing" [A6] introduced by Jack Phillips in 2003, more than 160 years after Olivier, is not questionable.

Modern Interpretation of Main Contributions to Mechanism Design

Olivier's work ranks at the top of what can be done by using essentially the means of descriptive geometry. He described many types of ruled surfaces in a purely geometric manner almost without using equations. The ruled surfaces

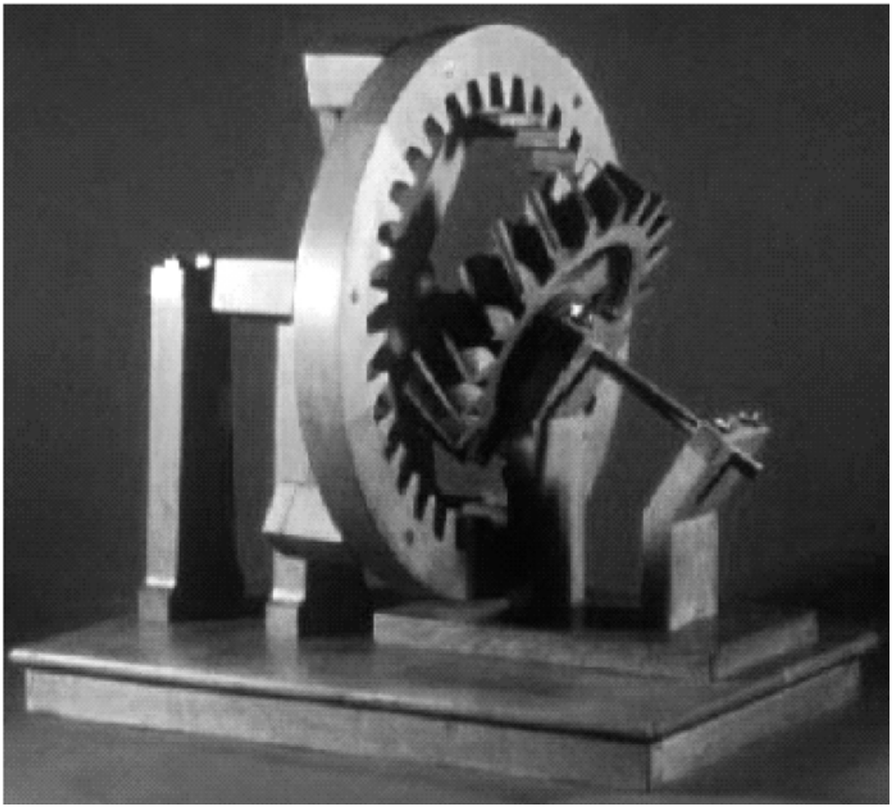


Fig. 11. Two gear wheels with a skew layout of their axes of rotation; internal gearing (Musée des Arts et Métiers, No. 5459-).

play a key role in kinematics and mechanism theory because of the theorem: any rigid-body motion (or displacement) is a screw motion. This theorem was stated for the first time by Giulio Mozzi in 1765 and it was found again by Michel Chasles in 1830. A screw has an axis and, therefore, when a rigid body moves with respect to another body, the locus of all the screw axes is a ruled surface that is called axode of the time-dependent movement. Consequently, Olivier's work can be considered as being a basic step in the edification of the *Screw Theory*, which was achieved by Robert Ball in 1900. Ball's work implements the mathematical tools of the geometry of straight lines, which was devised in 1869 by Julius Plücker. Hence, in a particular way, Olivier pioneered Plücker's work.

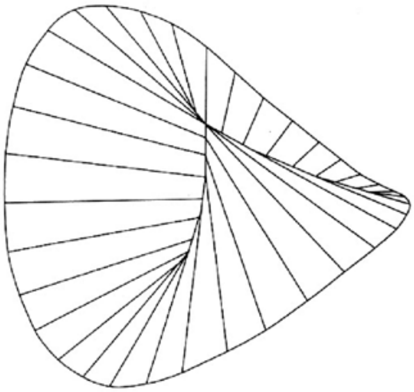


Fig. 12. Plücker conoid (or cylindroid) in modern kinematics [A5].

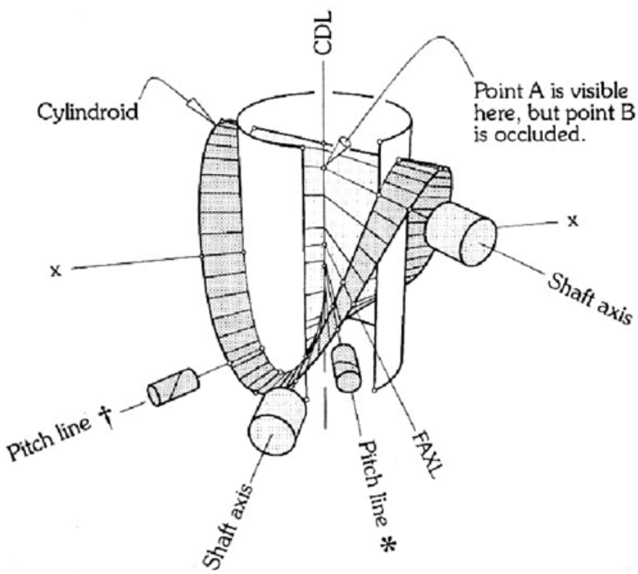


Fig. 13. Cylindroid (Plücker conoid) in recent advancement on gearing theory [A6].

Moreover, when evaluating Olivier's contribution, one has to be aware that the birth of vector calculation is contemporaneous with Olivier's work.

Figure 12 shows the Plücker conoid as presented in a modern book on robot kinematics (Selig, 2000, [A5]). The resultant screw (twist or wrench)

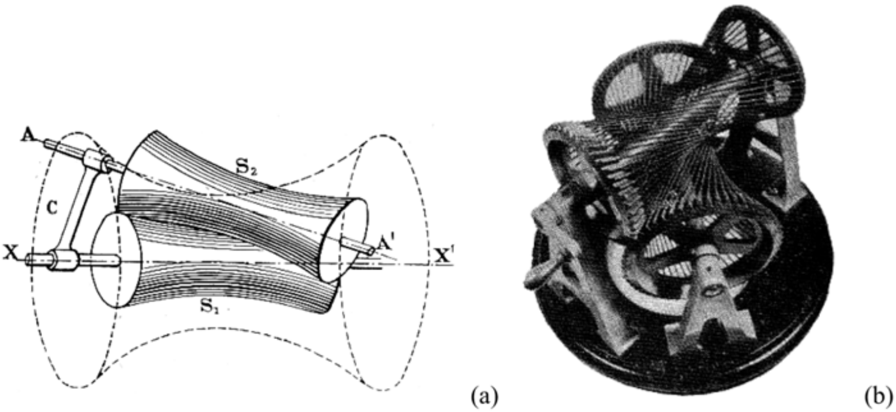


Fig. 14. Revolute hyperboloids (a) in a mechanism for transmitting the rotation; spatial gearing (b).

of two given screws is a third screw whose axis belongs to the cylindroid determined by the first two screw axes.

In a general skew arrangement of two gear wheels rotating around two fixed shafts with a given ratio of their angular velocities, the relative motion of one wheel with respect to the other one is a screw motion. Its screw axis is called *pitch line* in Phillips' book [A6], 2003, and belongs to the cylindroid of the two gear axes, (Figure 13).

A possible use of two revolute hyperboloids for transmitting rotation between two shafts with a constant velocity ratio is illustrated by the mechanism of Figure 14a that is excerpted from "Les mouvements mécaniques" by Marcel Nicaise, 1931 [A3]. The physical hyperboloids are in touch along a straight line. This line belongs necessarily to the set of straight lines constituting the cylindroid that is derived from the two twists in the fixed axes of the revolute pairs.

Cutting a slice in each of the two hyperboloids, adding teeth on the obtained revolute disks and thus making up a couple of cogged wheels, one obtains a rough design of a spatial system of gears (Figure 14b). Further considerations are needed for determining the adequate tooth shapes. Olivier proposed solutions to this complex problem more than 160 years ago. However, Olivier's work on spatial gearing was not definitely finished and was supplemented in 2003 by Jack Phillips who manifested his respect to Olivier calling

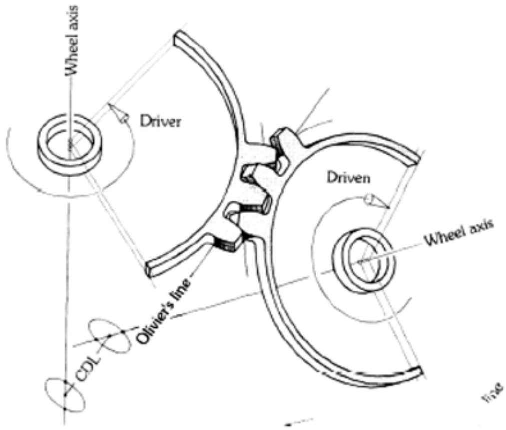


Fig. 15. Olivier's line in Phillips' book on the spatial involute gearing [A6].

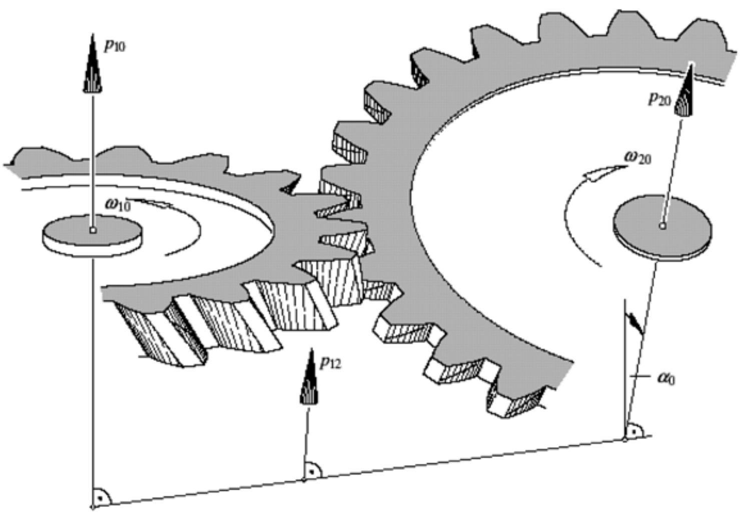


Fig. 16. Phillips'gearing as further explained by Stachel.

Olivier's line the rectilinear trajectory of the contact point in involute gears (Figure 15).

The general spatial involute gearing devised by Jack Phillips in 2003, [A6], has attracted the attention of present-day geometers. For instance, Hellmuth Stachel in 2004, [A7, A8] disclosed further explanations on the Phillips' gearing. In the present beginning of the twenty-first century, the facilities of electronic computers have replaced the manual drawings and the wooden models of Théodore Olivier who worked in the first half of the nineteth century. Amazingly, the virtual gears revealed by the use of modern computers look like Olivier models (Figure 16) [A7].

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[W5] Library of Ecole Polytechnique Former Student Biographical Notes, http://biblio.polytechnique.fr/F/YQ6AM3KVYEGQQI69KSNIAJ1HFQXVRD3QB7SBS89CH6HTD9Q7DA-00979?func=full-set-set&set_number=048446&set_entry=000013&format=999

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UFIMTSEV ANATOLY GEORGIEVICH (1880–1936)

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Abstract. This paper presents a description of some inventions of a talented Russian scientist-inventor who one hundred years ago suggested new ideas for the development of motors for airplanes, construction of electrical power stations and more than twenty different technological inventions. In particular,Ufimtsev offered an interesting idea for use of an inertia accumulator, which led to the realization of a wind-powered electrical power station.

Biographical Notes

Anatoly Georgievich Ufimtsev was born on the 24th of November, 1880 in Kursk, in the family of a land surveyor; he was a grandson of F. A. Semyonov (an eminent self-educated scientist, meteorologist-astronomer, mechanic, honored citizen of Kursk, and a corresponding member of the Russian Geographical Society). Since childhood he had exhibited the ability to devise and manufacture various hand-made articles that he had studied in elementary school. At 16 years of age he designed and built an "electro-copying pen" for plural copying and a high-speed typewriter.

Having entered into the struggle against the church, in 1898 Ufimtsev invented a bomb with a clock mechanism and blew up a "miraculous ikon" in Znamensky Cathedral in Kursk. After the explosion Ufimtsev was not found immediately, but in two years after this business he was arrested and banished for five years to Akmolinsk (Kazakhstan).

The famous Russian writer Maksim Gorky (1868–1936) became interested in the history of the icon explosion, found the inventor and provided him with material aid. On Gorky's money, Anatoly Ufimtsev equipped, within the prison of Akmolinsk, a small workshop to repair home appliances. In this

M. Ceccarelli (ed.), Distinguished Figures in Mechanism and Machine Science, 319–336.

Fig. 1. Anatoly Georgievich Ufimtsev (24 November 1880–10 July 1936).

workshop he started to make and sell kerosene lamps and oil lanterns with use of his original ideas in design.

Having returned after amnesty in 1906 to Kursk, he constructed and equipped, in his own manor on Semenovskaya Street in the centre of Kursk, a workshop for repair of sewing machines and bicycles, and also continued to work with kerosene lanterns. Lanterns of Ufimtsev?s design were eventually installed, and were used for many years, in the streets of Kursk, Sevastopol and other cities of Russia.

In 1909 Ufimtsev became interested in the design of flying machines. He constructed an unusual flying machine with a wing in the form of a spherical surface with a large radius; his UFO (Unknown Flying Object) was called a “sphereplane”. His sphereplane had an adaptation for ejection catapults, using compressed air, such as we now find in modern sea-aircraft. A three-wheeled chassis flying machine with tail wheel was built by Anatoly Ufimtsev for the first time in Russia at the same time as one was built by the American aircraft designer Curtis.

In the same year Ufimtsev created a double rotation aviation engine for the sphereplane. In 1910–1911 he had already constructed two new four-and six-cylinder birotational (double rotation) engines. Propellers rotated on opposite sides of coaxial shafts, one of which was hollow. In 1912 at the Second

International Aeronautics Exhibition in Moscow, Ufimtsev was awarded the Greater silver medal for his four-cylinder birotational engine. Unfortunately on the 11th of July, 1910 a hurricane and a strong storm destroyed the pre-production model of the flying machine.

In 1910 Ufimtsev improved the design of internal combustion engines and received a patent for the double rotation oil engine. At the beginning of the First World War he made serially linked engines for installation on threshers. In the common opinion of consumers, his engines were the most reliable among similar devices. During the First World War, Ufimtsev returned to the manufacture of birotational engines that were needed by military aircraft.

But the most significant technological contribution that Ufimtsev made after the war was the creation, for the first time in the world, of a reliably working wind-powered electro power station. He invented an inertial accumulator in 1918 – a flywheel and, in cooperation with the scientist Professor V. P. Vetchinkin, devised a new and unique wind-wheel with rotary blades and a variable corner of attack, as is now found in modern helicopters.

Ufimtsev and Vetchinkin identified full blossoming of the Russian energy industry with full use of wind power. They called it "continuous anemofication of Russia". They went so far as to compile statistical calculations on separate areas of the country which confirmed that all Russian power could be based on the use of wind power.

In April, 1923 the government of Russia allowed construction of a wind-driven power station in Kursk. It was built in a courtyard of Ufimtsev manor and began power production on the 4th of February, 1931. The wind-driven power station occupied a two-storied house, and also a part of the street, with the machine tools of the workshop placed in a cellar. Owing to an inertial flywheel, the power station produced some voltage within several hours even in windless weather. The design of Ufimtsev's station with a special wheel had outperformed the technology of the last hundred years.

On the 10th of July, 1936, in his 56th year and at the peak blossoming of his creative life, the inventor died.

Ufimtsev patented twenty-two inventions. His inquisitive mind and research talent had allowed him to make contributions in a widely varied range of technological advances.

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6. The copyright certificate No. 10092. The invention of devices for alignment of work of a wind-power plant. Under the application from March, 15th 1927. Leningrad, on October, 31st 1932.
7. The guarding certificate No. 69361. The invention of a boring machine for dot mass drilling and milling. On May, 13th 1916. The Ministry of Trade and the industries. Department of the industry. Committee on technical affairs.
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   On the invention of a special heated lamp. Under the application submitted on February, 14th 1906. St. Petersburg on October, 31st 1907.
10. The patent for the privilege No. 14971. On the invention of a special kerosene to heat a torch. Under the application submitted on December, 5th 1906. St. Petersburg on December, 31st 1908.
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12. The privilege which has been given out on January, 31st 1909. No. 15007, declared on December, 5th 1906. The description of kerosene for the heated lamp in automatic ignition.

Fig. 2. View of the wind-driven power station built by A. Ufimtsev (current photo).

13. The patent for the privilege No. 28318. On the two-contact oil engine. Under the application submitted on January, 31st 1914. Petrograd on December, 31st 1915.

Review of Main Works on Mechanism Design

Description of a Birotative Motor for an Airplane

In 1909 Ufimtsev designed a two-cylinders two-cycles rotary engine [1, 2, 14]. The motor today is kept in a permanent display of Ufimtsev's inventions in the Moscow Aerospace Centre (see Figures 3 and 4).

The motor was built by Ufimtsev and was tested in a Schetinin factory in 1908–1909 in Moscow Forest University where it registered maximal power around 30 kW.

This motor is kept in the Aerospace Centre in Moscow in very good condition.

In 1910–1911 he built two four- and six-cylinders birotative engines.

Fig. 3. General view of a permanent display of Ufimtsev's inventions in the in Moscow Aerospace Centre.

Fig. 4. General view of the birotative engine; diameter 1 m, length 0.85 m, weight 50 kg, average power 18 kW.

The general view of this engine is shown in Figure 2. In 1912 Ufimtsev was awarded the Silver medal at the Third Russian conference.

Description of the Invention of Ufimtzev's Airplane

Figure 5 shows the scheme of a Sphereplane No. 1. This picture shows inventor Ufimtzev with colleagues at a test flight of the sphereplane. This airplane had an original structure. The wing of the plane had a circular shape and three wheels, one of which was placed at the front of the airplane. This apparatus was driven by an engine with power around 15 kW which had been

Fig. 5. General view of Ufimtzev's Sphereplane.

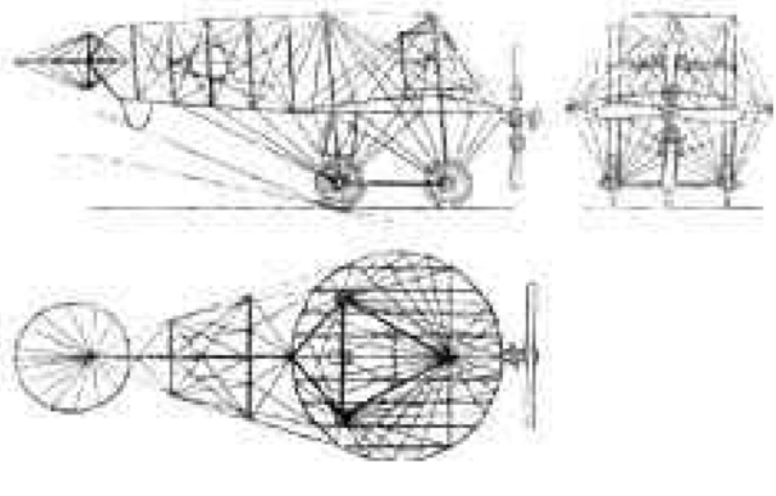

Fig. 6. Scheme of a Sphereplane No. 1.

designed by Ufimtzev. This three-wheel chassis was built for the first time in the world simultaneously with one built by the American researcher Curtis.

The wing was made in the form of a circular frame and had a diameter of 3, 4 m with an area of 9 m^2 and had six main frame pivots which provided durability of the wing. The steering system was maintained behind the screw. This sphereplane was tested successfully.

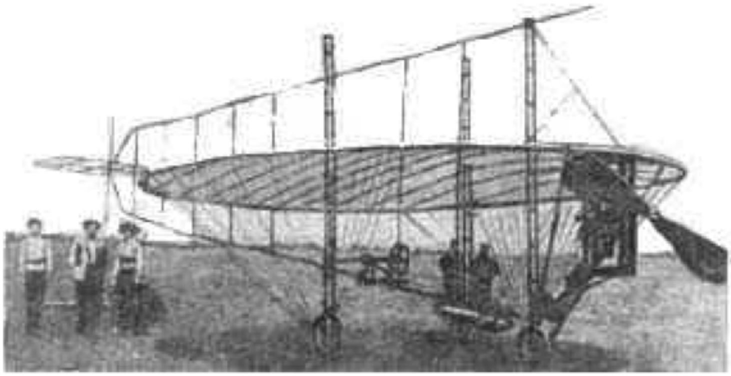

Fig. 7. The scheme of a Sphereplane No. 2.

The scheme and structure of Sphereplane 2 can be seen in Figure 7. The general scheme of this airplane was similar to the first one, but the dimensions were two times larger. The area of the wing was 36 m^2 and the steering system in the wing was integrated. This wing had 11 additional frame pivots for providing of adequate durability of this plane.

Description of a Wind-Driven Generator Invention

Ufimtsev looked into history for ideas about generating energy through wind power. Wind machines were used for grinding grain in Persia as early as 200 BC. The same type of machine was introduced into the Roman Empire by 250 AD. By the 14th century Dutch windmills were in use to drain areas of the Rhine River delta. In Denmark by 1900 there were about 2500 windmills for mechanical loads such as pumps and mills, producing an estimated combined peak power of about 30 MW.

The first windmill for electricity production was built in Cleveland, Ohio by Charles F. Brush in 1888, and in 1908 there were 72 wind-driven electric generators from 5 kW to 25 kW. The largest machines were on 24 m (79 ft) towers with four-bladed 23 m (75 ft) diameter rotors. Poul La Cour is considered as Urvater of the modern wind energy. The first wind-powered device to produce electricity was built in 1891. It used a number of already existing windmills around the country to capture the wind for conversion into electricity.

Ufimtsev analyzed this problem pragmatically. He created a unique device, never before built, called a kinetic inertia accumulator, which released an even flow of electrical energy. The accumulator provided a thousand incandescence lamps in the presence of a Special Commission. The invention of the accumulator created a real possibility for exploitation of wind generators and thus an economic boon to the country.

Ufimtsev's wind motor was made with the help of the famous aerodynamicist V. P. Vetchinkin. Their motor turned out to be half the weight of any previously known motor.

The Kursk wind-driven electrical station quickly became known for its simplicity of service and its outstanding operating qualities.

Patents defending some of Ufimtsev's basic inventions with respect to a wind-driven electrical power station are listed above [3–7]. Ufimtsev realized that full exploitation of wind power could only be achieved with such an accumulator. His device was represented as an object with a large symmetric mass rotating at high speed in a rarefied medium that would minimize the loss of energy due to resistance.

Figure 8 compares various constructions of inertia accumulators. Figure 8a shows an accumulator with a vertical axle and electrical transmission of energy. Figure 8b shows an accumulator with a lower fulcrum. Figure 8c shows the vertical section of a casing with some spring bearings. Figure 8d shows the chart of the disposition of roller bearings attached to a horizontal axle. Figure 8e shows the chart of the disposition of disk supports with a horizontal axle and Figure 8f shows an accumulator with the mechanical transmission of energy.

A rotating wind-wheel is connected with an axle of a dynamo machine. This machine gives a direct current irrespective of a changing frequency of revolutions and sends the current to the dynamo motor of the inertia accumulator.

Energy is stored in this accumulator in the form of kinetic energy of the rotating object. If it is necessary, this energy could be used to create electric power.

It was very important to decrease losses of energy that were due to resistance of the rotating body contacting with the environment and friction in the supports of the body axle. A crucial question was how to decrease the vibration of the rotating body. Solutions of these problems allowed preservation of energy in the inertia accumulator for long periods of time.

The accumulator of the prolonged operation (Figure 8) consisted of the rotating object (1) on the vertical or horizontal axle (2). The body was repres-

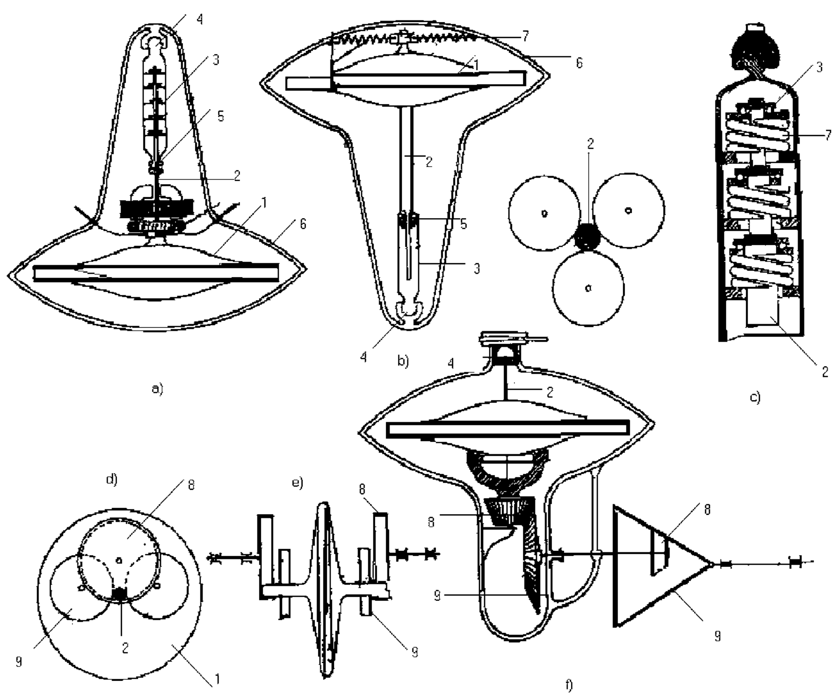

Fig. 8. Schemes of inertia accumulator: 1. rotating object; 2. axle of rotation; 3. bearings; 4. spherical fulcrum; 5. case; 6. casing; 7. spring; 8, 9. gearing.

ented as a disk with a constant resistance factor at any point. The vertical axle (2) of the disk is hung on one or more ball bearings (3) with spiral springs (7). Springs lean on circular projections of the cylindrical case, in which all bearings are located.

Bearings decrease the friction owing to the allocation of the weight of the disk (1) among all bearings. The case (5) is hung on the casing (6) with help of the universal hinge. The casing (6) contains all parts of the system that exist in the rarefied medium. The case (5) is separated from the other space with the help of the hydraulic bolt, because the pressure of oil vapor for bearings does not give the possibility to rarefy air in the space of the casing (6). The hydraulic bolt contains either mercury or an alloy of sodium and potassium and does not hinder the free rotation of the axle (2). The pumping out may be replaced the filling some light gas, for example a hydrogen, under conditions of the atmospheric pressure.

The armature (8, 9) of the dynamo motor is fastened to the axle (2) for the transmission of energy to the accumulator and from it. The dynamo motor will keep up the tension on the contact level, as the frequency of revolution changes over time. This property is attained by turning parts of the electromagnets automatically. Reversibility of the dynamo machine enables either rotation of the disk (accumulation of energy) or forming of the current due to this accumulation. The motor may be placed outside the casing on the axle (2), if the axle is put out, or on the other axle, which would have to connect the gear with the axle (2). The disk (1) of the accumulator may be placed above (Figure 8e), in which case the axle (2) is the supporting part, and bearings are placed below. The bearing (1) is fixed to the casing with help of springs for maintenance of the upper edge of the axle (2).

The bearing (1) is fixed to the casing with help of springs for the maintenance of the upper edge of the axle (2). Disk supports are represented wheels (8), (9) with sides, on which the end of the axle (2) lies on the internal side. Wheels are fixed with help of axles of auxiliary wheels.

The inertia accumulator (Figure 8f) has a mechanical transmission. This transmission is used in accumulation of energy to be used for motion (trams, cars, airplanes). The vertical axle (2) holds the disk (1), which is attached by conical gear (8, 9) and the next external gearing. The upper end of the axle is hung on the box with help of roller bearings (4), which are located in the casing (6).

After these mechanisms were proved to be possible, Ufimtsev worked on improving their effectiveness. He established the fact that when energy is transmitted from a wind generator to a dynamo machine, big problems arise in consequence of a huge difference between quantities of revolutions of these two mechanisms.

In transmission of energy, energy for the dynamo is taken on by the small wind motor, which is fastened to the wing of the wind motor and is connected with the axle of the dynamo. The small wind motor can take on a larger store of energy; and it gives a larger angular velocity when the wind motor is the propeller because the peripheral velocity at the end of the wing is very high.

The suggested wind-driven electrical generator (Figure 9) is a wind motor with dynamo machine (3) and the propeller (2), that is located on the axle of the dynamo.

The gear (Figure 9b) in the wind-driven electrical mechanism is modified because the transmission from the propeller (2) to the dynamo (3) is made

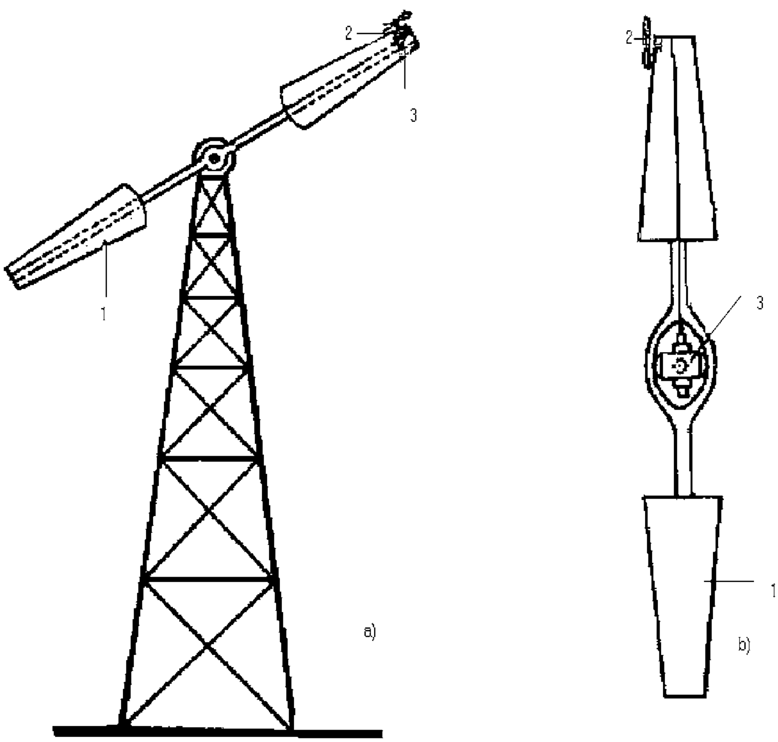

Fig. 9. Lay-outs of transmission of energy. (a) The dynamo motor is at the end of the wing; (b) the dynamo motor is at the center of the propeller. 1. propeller; 2. small wind motor; 3. dynamo machine.

with help of the couples of cogwheels, and the electrical generator does not connect with strokes of the motor.

The mechanism for alignment of work at the wind-driven electrical station (shown in Figure 10) was suggested by Ufimtsev and was announced in a patent application on March 15, 1925. The patent was issued on 29 June, 1929.

The development is represented in the following manner: The wind generator is fastened on the tower and transmits the motion mechanically to the generator of electrical power. The generator is connected with the buffer inertia accumulator (1), which stores the energy by use of the rapidly rotating object with a large mass in the rarefied medium that helps decrease loss of en-

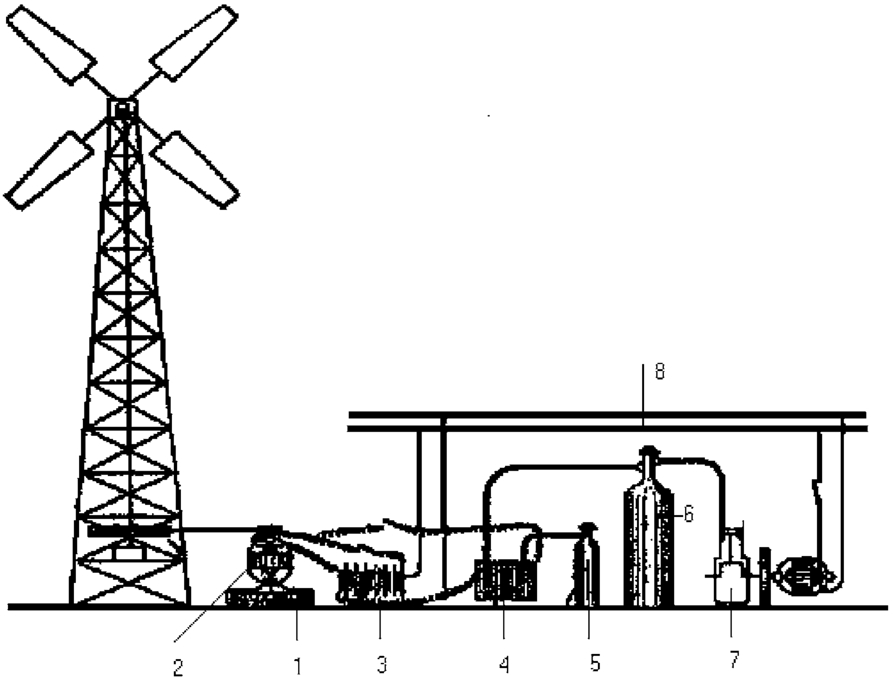

Fig. 10. Scheme of wind-driven electrical station. 1. inertia accumulator; 2. generator; 3. line of consumption; 4. electrical storage battery; 5. electrolyzer.

ergy from resistance. This part of the mechanism makes it possible to produce an even output over time.

The generator (2) directs the current to the line of consumption (3) and the surplus – to the electrical storage battery (4) with whose help the mechanism aligns temporary overloading.

When the storage battery is charged the current is directed to the electrolyzer (5), where water is decomposed into hydrogen and oxygen. The oxygen is eliminated from the gathering cistern, but the hydrogen is used as a power supply for the reserve motor. This motor is mechanically connected with the auxiliary generator, which is turned on parallel to the main generator and produces electrical energy without the wind.

Modern Interpretation of Main Contributions to Mechanism Design

Total analysis of Ufimtsev's inventions shows that all his creations solved concrete problems in different fields of technology during the beginning of the 20th century. Already one hundred years ago he suggested some new ideas for the development of motors for an airplane, wind-driven electrical power station and numerous other inventions besides. His creations can be divided into two categories. One category consists of inventions that had a direct impact on the early years of the 20th century. These are mainly such items as a boring machine for drilling and milling [8], a lathe [9], a special heated lamp [10, 11], a kerosene for the heated torch [12], kerosene of the heated lamp in automatic ignition [13] and many others[14]. These inventions had an important significance for that period of economical development in the world. The second category includes inventions and ideas that have now been distributed all over the world. First of all we will talk about wind-driven electrical power stations.

In the course of World War II the Danish company F. L. Smidth built several two- and three-blade wind-powered devices. Likewise at this time the two technical designers Smith and Putnam built a wind system north of New York. The tower height amounted to 33 meters and the whole construction had a diameter of more than 52 meters. One day a rotor blade flew off resulting in damages 230 meters away. The potential loss in repairs of such incidents would have been enormous, therefore the operators decided that this power station was to be shut down [15].

In the year 1956, J. Juul built a wind system that is considered as a predecessor of today's wind towers. It was a three-blade system with electromechanical wind adjusting. Juul's construction was the largest and the most reliable in the world for a long time.

The oil crisis in 1973 aroused interest in renewable energy in several countries. Denmark built many large systems, followed shortly by the USA, Sweden, Germany and Great Britain. The maintenance was however very expensive and the electricity tariffs were accordingly high. By 1980, sufficient wind- driven power stations had been designed with a production capacity of more than 50 kW. This had the consequence that electricity tariffs sank around approximately 50%. The Americans (especially the Californians) developed a proper "wind intoxication". Wind systems shot up like mushrooms from the

Fig. 11. Wind-driven farm in Neuenkirchen, Dithmarschen, Germany.

soil at this time in California. When however around 1985 the interest sank again, many wind-driven power stations disappeared overnight.

The tower of a wind-powered device carries the gondola and the rotor. Usually a high tower is used as with rising height the wind velocity increases. A modern wind-driven power station has a tower of 40–60 meters height. The gondola is not fixed in direction by its connection to the tower. It can align itself and the rotor to follow the wind and thus always profit from its optimal strength.

At present, the largest “wind market” exists in Germany but also in other countries the interest in this energy begins to grow again.

Wind-driven power is the conversion of wind energy into more useful forms, usually electricity using wind turbines. In 2005, worldwide capacity of wind-powered generators was 58,982 megawatts; although it currently produces less than 1% of worldwide electricity use, it accounts for 23% of electricity use in Denmark, 6% in Germany and approximately 8% in Spain. Globally, wind power generation more than quadrupled between 1999 and 2005.

Most modern wind power is generated in the form of electricity by converting the rotation of turbine blades into electrical current by means of an electrical generator. In windmills (a much older technology) wind energy is used to turn mechanical machinery to do physical work, like crushing grain or pumping water.

Wind power is used in large scale wind farms for national electrical grids as well as in small individual turbines for providing electricity to rural residences or grid-isolated locations.

Wind-driven energy is ample, renewable, widely distributed, clean, and mitigates the greenhouse effect if used to replace fossil-fuel-derived electricity [15].

An estimated 1 to 3% of energy from the Sun that hits the earth is converted into wind energy. This is about 50 to 100 times more energy than is converted into biomass by all the plants on Earth through photosynthesis. Most of this wind energy can be found at high altitudes where continuous wind speeds of over 160 km/h (100 mph) occur. Eventually, the wind energy is converted through friction into diffuse heat throughout the Earth's surface and atmosphere.

The origin of wind is simple. The Earth is unevenly heated by the sun resulting in the poles receiving less energy from the sun than the equator does. Also the dry land heats up (and cools down) more quickly than the seas do. The differential heating powers a global atmospheric convection system reaching from the Earth's surface to the stratosphere which acts as a virtual ceiling.

As the wind-driven turbine extracts energy from the air flow, the air is slowed down, which causes it to spread out and diverts it around the wind-driven turbine to some extent. Albert Betz, a German physicist, determined in 1919 (see Betz' law) that a wind-driven turbine can extract at most 59% of the energy that would otherwise flow through the turbine's cross section. The Betz limit applies regardless of the design of the turbine.

Because so much power is generated by higher wind speed, much of the average power available to a windmill comes in short bursts. The 2002 Lee Ranch sample is telling; half of the energy available arrived in just 15% of the operating time. The consequence of this is that wind energy is not as evenly available as in the case of fuel-fired power plants; additional output cannot be supplied in response to load demand. Since wind speed is not constant, a wind-driven generator's annual energy production is never as much as its nameplate rating multiplied by the total hours in a year. The ratio of actual productivity in a year to this theoretical maximum is called the capacity factor. A well-sited wind-driven generator will have a capacity factor of as much as 35%. This compares to typical capacity factors of 90% for nuclear plants, 70% for coal plants, and 30% for oil plants [8]. When comparing the size of wind-driven turbine plants to fueled power plants, it is important to note that 1000 kW of wind-driven turbine potential power would be expected to produce as much energy in a year as approximately 500 kW of coal-fired generation. Though the short-term (hours or days) output of a wind-driven plant is not completely predictable, the annual output of energy tends to vary only a few percent points between years. When storage, such as with pumped hydroelectric storage, or other forms of generation are used to "shape" wind power (by assuring constant delivery reliability), commercial delivery represents a cost increase of about 25%, yielding viable commercial performance. Electricity consumption can be adapted to production variability to some extent with Energy Demand Management and smart meters that offer variable market pricing over the course of the day. For example, municipal water pumps that feed a water tower do not need to operate continuously and can be restricted to times when electricity is plentiful and cheap. Consumers could choose when to run the dishwasher or charge an electric vehicle.

Summary

The inventions which were described in this paper allowed construction of the first wind-driven electrical station. It has been operational in Kursk for a long time. This station showed that Ufimtsev's ideas were correct and made a convincing argument for utilization of wind-driven power based on economics and ecological responsibility.

Acknowledgements

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JAMES WATT (1736–1819)

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Abstract. It is generally accepted that the birth of James Watt was destined to bring about a revolution in the utilization of power. Watt, see Fig. 1, is regarded by many to be the progenitor of the science of thermodynamics. He was not only the inventor of the separate condenser and many other parts of the steam engine, but he was the first to study the steam engine scientifically. He also made distinguished contributions to the development of workshop practice. His scientific examination of heat losses in engines led him to recognition of the influence of latent heat on steam engine economy. Watt was at one and the same time a scientist, an inventor, and a producer. He put numbers to the concept of horsepower and is credited with inventing the centrifugal governor for automatic control of the speed of the steam engine, a rotary motion device for the steam engine, a pressure gauge, a smoke-consuming furnace, and a letter-copying device based on the link transfer process. Watt also invented an approximate straight-line mechanism for his famous double-acting steam engine, thereby creating a whole new family of linkages. This brief article will focus on Watt as the inventor of parallel motion, the basis of many machines. It is interesting to note that at the age of 72, he wrote to his son [1]: "Though I am not over-anxious after fame, yet I am more proud of the parallel motion than of any other mechanical invention I have ever made." Watt was rightfully proud of the parallel motion four-bar linkage. This mechanical invention is believed to be the beginning of an ordered and an advanced synthetic process [2].

Biographical Notes

James Watt was born on the 19th of January 1736, the fourth son to James and Agnes Watt. He was born near the port of Greenock, on the southern bank of the River Clyde, not far from where the Clyde turns south into the Firth and about twenty-five miles west of the City of Glasgow. From a young age Watt showed signs of the chronic ill-health that was going to torment him through the greater part of his life. His mother was devoted to him, and, rather than

M. Ceccarelli (ed.), Distinguished Figures in Mechanism and Machine Science, 337–369.

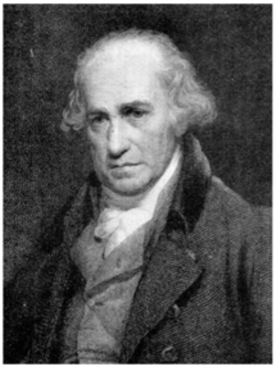

Fig. 1. James Watt in later years.

send him to a school where he might not be properly looked after, she kept him for a time under her own care at home and gave him his first lessons. By the time he was let out of the family circle into a wider world, his individuality and originality were already well developed, and he never showed any tendency to adapt himself to the type that was most admired by his schoolfellows. He went his own way and took the consequences, and they must have been severe. Watt was slow and awkward, and fell below the ordinary standard demanded by the common routine of school lessons. In fact, Watt was thought rather dull at his lessons. However, when he adapted to his new surroundings and found work that was congenial to him, his genius peeped through the veil of his childishness. His abilities began to appear when, at the age of about fourteen, he was put into a mathematical class, and made rapid progress. For a more detailed account of Watt's schooldays, the reader is referred to [3–5].

In 1753, when Watt was seventeen, his mother died. It was probably his mother's devotion to him that had kept Watt so long at home when other boys of his age were away earning their living. It is believed her death broke up the family life at Greenock. In June of the following year he went to Glasgow to learn the craft of a mathematical instrument maker. It was a profession closely allied to those of his father and his grandfather, and it gave more scope to his mechanical dexterity than he would have obtained by following either of their trades. The prospects, too, were good. It was described, at this date, as a "very ingenious and profitable business," and was by no means overstocked with labor.

When Watt arrived in Glasgow, however, he found there was no one who could teach him. He spent a year there, working under a nondescript mechanic who called himself an "optician," until he attracted the attention of Dr. Dick, Professor of Natural Philosophy at the University of Glasgow. Dick realized that here was a first-class talent going to waste, and advised him to go to London and obtain the best training that was available. Watt asked the permission of his father to go, and it was granted. It was a momentous decision, for this was surely the first time in the history of the Watt family that a member had proposed to cross the border from Scotland to England. Also, London was a long way from Glasgow when traveling on horseback. In addition, there was the expense to consider. Apparently Watt's father had either overreached himself in his speculations or had suffered losses at sea; for, although he had once been quite well-to-do, he was now obliged to leave his son to make his own way in the world, giving him only the most meager of allowances while he was obtaining his training. In spite of all these difficulties the adventure was accepted, and on June 7th, 1755, Watt set off for London, with a letter of introduction from Dr. Dick.

It took Watt approximately twelve days to reach London, and immediately he encountered some difficulties. The city was still clinging to ancient customs and privileges, chief among which was the right to keep all of its trade in the hands of the native-born townsmen, and to forbid anyone from another town to settle down within the city walls to earn a living. The time was long past when any town could preserve this monopoly intact, or indeed wanted to, but the right remained in theory, and could be used discreetly to remove undesirables. The vagrant, who seemed likely to become a pauper, and the skilled craftsman, who might prove a dangerous competitor for the custom of the townspeople, were refused admission. However, the wealthy merchant and the honest, non-enterprising laborer went unmolested. The initiative in these matters came generally from the Guilds and Companies which controlled the various trades carried on in the City. They were afraid of competition, and anxious to keep down the number of tradesmen among whom the available custom had to be divided. The chief principles which the Guilds had inherited from the Middle Ages were as follows; (i) all regulations affecting the trade were made by the Masters who ruled the Guild; (ii) no person could set up in business on his own unless he was a Master and had been admitted as such into the Guild, and (iii) the normal way of becoming a Master was by serving an apprenticeship of seven years under a Guildsman, and then paying the fees for admission to the rank and privileges of Mastership. In this way, the trade was protected against an influx of inferior and irresponsible labor which might lower the standard of work, and, by competing for employment in the restricted market of the town, lower the level of the earnings of the craftsman.

Society in the reign of George II, however, was anything but medieval. Little was left of the elaborate system of industry based on the Guild. At the top of the industrial scale was a class of wealthy men, merchants or employers of labor, who had no patience with rules of this kind. They ran their business as they thought best, advanced boldly into any field that looked profitable, respecting the preserves of nobody, and they had no intention of teaching the secrets of their trade to any one except their own sons. At the other end of the scale were the laborers in common trades where the required degree of skill was small. Such men were not likely to go through a long period of apprenticeship when they could learn their job well enough without it, and nothing awaited them at the end of it but a fight for existence in an overstocked labor market in which they had no special advantage. Between these two classes came the highly skilled handicrafts, and there conditions were often quite different. As a long period of training was essential, apprenticeship had some meaning, and when it was over the craftsman was ready to start a business on his own. The Masters, in a trade of this kind, were in a commanding position. They had no employers over them with power to dictate terms; they had nothing to fear from the competition of upstart unqualified workmen; and they had a monopoly in training recruits to the craft. Whenever there were enough of them in a town to have an organization of their own they made strict rules for the training of novices and their admission to the status of Master, and no one who had not qualified according to these rules was permitted to open shop within the town.

The clockmakers of London were a trade of this kind. The company was not medieval in origin; it had been founded in 1631. But it was by nature suited to the medieval type of organization. The mathematical instrument makers were a branch of the Company of Clockmakers and followed the same rules. Watt, apparently, had not thought of this difficulty. His case was exactly that for which apprenticeship rules were designed. He wanted to be trained in order to become a Master and start a business on his own. His only proper course of action was to bind himself by a legal contract as an apprentice to a member of the trade. He was in no position, however, to conform to the ordinary regulations. In the first place he was too old; in the second place he was a "foreigner" and had no right to work in the City; and in the third place he could not afford to undertake to serve the full term of seven years. He must find a Master who was prepared to break the rules. The fact that he was a foreigner who had no intention of setting up shop in the city was a point in his favor, for London was not afraid of possible rivals in Glasgow. To teach such a man the mysteries of the craft was a breach of the letter of the law only, not of the spirit of the law. It took Watt some three weeks to find Mr. John Morgan of Cornhill, a man who was willing to take him for a year and teach him all he wanted to learn. During that time Watt was to give his labor free, and as the engagement was quite irregular, he had to pay the large sum of twenty guineas to compensate his master for the trouble he was causing to his conscience.

Watt settled down to do the seven years of work in a single year. He put in ten hours of work a day, five days each week, but it was difficult to avoid wasting time. The workmen in the shop were specialists on some particular instrument; Watt wanted to learn to make them all, and so he worked with each in turn. But if the man he wanted happened to be busy, or away for a time, Watt was interrupted in his course of progress. In six weeks, however, he had outstripped a fellow-apprentice who had been in the shop for two years; in nine months he was as skillful as a fully trained and experienced workman, and could cover a wider field. All this time he hardly ever went out. When he finished work in the evening he was much too tired to think of amusements, and anyhow he could not afford them. But he had another reason for staying indoors. England was enjoying a short interval of peace, recovering from the strain of fighting with Austria against Prussia, before she embarked on a new war with Prussia against Austria. Some fifteen years before, to the strains of the popular new song, "Rule Brittania!" the British fleet had sailed out to defend their precious monopoly in the slave trade. Now, while the people of London were still proclaiming that "Britons never, never, never will be slaves," the officers of the Press-gang were lurking around the corner ready to pounce on any young Englishman who had faith in the freedom of his country as to walk the streets of the capital after dark. This was a serious danger to Watt, for, as he was a stranger with no rights in the City, he could not claim the protection of the civil authorities. In the spring of 1756, the Press-gang became very active. A fleet had to be manned in a hurry for Admiral Byng to take out, to disgrace itself at Minorca. It is believed that a thousand men were taken in one night. “They now press anybody they can get,” Watt wrote to his father, “landsmen as well as seamen, except it be in the Liberties of the City, where they are obliged to carry them before my Lord Mayor first; and unless one be either ’prentice or a creditable tradesman, there is scarce any getting off again. And if I was carried before my Lord Mayor, I durst not avow that I wrought in the City, it being against their laws for any unfreeman to work, even as a journeyman, within the Liberties.” Fortunately, Watt escaped the clutches of the Press-gang.

All this time, Watt was working much too hard and not getting enough to eat. He cut his expenditure on food down to eight shillings a week, and could get it no lower without “pinching his belly.” The strain was too much for his fragile constitution. When his year was up his health gave way, and he suffered from violent attacks of rheumatism. He longed to get back to the fresh air of the Scottish countryside. In August 1756, he found the courage to face the weary journey, and mounting his horse, he turned his back on London. After a short stay at Greenock that restored his health and his spirits, he traveled on to Glasgow, with the outfit of tools bought in London, to offer his newly-won skill to the world. Watt, however, met with the same difficulties in Glasgow as he had faced in London. Here, too, he was a “foreigner,” and a dangerous “foreigner,” because he did not wish to only study the craft in the shop of a Master, but had every intention of setting up shop for himself. His trade came under the jurisdiction of the Incorporation of Hammermen, and its collection of industrial autocrats, worthy men, no doubt, but intellectually hammers indeed as compared with Watt’s gimlet. They refused him permission to work within the town in any capacity whatsoever. This was in spite of the fact that there was not one of them who pretended to understand the rudiments of his particular craft. Watt was saved by one of those odd coincidences that crop up from time to time in the ages of history. Within a month of his arrival in Glasgow, the University received a present of a case of astronomical instruments from Alexander Macfarlane, a merchant living in Jamaica. Classes in physical astronomy had recently commenced, and the gift was most opportune, but the sea voyage had thrown the delicate instruments out of gear, and they needed overhauling by an expert. Dr. Dick, in whose charge they were placed, remembered his young friend and asked him to undertake the work. Watt was delighted to have this chance of proving his skill, and soon put the whole collection into perfect order, for which service the University voted him the sum of five pounds. When, shortly afterwards, it was learned that he had been refused leave to have a workshop in the town, the University took him under its protection and gave him a room within the walls of the College, where the writ of the Hammermen did not have jurisdiction.

This may have been the turning point in the life of James Watt. Watt was already a brilliant mechanic [6–10], but it is reasonable to assume that he would never have won fame as an engineer if he had not also become a great scientist. That side of his genius had hitherto been starved. In the University, he found himself for the first time in the society of men who were his equals in intellect and his superiors in scientific experience. Also, these men, being pioneers in an unconquered territory, had none of the pride that makes the professional refuse to associate with the amateur, nor did they, like some jealous guardians of accumulated knowledge, feel proprietary about their science and resentful against trespassers. It was as the mathematical instrument maker to the University of Glasgow that Watt gained admission to the precincts of the College in the summer of 1757, but as soon as his remarkable gifts were recognized, he was treated by both Professors and students as a friend and colleague rather than as an employee. The initial steps were made easy for him by the fact that he was already known personally to some of the University staff. Professor Muirhead, a relative of his mother, who had first introduced him to Dr. Dick was still there; and when Dick died, early in 1757, his successor as Professor of Natural Philosophy was a man named Anderson, the brother of one of Watt's school friends. Anderson was a young man, not more than eight years senior to Watt, and provided an excellent channel of approach to the keener scientists both of the older and the younger generation. Watt's workshop was in the inner court of the College and connected to the premises occupied by the Natural Philosophy department. Teachers and students would come into the workshop, as they were leaving or returning to their work, to consult him about some piece of apparatus or to give him an instrument to repair. His friends dropped in to chat with him and brought their friends. Before long they were discussing with him not only the intricacies of apparatus but the scientific problems on which they were engaged in research. Watt's workshop became the regular meeting-place for those who were doing original work and could accept criticism of the theories suggested to them by the results of their experiments. More than once a Professor received a valuable hint from some swift thought hatched in the brain of the young craftsman.

Of all the friends that Watt made at this time the two who most deeply influenced his future were Joseph Black and John Robison. Black was a scientific genius of the first order. He had that rare gift of imaginative insight that is not afraid to leap into a new world of speculation, finding, as it were by inspiration, a fresh significance in facts that have long been known to all. But he was not one to make wild guesses. "No man," said Adam Smith, who knew him well, "has less nonsense in his head than Dr. Black," and he combined this freedom of vision with an unrivalled lucidity of exposition and accuracy of experiment. Lord Brougham had heard him lecture and wrote of him, "I have heard the greatest understandings of the age giving forth their efforts in its most eloquent tongues, but I should, without hesitation, prefer, for mere intellectual gratification, to be once more allowed the privilege which I in those days enjoyed of being present while the first philosopher of his age was the historian of his own discoveries." Black had come across Watt when he was at work on Macfarlane's instruments. He would come and stand in the shop toying with a quadrant and whistling softly to himself. But it was not until later, when he had Watt make him some apparatus for his experiments, that he became aware of Watt's genius. "I found him," he says, "to be a young man possessing most uncommon talents for mechanical knowledge and practice, with an originality, readiness and copiousness of invention which often surprised and delighted me in our frequent conversations together." The two men became close friends, and Black's affection for Watt lasted to the end of his life. When he was an old man a friend brought him news of Watt's triumph at law over an infringer of his patent. The old scientist, weakened by years of illness, wept with joy; and then apologized. "It is very foolish, but I can't help it, when I hear of anything good to Jamie Watt." Watt profited immeasurably from his contact with this inspiring mind, and was also kept in touch with the most advanced scientific thought of the day. He realized his debt to Black. "To him I owe," he said, "in great measure my being what I am; he taught me to reason and experiment in natural philosophy, and was always a true friend and adviser."

Robison was a younger man, who had just graduated when Watt arrived at the University. Though an able scientist, good enough to be elected Professor both in Glasgow and in Edinburgh, he was not the same caliber as Black. But he had great vitality and enthusiasm, qualities which made him an ideal companion for Watt when his bouts of ill-health made him talk of giving up work altogether. Robison quickly recognized that Watt was his superior, and always generously admitted it. He has described his first conversation with Watt in his workshop in the College: "I saw a workman, and expected no more; but was surprised to find a philosopher, as young as myself, and always ready to instruct me. I had the vanity to think myself a pretty good proficient in my favorite study, and was rather mortified at finding Mr. Watt so much my superior." They became friends, but Robison's adventurous tastes carried him away to sea soon afterwards. Several years later he returned, and renewed his friendship with Watt. He found that, thanks to his more systematic training, he could help Watt by testing and analyzing "the random suggestions of his inquisitive and inventive mind." But Watt was undoubtedly the leader, and was continually striking out into untrodden paths, where Robison was always obliged to be a follower. Watt had, by this time, gained a wide reputation. The young enthusiasts clustered round him. Whenever any puzzle came their way, they went to Watt. He needed only to be prompted; everything became to him the beginning of a new and serious study; everything became science in his hands.

Meanwhile Watt's business was doing very well. The University, when granting him quarters, had not stipulated that he should work only for them. On the contrary, he was provided with a room fronting the street, where he could offer for sale to the public the instruments he made in his workshop. In order to develop this side of the business he went into partnership, in 1759, with a man named Craig, who undertook to provide most of the capital needed for expansion, and to do all the commercial transactions, which Watt, then as ever afterwards, detested. They started with stock and cash worth £200, and about five years later were making gross sales up to £600 a year, and kept a staff of sixteen men at work.

It was Watt's reputation as a universal mechanical expert that brought so much custom to his shop. When anything had to be done and there was no one in Glasgow who knew how to do it – which was often – it was taken to Watt. He was always ready to try. If the instrument to be repaired was one that he had never seen before, he set to work to master its principles with what help he could find at the library, and was not satisfied until he had put it to rights. And what he learned he never forgot. In this way he repaired and afterwards made, fiddles, guitars, and flutes, although he could not tell one note of music from another. When a Masonic Lodge in Glasgow wanted an organ, the officers went to Watt. They imagined that Watt could do anything, and they asked him to build the organ. He sat down to study the theory of music, thoroughly examined the mechanism of the best organ he could find, and devised an exact method by which he could tune the pipes by observing "the beats of imperfect consonances." By the time the work was completed, Watt had made substantial contributions, not only to the mechanics of organ design, but also to the theory of sound. Soon after he formed his partnership with Craig, Watt opened a shop in the town, though still living in the College. In 1763, at the age of 27, Watt became engaged to be married to his cousin, Margaret Miller, and so took a house, into which he moved in the following year. He was married in 1765 and had four children. Only two of the children survived their mother who died in childbirth in the fall of 1773. Watt then remarried in 1776. His second marriage was to Ann Macgregor, the daughter of a prosperous Glasgow merchant. There were two children to this marriage, Gregory and Janet, both of whom predeceased their father. Before both of his marriages, however, Watt had begun his pioneer work on the improved steam engine.

Review of Main Work on Mechanism Design

Although Watt had no formal study of mechanisms he became a highly gifted designer of mechanisms. The windmill flyball governor for regulating the gap between millstones was adapted by Watt as an engine speed regulator giving the first closed-loop servomechanism. Watt, instrument maker and engineer, was concerned with the synthesis of movement. Watt's rotative engine was the first engine to produce power directly on a shaft without the intervention of a water-wheel fed by a reciprocating pumping engine. He took out a patent in April 1784, which described various methods of converting angular motion into rectilinear motion. Of the methods described in this patent, the one that he developed was the parallel motion linkage.

Watt's linkage was a good solution to the practical problem. However, his solution did not satisfy mathematicians who knew that all four-bar straight-line linkages (that have no sliding pairs) can only trace an approximate straight line. An exact straight-line planar linkage was not known until much later, about 1864, when the French captain Charles-Nicholas Peaucellier finally synthesized the exact straight-line linkage that bears his name [11, 12]. The Peaucellier straight-line linkage is a more complex linkage than the four-bar and has eight members and six joints, four of which are ternary joints. Four-bar linkages were in widespread use by the sixteenth century, however, they probably originated as early as the thirteenth century. Some drawings of that period indicate that a four-bar linkage was used in an up-and-down sawmill. Also, Leonardo da Vinci (1452–1519) described a crank and slider mechanism for a sawmill machine. The conversion of rotary to reciprocating motion (an oscillation through a small circular arc) using rigid links can be found in the sixteenth century. Although at that time the conversion of rotary to reciprocating motion was more frequently accomplished by cams and intermittent gearing. Nevertheless, the idea of linkages was a firmly established part of the repertory of the machine builder before 1600. In 1588, Agostino Ramelli published his book on machines where linkages were widely used [13]. The book exhibits more than 200 machines of various degrees of complexity and ingenuity. In reading this book, one might wonder if linkages had not reached their ultimate stage of development. However, it is important to note that there is a vast difference, both in conception and execution, between the linkages of Ramelli and those of Watt some 200 years later.

Designers of the four-bar linkage before Watt had confined their attention to the motions of the links attached to the frame (or ground). Watt, however, focused his attention on the motion of a point on the coupler link of the four-bar linkage. The year was 1784 and the application of this idea allowed Watt to build a double-acting steam engine. The earlier chain connecting piston and beam was now replaced by a linkage that was able to transmit force in two directions instead of only one. Watt had discovered coupler-point motion, although its definition in these terms lay well in the future. It was a singular achievement, one could almost say a pivotal point, along the road to kinematic synthesis. It took Watt several years to design the straight-line linkage that would change motion from straight-line to circular. In a letter to Matthew Boulton (a partner and machine builder who built engines in his works in Soho, a district of Birmingham, England) he wrote [7]: "I have got a glimpse of a method of causing a piston-rod to move up and down perpendicularly, by only fixing it to a piece of iron upon the beam, without chains, or perpendicular guides, or untowardly frictions, arch-heads, or other pieces of clumsiness … I have only tried it in a slight model yet, so cannot build upon it, though I think it a very probable thing to succeed, and one of the most ingenious simple pieces of mechanisms I have contrived, …" Watt was responsible for initiating profound changes in mechanical technology, but it should be recognized that the art of mechanics had, through centuries of slow development, reached the state where his genius could flourish. The knowledge and ability to provide the materials and tools necessary for Watt's research were at hand, and through the optimism and patient encouragement of his partner Boulton at the Soho Works, they were placed at his disposal.

The genius of Watt was nowhere more evident than in his synthesis of linkages [14]. An essential ingredient in the success of Watt's linkages, however, was his partner's appreciation of the entirely new order of refinement that the linkages required. Boulton, who had been a successful manufacturer of buttons and metal novelties long before his partnership with Watt was formed, had recognized at once the need for care in the building of Watt's steam engine. On February 7, 1769, he wrote to Watt, "I presumed that your engine would require money, very accurate workmanship and extensive correspondence to make it turn out to the best advantage and that the best means of keeping up the reputation and doing the invention justice would be to keep the executive part of it out of the hands of the multitude of empirical engineers, who from ignorance, want of experience and want of necessary convenience, would be very liable to produce bad and inaccurate workmanship; all of which deficiencies would affect the reputation of the invention." Boulton expected to build the engines in his shop "with as great a difference of accuracy as there is between the blacksmith and the mathematical instrument maker." The Soho Works solved the problem of producing the mechanisms (in sizes large enough to be useful in steam engines) that Watt devised [15]. The contributions of Boulton and Watt to practical mechanics cannot be overestimated. There were, in the eighteenth century, instrument makers and makers of timekeepers who had produced astonishingly accurate work, but such work comprised relatively small items, all being within the scope of a bench lathe, hand tools, and superb handwork. The rapid advancement of machine tools, which greatly expanded the scope of the machine- building art, began during the Boulton and Watt partnership from 1775 to 1800.

In April 1775 an event occurred that marked the beginning of a new era of technological advance. Boulton wrote to his partner and commented upon receiving the cast-iron steam engine cylinder that had been finished in Wilkinson's new boring mill: "it seems tolerably true, but is an inch thick and weighs about 10 cwt (approximately 1100 lbs). The diameter is about as much above 18 inches as the tin one was under, and therefore, it has become necessary to add a brass hoop to the piston, which is made almost two inches broad." This cylinder indeed marked the turning point in the discouragingly long development of the Watt steam engine, which for 10 years had occupied nearly all of Watt's thoughts and all the time he could spare from the requirements of earning a living. Although there were many trials ahead for the firm of Boulton and Watt in further developing and perfecting the steam engine, the crucial problem of leakage of steam past the piston in the cylinder had now been solved by the boring mill. This tool was the first large machine tool capable of boring a cylinder both round and straight and the first of a new class of machine tools that, over the next 50 or 60 years, came to include nearly all of the basic types of heavy chip-removing tools that are in use today. The development of tools was accelerated by the inherent accuracy required of the linkages that were originated by Watt. Once it had been demonstrated that a large and complex machine, such as the steam engine, could be built sufficiently accurately so that its operation would be relatively free of trouble, many outstanding minds became engaged in the development of machines and tools. It is interesting, however, to see how Watt grappled with the solutions of problems that resulted from the advance of the steam engine.

During the 1770s the demand for continuous, dependable power applied to a rotating shaft was becoming insistent, and much of the efforts of Boulton and Watt was directed toward meeting this demand. Mills of all kinds used water or horses to turn "wheel-work," but, while these sources of power were adequate for small operations, the quantity of water available was often limited, and the use of enormous horse-whims was frequently impracticable. The only type of steam engine then in existence was the Newcomen beam engine, which had been introduced in 1712 by Thomas Newcomen. This type of engine was widely used, mostly for pumping water out of mines but occasionally for pumping water into a reservoir to supply a waterwheel. It was arranged with a vertical steam cylinder located beneath one end of a large pivoted working beam and a vertical plunger-type pump beneath the other end. Heavy, flat chains were secured to a sector at each end of the working beam and to the engine and pump piston rods in such a way that the rods were always tangent to a circle whose center was at the beam pivot. The weight of the reciprocating parts pulled the pump end of the beam down; the atmosphere, acting on the open top of the piston in the steam cylinder, caused the engine end of the beam to be pulled down when the steam beneath the piston was condensed. The chains would, of course, only transmit force from piston to beam when in tension.

A connecting rod, a crank, and a sufficiently heavy flywheel could have been used in a conventional Newcomen engine in order to supply power to a rotating shaft, but contemporary evidence suggests that this solution was by no means obvious to Watt. At the time of his first engine patent, in 1769, Watt had devised a "steam wheel," or rotary engine, that used liquid mercury in the lower part of a toroidal chamber to provide a boundary for steam spaces successively formed by flap gates within the chamber. The practical difficulties of construction ruled out this solution to the problem of a rotating power source, but not until after considerable effort and money was spent on the idea. In 1777, a speaker before the Royal Society in London observed that in order to obtain rotary output from a reciprocating steam engine, a crank "naturally occurs in theory," but that in fact the crank is impractical because of the irregular rate of running of the engine and its variable length of stroke. He said that on the first variation of length of stroke the machine would be "either broken to pieces, or turned back." John Smeaton, in the front rank of English steam engineers of his time, was asked in 1781 by His Majesty's Victualling Office for his opinion as to whether a steam-powered grain mill ought to be driven by a crank or by a waterwheel supplied by a pump. His conclusion was that the crank was quite unsuited to a machine in which regularity of operation was a factor. "I apprehend," he wrote, "that no motion communicated from the reciprocating beam of a fire engine can ever act perfectly equal and steady in producing a circular motion, like the regular efflux of water in turning a waterwheel." He recommended, incidentally, that a Boulton and Watt steam engine be used to pump water to supply the waterwheel. Smeaton had thought of a flywheel, but he reasoned that a flywheel large enough to smooth out the halting, jerky operation of the steam engines that he had observed would be more of an encumbrance than a pump, reservoir, and waterwheel.

The simplicity of the eventual solution of the problem was not clear to Watt at this time. He was not, as tradition has it, blocked merely by the existence of a patent for a simple crank and thus forced to invent some other device as a substitute. Wasbrough, the engineer commonly credited with the crank patent, made no mention of a crank in his patent specification, but rather intended to make use of "racks with teeth," or "one or more pulleys, wheels, segments of wheels, to which are fastened rotchets and clicks or palls." He did, however, propose to "add a fly or flies, in order to render the motion more regular and uniform." Unfortunately, he submitted no drawings with his patent specification. James Pickard, a button maker in Birmingham, patented a counterweighted crank device in 1780 that was expected to remove the objection of a crank. The device operated with changing leverage and, therefore, irregular power. The counterweighted wheel, revolving twice for each revolution of the crank, allowed the counterweight to descend while the crank passed the dead center position and would be raised while the crank had maximum leverage. No mention of a flywheel was made in this patent.

Wasbrough, finding that his "rotchets and clicks" did not serve, actually used a crank with a flywheel in 1780. Watt was aware of this, but he remained unconvinced of the superiority of the crank over other devices and did not immediately appreciate the regulating ability of a flywheel. In April 1781, Watt wrote to Boulton, "... I know from experiment that the other contrivance, which you saw me try, performs at least as well, and has in fact many advantages over the crank" [10]. The "other contrivance" probably was his swash wheel which he built and which appeared on his next important patent specification. Also in this patent were four other devices, one of which was easily recognizable as a crank, and two of which were eccentrics. The fourth device was the well-known sun-and-planet gearing. In spite of the similarity of the simple crank to the several variations devised by Watt, this patent drew no fire from Wasbrough or Pickard, perhaps because no reasonable person would contend that the crank itself was a patentable feature, or perhaps because the similarity was not at that time so obvious. However, Watt steered clear of directly discernible application of cranks because he preferred to avoid a suit that might overthrow his or other patents. For example, if the Wasbrough and Pickard patents had been voided, they would have become public property; and Watt feared that they might fall into the hands of men more ingenious, who would give Boulton and Watt more competition than Wasbrough and Pickard. The sun-and-planet arrangement, with gears of equal size, was adopted by Watt for nearly all the rotative engines that he built during the term of the "crank patents." This arrangement had the advantage of turning the flywheel through two revolutions during a single cycle of operation of the piston. This required a flywheel only one-fourth the size of the flywheel needed if a simple crank were used.

From the first, the rotative engines were made double-acting; i.e., work was done by steam alternately in each end of the cylinder. The double-acting engine, unlike the single-acting pumping engine, required a piston rod that would push as well as pull. It was in the solution of this problem that Watt's originality and sure judgment were most clearly demonstrated. A rack and sector arrangement was used on some engines. The first one, according to Watt, "has broke out several teeth of the rack, but works steady." A little later he told a correspondent that his double-acting engine, “acts so powerfully that it has broken all its tackling repeatedly. We have now tamed it, however.” It was about a year later that the straight-line linkage was thought out. “I have started a new hare,” Watt wrote to his partner. “I have got a glimpse of a method of causing the piston-rod to move up and down perpendicularly, by only fixing it to a piece of iron upon the beam, without chains, or perpendicular guides, or untowardly frictions, archheads, or other pieces of clumsiness. I have only tried it in a slight model yet, so cannot build upon it, though I think it a very probable thing to succeed, and one of the most ingenious simple pieces of mechanism I have contrived.”

This section is based on the tribute to James Watt by Eugene S. Ferguson, 1916–2004, a truly outstanding professional historian of technology whose detailed and insightful monograph [2] encouraged the author to attempt the writing of this brief article.

On the Circulation of Works

Watt’s marvelously simple straight-line linkage was incorporated into a large beam engine almost immediately, and the inventor was elated when he told Boulton: “new central perpendicular motion answers beyond expectation, and does not make the shadow of a noise.” The parallel motion linkage was included in an extensive patent submitted by Watt (British Patent 1321, 1782). Figure 2a is a drawing of the Watt engine. The engine had a 30-inch diameter cylinder and a stroke of 8 feet.

Figure 2a is Plate 15 in the book by J.P. Muirhead, *The Origin and Progress of the Mechanical Inventions of James Watt*, Vol. 3, London, England, 1854 [7]. A drawing of the Watt double acting steam engine that appeared in the work of Lardner [16] is reproduced here in Figure 2b.

Watt considered several alternative devices for the conversion of reciprocating motion to rotating motion in the steam engine. The device that he finally employed in the Watt and Boulton large beam engines is the sun-and-planet gearing that is shown in Figure 3.

Figure 3 is Plate 7 in the book by J.P. Muirhead, *The Origin and Progress of the Mechanical Inventions of James Watt*, Vol. 3, London, England, 1854 [7].

As brilliant as the conception of the parallel motion linkage was, it was followed up by a synthesis that is very little short of incredible. In order to

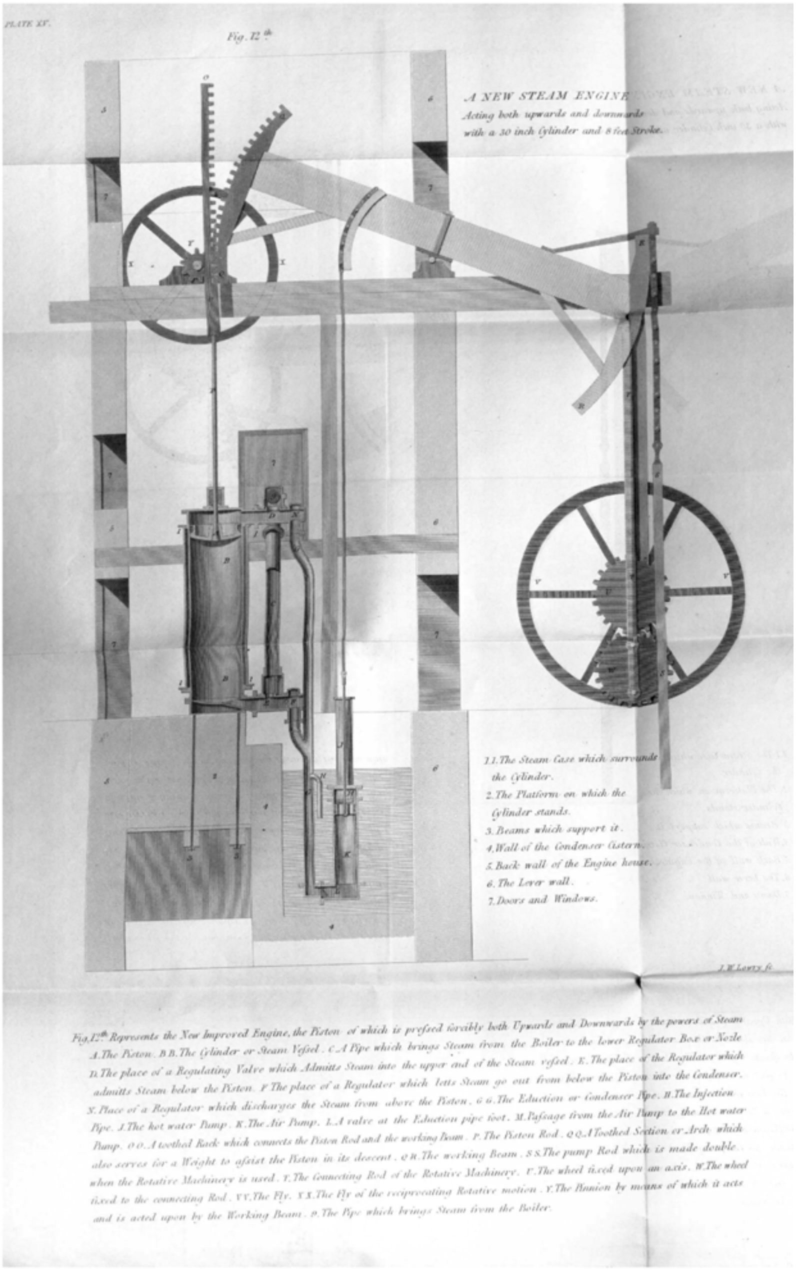

Fig. 2a. The Watt engine (British Patent 1321, March 12, 1782).

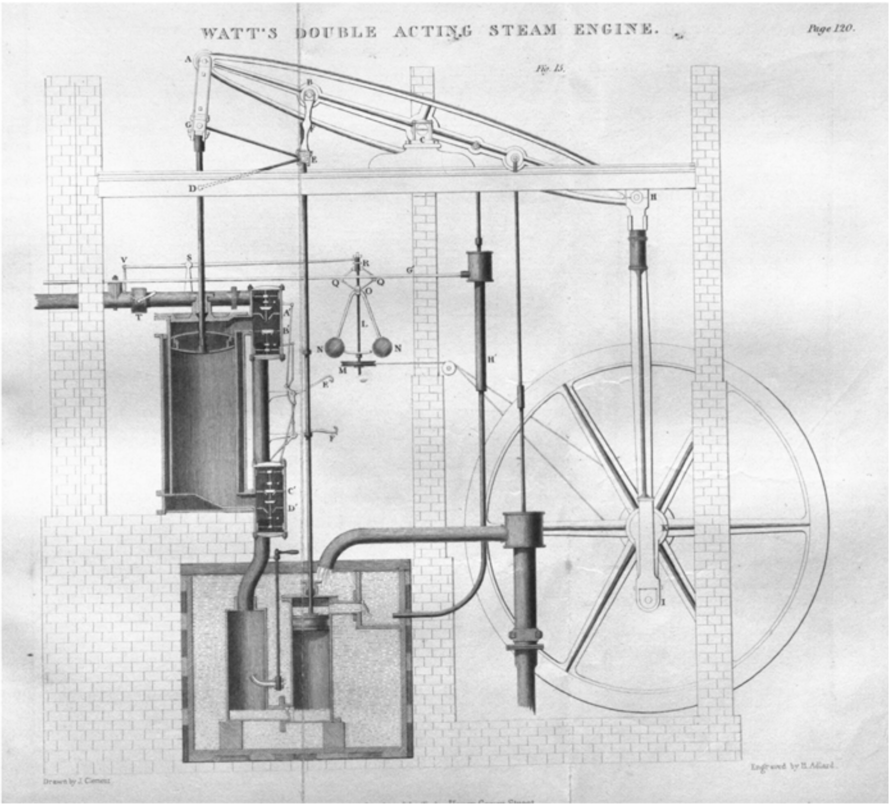

Fig. 2b. The Watt double acting steam engine.

make the linkage attached to the beam of his engines more compact, Watt plumbed the depths of his experience for ideas. This experience yielded up the work that was completed much earlier on a drafting machine that made use of a pantograph. Watt combined his straight-line linkage with a pantograph, one link becoming a member of the pantograph. This pantograph mechanism [16], denoted as ABEG, is shown in Figure 4.

With this design, the length of each oscillating link of the straight-line linkage was reduced to one-fourth instead of one-half the beam length. The entire mechanism could then be constructed so that it would not extend beyond the end of the working beam. This arrangement soon came to be known as Watt's parallel motion linkage, denoted as O_2ABO_4 in Figure 5.

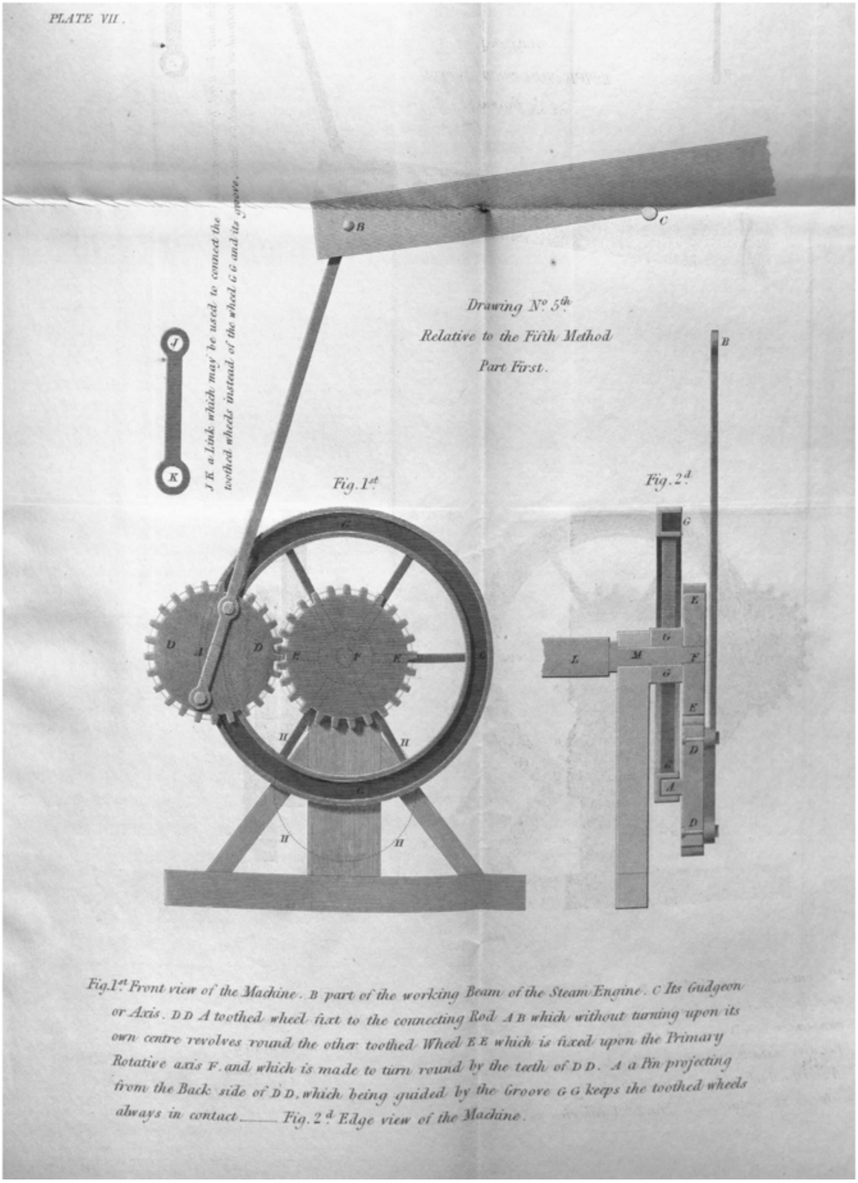

Fig. 3. The sun-and-planet gearing. (British Patent 1306, October 25, 1781).

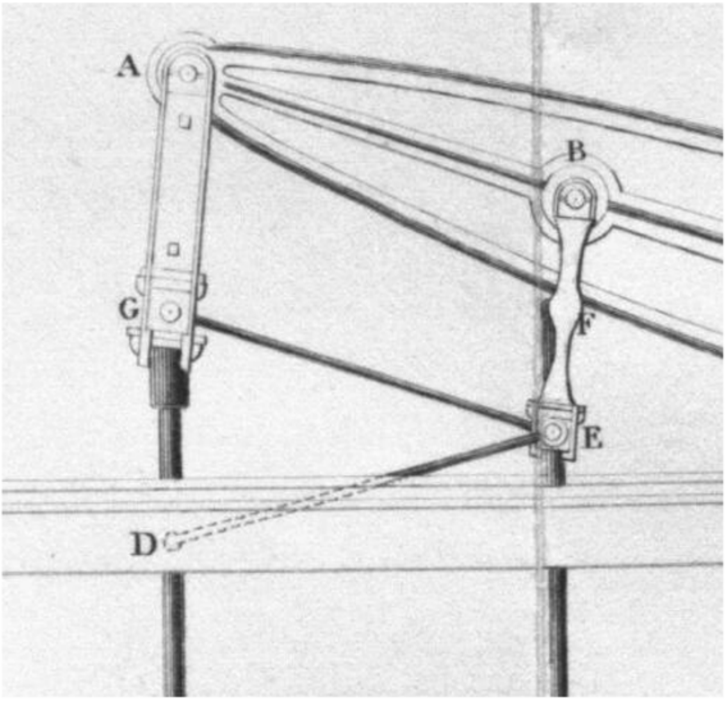

Fig. 4. The pantograph mechanism.

Through insight we can detect in this straight-line linkage the birth of a very ordered and advanced synthetic process. The kinematic analysis of the Watt four-bar linkage, see Figure 6a, and the geometry of the path of point M fixed in the coupler link AB (link 3) can be investigated using the method of kinematic coefficients [17].

The vectors that are required for the kinematic analysis of the Watt four-bar linkage are shown in Figure 6b.

Modern Interpretation of Main Contribution to Mechanism Design

The vector loop equation for the four-bar linkage can be written as

$$\overset{\surd\mathrm{I}}{\overline{R}}_2 + \overset{\surd ?}{\overline{R}}_3 - \overset{\surd ?}{\overline{R}}_4 + \overset{\surd\surd}{\overline{R}}_1 = 0, \qquad (1)$$

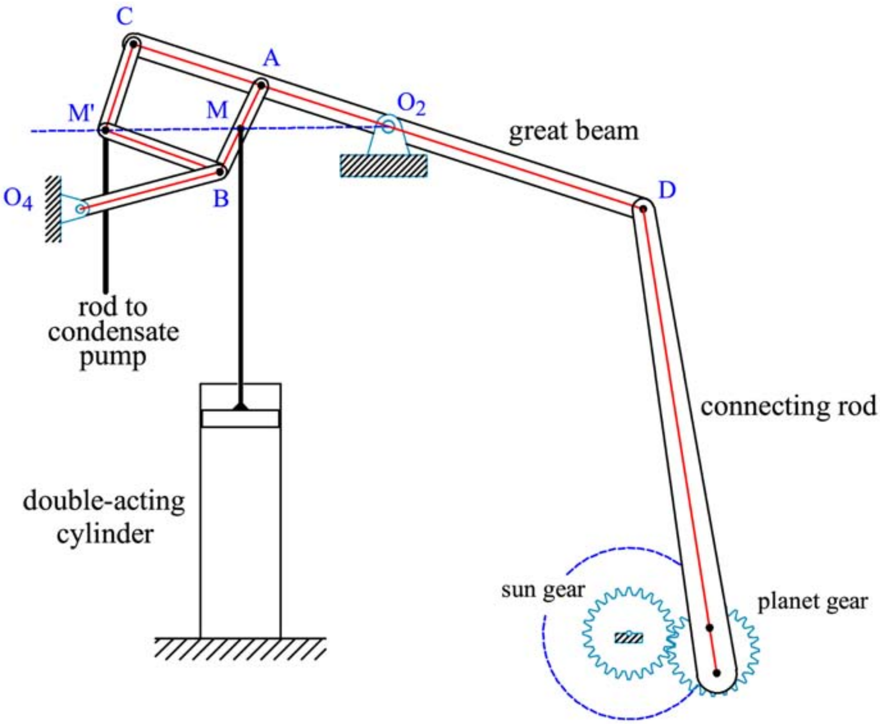

Fig. 5. The Watt parallel motion linkage.

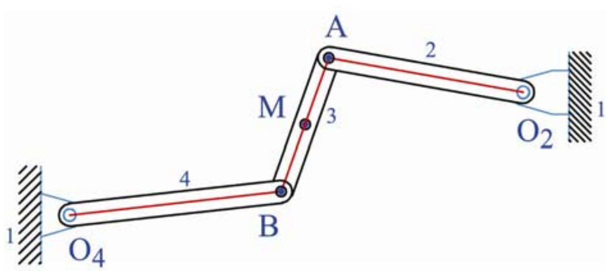

Fig. 6a. The Watt four-bar linkage.

where the first symbol above each vector indicates its magnitude and the second symbol indicates its direction. The known quantities are denoted by $\surd$ the unknown variables are denoted by ?, and the independent variable is denoted by I. Without loss in generality, the independent variable is assumed to be the angular position of link 2 and the unknown variables are the angular

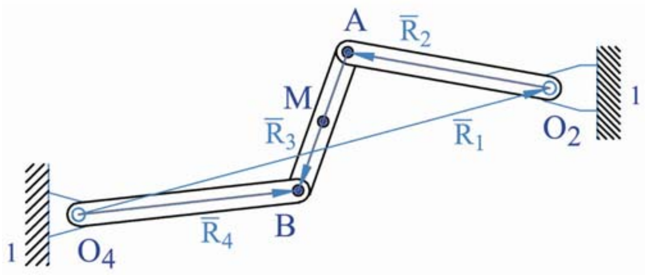

Fig. 6b. The vectors for the Watt four-bar linkage.

positions of the coupler link 3 and the side link 4. The X and Y components of Equation (1) are

$$R_2 \cos\theta_2 + R_3 \cos\theta_3 - R_4 \cos\theta_4 + R_1 \cos\theta_1 = 0 \tag{2a}$$

and

$$R_2 \sin\theta_2 + R_3 \sin\theta_3 - R_4 \sin\theta_4 + R_1 \sin\theta_1 = 0. \tag{2b}$$

Differentiating Equations (2) with respect to the independent variable θ_2 gives

$$-R_2 \sin\theta_2 - R_3 \sin\theta_3\theta_3' + R_4 \sin\theta_4\theta_4' = 0 \tag{3a}$$

and

$$R_2 \cos\theta_2 + R_3 \cos\theta_3\theta_3' - R_4 \cos\theta_4\theta_4' = 0, \tag{3b}$$

where $\theta_3' = \mathrm{d}\theta_3/\mathrm{d}\theta_2$ and $\theta_4' = \mathrm{d}\theta_4/\mathrm{d}\theta_2$ are referred to as the first-order kinematic coefficients of links 3 and link 4, respectively. Then writing Equations (3) in matrix form gives

$$\begin{bmatrix} -R_3 \sin\theta_3 & R_4 \sin\theta_4 \\ R_3 \cos\theta_3 & -R_4 \cos\theta_4 \end{bmatrix} \begin{bmatrix} \theta_3' \\ \theta_4' \end{bmatrix} = \begin{bmatrix} R_2 \sin\theta_2 \\ -R_2 \cos\theta_2 \end{bmatrix}. \tag{4}$$

The determinant of the coefficient matrix in Equation (4) can be written as

$$\mathrm{DET} = \begin{vmatrix} -R_3 \sin\theta_3 & R_4 \sin\theta_4 \\ R_3 \cos\theta_3 & -R_4 \cos\theta_4 \end{vmatrix} = R_3 R_4 \sin(\theta_3 - \theta_4). \tag{5a}$$

Note that the determinant is zero when

$$\theta_3 = \theta_4 \quad \text{or} \quad \theta_3 = \theta_4 + 180°, \tag{5b}$$

i.e., the mechanism is in a special position (links 3 and 4 are either aligned or folded on top of each other). Using Cramer's Rule, the first-order kinematic coefficient of link 3, from Equation (4), can be written as

$$\theta_3' = \frac{\begin{vmatrix} R_2 \sin\theta_2 & R_4 \sin\theta_4 \\ -R_2 \cos\theta_2 & -R_4 \cos\theta_4 \end{vmatrix}}{\text{DET}} = \frac{R_2 R_4 \sin(\theta_4 - \theta_2)}{\text{DET}} \tag{6a}$$

and the first-order kinematic coefficient of link 4 can be written as

$$\theta_4' = \frac{\begin{vmatrix} -R_3 \sin\theta_3 & R_2 \sin\theta_2 \\ R_3 \cos\theta_3 & -R_2 \cos\theta_2 \end{vmatrix}}{\text{DET}} = \frac{R_2 R_3 \sin(\theta_3 - \theta_2)}{\text{DET}}, \tag{6b}$$

where the determinant is given by Equation (5a).

Differentiating Equations (3) with respect to the independent variable θ_2 gives

$$-R_2 \cos\theta_2 - R_3 \cos\theta_3 \theta_3'^2 - R_3 \sin\theta_3 \theta_3'' + R_4 \cos\theta_4 \theta_4'^2 + R_4 \sin\theta_4 \theta_4'' = 0 \tag{7a}$$

and

$$-R_2 \sin\theta_2 - R_3 \sin\theta_3 \theta_3'^2 + R_3 \cos\theta_3 \theta_3'' + R_4 \sin\theta_4 \theta_4'^2 - R_4 \cos\theta_4 \theta_4'' = 0, \tag{7b}$$

where $\theta_3'' = \mathrm{d}^2\theta_3/\mathrm{d}\theta_2^2$ and $\theta_4'' = \mathrm{d}^2\theta_4/\mathrm{d}\theta_2^2$ are referred to as the second-order kinematic coefficient of links 3 and 4, respectively. Then writing Equations (7) in matrix form gives

$$\begin{bmatrix} -R_3 \sin\theta_3 & R_4 \sin\theta_4 \\ R_3 \cos\theta_3 & -R_4 \cos\theta_4 \end{bmatrix} \begin{bmatrix} \theta_3' \\ \theta_4' \end{bmatrix} = \begin{bmatrix} R_2 \cos\theta_2 + R_3 \cos\theta_3 \theta_3'^2 - R_4 \cos\theta_4 \theta_4'^2 \\ R_2 \sin\theta_2 + R_3 \sin\theta_3 \theta_3'^2 - R_4 \sin\theta_4 \theta_4'^2 \end{bmatrix}. \tag{8}$$

Note that the coefficient matrices in Equations (4) and (8) must be the same, which is a useful check of the differentiation. Using Cramer's rule, the second-order kinematic coefficient of link 3, from Equation (8), is

$$\theta_3'' = \frac{\begin{vmatrix} R_2 \cos\theta_2 + R_3 \cos\theta_3 \theta_3'^2 - R_4 \cos\theta_4 \theta_4'^2 & R_4 \sin\theta_4 \\ R_2 \sin\theta_2 + R_3 \sin\theta_3 \theta_3'^2 - R_4 \sin\theta_4 \theta_4'^2 & -R_4 \cos\theta_4 \end{vmatrix}}{\text{DET}} \tag{9a}$$

and the second-order kinematic coefficient of link 4 is

$$\theta_4'' = \frac{\begin{vmatrix} -R_3 \sin\theta_3 & R_2 \cos\theta_2 + R_3 \cos\theta_3 \theta_3'^2 - R_4 \cos\theta_4 \theta_4'^2 \\ R_3 \cos\theta_3 & R_2 \sin\theta_2 + R_3 \sin\theta_3 \theta_3'^2 - R_4 \sin\theta_4 \theta_4'^2 \end{vmatrix}}{\text{DET}}, \tag{9b}$$

where the determinant is given by Equation (5a).

From the definition of the first-order kinematic coefficients, the angular velocities of links 3 and 4 can be written, respectively, as

$$\omega_3 = \theta_3' \omega_2 \quad \text{and} \quad \omega_4 = \theta_4' \omega_2. \tag{10}$$

Differentiating Equations (10) with respect to time, the angular accelerations of links 3 and 4 can be written, respectively, as

$$\alpha_3 = \theta_3'' \omega_2^2 + \theta_3' \alpha_2 \quad \text{and} \quad \alpha_4 = \theta_4'' \omega_2^2 + \theta_4' \alpha_2. \tag{11}$$

Now that the kinematic analysis of the linkage is complete, the kinematics of the coupler point M and the geometry of the path traced by point M can be investigated. The vectors that are required for the kinematic analysis of point M are as shown in Figure 7.

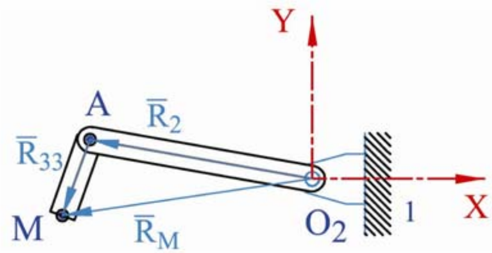

Fig. 7. The vectors for the coupler point M.

The vector equation for coupler point M can be written as

$$\bar{R}_M = \bar{R}_2 + \bar{R}_{33}. \tag{12}$$

The X and Y components of this equation are

$$X_M = R_2 \cos\theta_2 + R_{33} \cos\theta_3 \tag{13a}$$

and

$$Y_M = R_2 \sin\theta_2 + R_{33} \sin\theta_3. \tag{13b}$$

Differentiating Equations (13) with respect to the independent variable θ_2, the first-order kinematic coefficients of coupler point M are

$$X'_M = -R_2 \sin\theta_2 - R_{33} \sin\theta_3 \theta'_3 \tag{14a}$$

and

$$Y'_M = R_2 \cos\theta_2 + R_{33} \cos\theta_3 \theta'_3, \tag{14b}$$

where θ'_3 (i.e., the first-order kinematic coefficient of coupler link 3) is known from the kinematic analysis of the four-bar linkage, see Equation (6a).

Differentiating Equations (14) with respect to the independent variable θ_2, the second-order kinematic coefficients of coupler point M are

$$X''_M = -R_2 \cos\theta_2 - R_{33} \cos\theta_3 \theta'^2_3 - R_{33} \sin\theta_3 \theta''_3 \tag{15a}$$

and

$$Y''_M = -R_2 \sin\theta_2 - R_{33} \sin\theta_3 \theta'^2_3 + R_{33} \sin\theta_3 \theta''_3, \tag{15b}$$

where θ''_3 (i.e., the second-order kinematic coefficient of coupler link 3) is known from the kinematic analysis of the four-bar linkage, see Equation (9a).

The velocity and acceleration of coupler point M can be written, respectively, as

$$\bar{V}_M = (X'_M \hat{i} + Y'_M \hat{j})\omega_2 \tag{16a}$$

and

$$\bar{A}_M = (X''_M \hat{i} + Y''_M \hat{j})\omega_2^2 + (X'_M \hat{i} + Y'_M \hat{j})\alpha_2. \tag{16b}$$

The geometry of the path traced by coupler point M can be investigated as follows. The unit tangent vector and the unit normal vector to the path of point M can be written, respectively, as

$$\hat{u}_t = \frac{X'_M \hat{i} + Y'_M \hat{j}}{R'_M} \tag{17a}$$

and

$$\hat{u}_n = \hat{k} \times \hat{u}_t = \frac{-Y'_M \hat{i} + X'_M \hat{j}}{R'_M}, \tag{17b}$$

where

$$R'_M = \pm\sqrt{(X'_M)^2 + (Y'_M)^2}. \tag{18}$$

Sign Convention: The positive sign is used in Equation (18) if the change in the independent variable is positive (i.e., counterclockwise) and the negative sign is used if the change in the independent variable is negative (i.e., clockwise).

The radius of the curvature of the path traced by coupler point M can be written as

$$\rho_M = \frac{V_M^2}{A_M^n}, \tag{19a}$$

where the normal acceleration of coupler point M can be written as

$$A_M^n = \bar{A}_M \cdot \hat{u}_n. \tag{19b}$$

Substituting Equations (16b) and (17b) into Equation (19b) and performing the dot product, the normal acceleration of coupler point M can be written as

$$A_M^n = \frac{(X'_M Y''_M - Y'_M X''_M)\omega_2^2}{R'_M}. \tag{20}$$

Then substituting Equations (16a) and (20) into Equation (19a), and using Equation (18), the radius of the curvature of the path traced by the coupler point M can be written as

$$\rho_M = \frac{R'^3_M}{X'_M Y''_M - Y'_M X''_M}. \tag{21}$$

Finally, the Cartesian coordinates of the center of the curvature of the path traced by coupler point M can be written as

$$Xcc = X_M + \rho_M (u_n)_x \tag{22a}$$

and

$$Ycc = Y_M + \rho_M (u_n)_y. \tag{22a}$$

Substituting Equation (17b) into Equations (22), the Cartesian coordinates of the center of the curvature of the path traced by coupler point M can be written as

$$Xcc = X_M + \rho_M \left[\frac{-Y'_M}{R'_M} \right] \tag{23a}$$

and

$$Ycc = Y_M + \rho_M \left[\frac{X'_M}{R'_M} \right]. \tag{23b}$$

In general, an arbitrary coupler point of a general four-bar linkage will trace a curve which is described as a tricircular sextic [18–20]. However, coupler point M of the Watt four-bar linkage traces a special curve which is best described as a figure-eight-shaped curve, as shown in Figure 8.

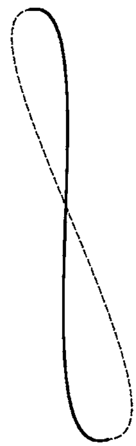

Fig. 8. The curve traced by coupler point M.

This curve is commonly referred to as a lemniscate and has two segments that approximate straight lines [21]. By means of the pantograph mechanism (see Figure 4), the path traced by point M' (see Figure 5) is similar to the path traced by coupler point M.

The Watt four-bar linkage was employed some 75 years after its inception by Richards when, in 1861, he designed his first high-speed engine indicator. The Richards indicator, which was introduced into England the following year, was an immediate success, and many thousands were sold over the next several decades. In considering the order of synthetic ability required to design the straight-line linkage and to combine it with a pantograph, it should be kept in mind that this was the first one of a long line of such mechanisms. Once the idea was abroad, it was only to be expected that many variations and alternative solutions should appear. One could wonder, however, what direction the subsequent work would have taken if Watt had not so clearly pointed the way.

Farey, in an exhaustive study of the steam engine, wrote perhaps the best contemporary view of Watt's work in 1827. As a young man, Farey had talked several times with the aging Watt, and he had reflected upon the nature of the intellect that had caused Watt to be recognized as a genius, even within his own lifetime. In attempting to explain Watt's genius, Farey set down some observations that are pertinent not only to kinematic synthesis but to the currently fashionable term "creativity." In Farey's opinion, Watt's inventive faculty was far superior to that of any of his contemporaries; but his many and various ideas would have been of little use if he had not possessed a very high order of judgment, that "faculty of distinguishing between ideas; decomposing compound ideas into more simple elements; arranging them into classes, and comparing them together." Farey was of the opinion that while a mind like Watt's could produce brilliant new ideas, still the "common stock of ideas which are current amongst communities and professions, will generally prove to be of a better quality than the average of those new ideas, which can be produced by any individual from the operation of his own mind, without assistance from others." Farey concluded with the observation that "the most useful additions to that common stock, usually proceed from the individuals who are well acquainted with the whole series."

During most of the century after Watt had produced his parallel motion, the problem of devising a linkage, one point of which would describe a straight line, was one that engaged the minds of mathematicians, ingenious mechanics, and of gentlemanly dabblers in ideas. The quest for a straight-line mechanism more accurate than that of Watt far outlasted the pressing practical need for such a device. Large metal planning machines were well known by 1830, and by mid-century crossheads and crosshead guides were used on both sides of the Atlantic in engines with and without working beams. By 1819, Farey had observed quite accurately that, in England at least, many other schemes had been tried and found wanting and that "no methods have been found so good as the original engine; and we accordingly find, that all the most established and experienced manufacturers make engines which are not altered in any great feature from Mr. Watt's original engine."

Two mechanisms for producing a straight line were introduced before the Boulton and Watt monopoly ended in 1800. The first was by Cartwright (1743–1823), who is said to have had the original idea for a power loom. This geared device was characterized, somewhat patronizingly, by a contemporary American editor as possessing "as much merit as can possibly be attributed to a gentleman engaged in the pursuit of mechanical studies for his own amusement." However, only a few small engines were made under the patent. The properties of a hypocycloid were recognized by White, an English engineer, in his geared design which employed a pivot located on the pitch circle of a spur gear revolving inside an internal gear. The diameter of the pitch circle of the spur gear was one-half that of the internal gear, with the result that the pivot, to which the piston rod was connected, traced out a diameter of the large pitch circle. White received a medal from Napoleon Bonaparte in 1801 for this invention when it was exhibited in Paris at an industrial exposition. Some steam engines employing White's mechanism were built, but without conspicuous commercial success. White himself agreed that while his invention was "allowed to possess curious properties, and to be a pretty thing, opinions do not all concur in declaring it, essentially and generally, a good thing."

The first of the non-Watt four-bar linkages appeared shortly after 1800. The origin of the grasshopper beam motion is somewhat obscure, although it came to be associated with Evans, the American pioneer, in the employment of high-pressure steam. A similar idea, employing an isosceles linkage, was patented in 1803 by Freemantle, an English watchmaker. This is the linkage that was attributed much later to Russell (1808–1882), the prominent naval architect. An inconclusive hint that Evans had devised his straight-line linkage by 1805 appeared in a plate illustrating his *Abortion of the Young Steam Engineer's Guide* (Philadelphia, 1805), and it was used on his Columbian engine, which was built before 1813. The Freemantle linkage, in modified form, appeared in Rees's *Cyclopaedia* of 1819, but it is doubtful whether even this would have been readily recognized as identical with the Evans linkage, because the connecting rod was at the opposite end of the working beam from the piston rod, in accordance with established usage, while in the Evans linkage the crank and connecting rod were at the same end of the beam. It is possible that Evans obtained his idea from an earlier English periodical, but concrete evidence appears to be lacking. If the idea did in fact originate with Evans, it is strange that he did not mention it in his patent claims, or in the descriptions that he published of his engines.

For more detailed information on the history of the problem to convert circular motion into straight line motion, the reader is referred to several references [see 22–24].

Concluding Remarks

James Watt died at Heathfield in Staffordshire on August 19th, 1819 and was buried in the grounds of St. Mary's Church, Handsworth, in Birmingham. He had been elected a Fellow of the Royal Society of Edinburgh in 1784, and a Fellow of the Royal Society of London in 1785. At the age of 70, he was granted the degree of LL.D. (Honorary Doctor of Law) by the University of Glasgow in 1806. Watt was made corresponding member of the Institute of France in 1808, and one of the eight Foreign Associates of the Academie des Sciences in 1814. He was offered a baronetcy, late in his life, but declined this honor. Perhaps this was evidence, to support his claim, that he was not over-anxious for fame. However, in spite of his humility, a marble monument to Watt was erected in Westminster Abbey, London, in 1824.

Watt, the man, can best be described by recounting the words of the famous Scottish novelist and poet, Sir Walter Scott (1771–1832) who wrote these words sometime after Watt's eightieth birthday: "The alert, kind, benevolent old man had his attention alive to every one's question. His information at every one's command – the man whose genius discovered the means of multiplying our national resources to a degree perhaps even beyond his own stupendous powers of calculation and combination. This potent commander of the elements, this abridger of time and space, this magician, whose cloudy machinery has produced a change on the world, the effects of which, extraordinary as they are, are perhaps only now beginning to be felt, was not only the most profound man of science, the most successful combiner of powers and calculator of numbers as adapted to practical purposes, was not only one of the most generally well-informed but one of the best and kindest of human beings."

To commemorate the bicentenary of the birth of Watt, the Institution of Mechanical Engineers of Great Britain decided to award every two years a Gold Medal to an engineer of any nationality who is deemed worthy of the highest award the Institution can bestow and that a mechanical engineer can receive. In making this award, the Institution sought the co-operation and advice of engineering Institutions and Societies in all parts of the world. In the long list of those who, by the practice of mechanical engineering, have added to the comfort, well-being and prosperity of mankind there is no man who holds a higher place in universal estimation than James Watt.

The eighth oldest higher education institution in the United Kingdom is the Heriot-Watt University. The history of this great university can be traced back to the School of Arts of Edinburgh, which was founded in 1821 as the first mechanics institute in Great Britain. In 1852 the School incorporated the funds raised by public subscription to erect a monument to the memory of James Watt and was renamed the Watt Institution and School of Arts. In 1869, in response to a campaign by female subscribers and their supporters, the Governors took the radical step of allowing women to attend classes on equal terms with men. This placed the School in the vanguard of Scottish Higher Education Institutions. It was not until the Universities (Scotland) Act of 1889 that Universities were empowered to admit women to graduation.

In 1885, the School merged with the trust bequeathed to Edinburgh in 1623 by George Heriot, who had been a goldsmith and financier to King James VI (James I of England), and was renamed Heriot-Watt College. The first professors of the new technical college were appointed in 1887. In recognition of the teaching quality, the College became a Central Institution in 1902, partly funded by the Scottish Education Department. From 1928, an independent Board of Governors assumed responsibility for the College but continued to receive financial support from George Heriot's Trust. The College continued to enhance its reputation in the fields of science and engineering and, on the recommendation of the Royal Commission on Higher Education chaired by Lord Robbins, became Heriot-Watt University in 1966. In 1969, Midlothian Council gifted the University with the Riccarton estate in southwest Edinburgh. This provided the vital spring-board for Heriot-Watt University to expand on a new purpose-built campus and to develop leading-edge strengths in teaching and research. An integral part of the campus is Europe's first Research Park, founded in 1971. In 1998, the university merged with the Scottish College of Textiles to create the Scottish Borders Campus in Galashiels. Heriot-Watt University has a long tradition of innovative teaching, learning and research, geared to the needs of modern industry, business and society. The University specializes in the built environment, engineering and physical sciences, mathematics and informatics, computer science, business management, finance, languages and textiles. Today, the University is an internationally renowned center for innovative education, enterprise and cutting-edge research. With campuses in Scotland and Dubai and over 18,000 students registered on courses from over 153 countries worldwide, Heriot-Watt University can rightfully claim to be Scotland's international university.

Acknowledgements

The author wishes to acknowledge the many outstanding scholars who have written such fascinating historical accounts of the life and times of James Watt. Unfortunately, space does not permit the author to name them individually but their books, treatises, and papers are the basis for this brief article. The article written by Eugene S. Ferguson [2] on the kinematics of mechanisms from the time of Watt was of particular inspiration. The author also wishes to thank the Institution of Mechanical Engineers, Great Britain, and Heriot-Watt University, Scotland, for the information they so willing provided on the significant contributions of James Watt. Surely, he was a profound man of science.

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Abstract. Walter Wunderlich was one of the most influential Austrian kinematicians in the 20th century. He wrote more than 200 scientific papers in the fields of mathematics, geometry and kinematics. Because of his influence, kinematic geometry is still an important subject in the curricula of geometry teachers' education in Austria.

Biographical Notes

Walter Wunderlich was born in Vienna on March 6th, 1910.[1] His father was an engineer. His ancestors came from different parts of the former Austrian empire. He had relatives in today's Slovenia, Bohemia and Hungary. These relatives were very important in his youth. Many of his holidays were spent in Hungary or in Bohemia, where he learned the Hungarian and Czech languages, both of which he spoke fluently.

After graduation from the "Realschule" (a natural science oriented high school) he studied civil engineering, but after finishing the undergraduate courses he changed to mathematics and descriptive geometry. He wanted to become a high school teacher in these subjects. At the University of Vienna he had excellent and famous professors as teachers. In mathematics: P. Furtwängler (founder of the Vienna school of number theory), H. Hahn (one of the founders of functional analysis, "theorem of Hahn–Banach"), K. Mayrhofer and W. Wirtinger (functional analysis, Wirtinger was the third winner of the Cayley prize after Cantor and Poincaré) and in descriptive geometry L. Eckhart, J. Krames, E. Kruppa, Th. Schmid and L. Schrutka. His master's

[1] Biographical notes are taken from Stachel (1999).

M. Ceccarelli (ed.), Distinguished Figures in Mechanism and Machine Science, 371–392.

Fig. 1. Walter Wunderlich in March 1990. (Photo taken from Stachel (1999) with permission from the author)

thesis (Diplomarbeit) under supervision of E. Kruppa on "Nichteuklidische Schraubungen" (Non-Euclidean Screw Motions) was finished in 1933 and already one year later he submitted his PhD. thesis (Doktorarbeit) having the title "Über eine affine Verallgemeinerungen der Lyon'schen Grenzschraubungen" (An affine generalization of Lyon's limit screw-motions).

Surrounded by many problems, mainly due to the political situation in Austria, he started a scientific career, got a part-time position as a tutor and in 1934 became assistant to L. Eckhart. In 1939 he submitted a habilitation thesis with the title "Darstellende Geometrie nichteuklidischer Schraubflächen" (Descriptive geometry of non-Euclidean screw surfaces). Immediately after the habilitation exam he was conscripted to the army. This fact is mentioned because it was during the war that he wrote his first scientific papers. Three of his papers even bear the affiliation "in the battlefield". In 1942 he was released from military service and went as a private researcher to a military research institution. He worked in the unit "physics of blasting" and wrote a book titled *Introduction to Under-Water Blasting*, which was finished after the World War under American supervision.[2] In 1943 he became university docent in Berlin, but because of problems due to the war he never started teaching. After the war he and his wife were in British internment

[2] He himself never mentions this book in the list of publications.

camps, where also their first son was born. In 1946, after coming back to Vienna, he immediately was re-employed at TU Vienna because he never was politically active. He became associate professor at the Technical University in Vienna and in 1951 he was appointed full professor. From that time he always stayed in Vienna, although many universities offered him challenging positions: Karlsruhe, Aachen and Munich. He was Dean of Natural Sciences faculty and Rector Magnificus of TU Vienna. He retired in 1980, but wrote more than 40 high standard scientific papers as a professor emeritus. Ten years before his death he suffered a retina ablation and died almost blind in 1998.

Walter Wunderlich was a member of the Austrian academy of science and for more than 25 years honorary editor of the IFToMM journal Mechanism and Machine Theory.

List of Main Works

Books

Wunderlich, W., *Ebene Kinematik*.
Wunderlich, W., *Darstellende Geometrie I*, 1966.
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Contributions to Non-Euclidean Kinematics

Wunderlich, W., Über eine affine Verallgemeinerung der Grenzschraubung, 1935.
Wunderlich, W., Darstellende Geometrie nichteuklidischer Schraubflächen, 1936.

Contributions to CAM Theory

Wunderlich, W., Contributions to the geometry of cam mechanisms with oscillating followers, 1971.
Wunderlich, W., Kurven mit isoptischem Kreis, 1971.
Wunderlich, W., Kurven mit isoptischer Ellipse, 1971.
Wunderlich, W., Single-disk cam mechanisms with oscillating double roller follower, 1984.

Contributions to Gearing Theory

Wunderlich, W. and Zenow, P., Contribution to the geometry of elliptic gears, 1975.
Wunderlich, W., Zur Triebstockverzahnung, 1943.
Wunderlich, W., Über abwickelbare Zahnflanken und eine neue Kegelradverzahnung, 1948.

Contributions to Singular Mechanisms

Wunderlich, W., Starre, kippende, wackelige und bewegliche Achtflache, 1965.
Wunderlich, W., Starre, kippende, wackelige und bewegliche Gelenkvierecke im Raum, 1971.
Wunderlich, W., Fokalkurvenpaare in orthogonalen Ebenen und bewegliche Stabwerke, 1976.
Wunderlich, W., Über die gefährlichen Örter bei zwei Achtpunktproblemen und einem Fünfpunktproblem, 1977.
Wunderlich, W., Bewegliche Stabwerke vom Bricardschen Typus, 1977.
Wunderlich, W., Gefährliche Annahmen der Trilateration und bewegliche Fachwerke I, 1977.
Wunderlich, W., Gefährliche Annahmen der Trilateration und bewegliche Fachwerke II, 1977.
Wunderlich, W., Über gefährliche Annahmen beim Clausenschen und Lambertschen Achtpunktproblem, 1978.
Wunderlich, W., Eine merkwürdige Familie von beweglichen Stabwerken, 1979. Wunderlich, W., Snapping and shaky antiprismas, 1979.
Wunderlich, W., Neue Wackelikosaeder, 1980.
Wunderlich, W., Wackelige Doppelpyramiden, 1980.
Wunderlich, W., Zur projektiven Invarianz von Wackelstrukturen, 1980.
Wunderlich, W., Wackeldodekaeder, 1980.
Wunderlich, W., Projective invariance of shaky structures. 1982.
Wunderlich, W., Kipp-Ikosaeder I, 1981.
Wunderlich, W., Kipp-Ikosaeder II, 1982.
Wunderlich, W., Ringartige Wackelpolyeder, 1982.
Wunderlich, W., Wackeldodekaeder. 1982.
Wunderlich, W., and Schwabe, Ch., Eine Familie von geschlossenen gleichflächigen Polyedern die fast beweglich sind, 1986.

Wunderlich, W., Fast bewegliche Oktaeder mit zwei Symmetrieebenen, 1987.
Wunderlich, W., Shaky polyhedra of higher connection, 1990.

A Review of Walter Wunderlich's Scientific Work

Walter Wunderlich's scientific work comprises 205 papers including three books. Almost all of the papers are journal papers and single authored. The main part of his scientific work is devoted to kinematics. More than 60 papers including one book are from this field. His other interests were in descriptive geometry, the theory of special curves and surfaces, the classical differential geometry and the application of geometry in surveying, civil and mechanical engineering.

It is worth noting that Walter Wunderlich did not develop an outstanding scientific theory, he contributed self-contained solutions to a large variety of problems in the above mentioned fields. All the time his unique geometric intuition was basic to his approach and sometimes the reader of his papers needs a lot of geometric knowledge to follow his elegant arguments and his stringent style of proofs. In the following review of Wunderlich's scientific achievements we will restrict ourselves to kinematic papers, the other fields will be mentioned only briefly in one subsection.

The first papers published by Wunderlich are devoted to kinematic problems in Non-Euclidean geometries. After F. Klein's Erlangen program from 1872, studies in different Non-Euclidean geometries became very popular. It was quite natural that also kinematic theory was developed in Non-Euclidean settings. The most important mathematicians and geometricians contributing to this field were W. Blaschke, E. Study, and F. von Lindemann. In W. Wunderlich's first scientific papers, essentially coming from his master's thesis, his dissertation and his habilitation thesis, he studies screw motions in elliptic space, hyperbolic space and the isotropic space which can be obtained from the elliptic space by some limit process.[3] Most of the time Wunderlich uses geometric or descriptive geometric methods to determine invariant properties of motions. To obtain graphic solutions he uses a Clifford parallel projection, which is quite natural in these Non-Euclidean Geometries. Most important for the classical kinematics in the plane are his results on screw surfaces in a

[3] This limiting case and the screw motion were also studied later on by Husty (1983) in his Dissertation.

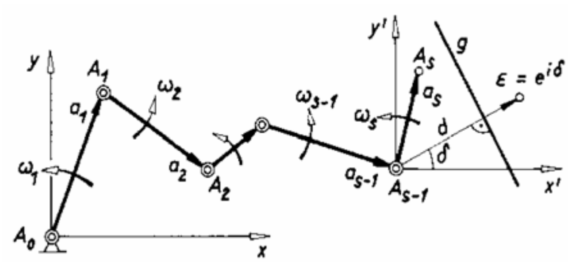

Fig. 2. Higher cycloidal motions.

quasi-elliptic space, because this Non-Euclidean space is the kinematic image space of planar motions.

In 1947 he published a paper dealing with a generalization of cycloidal motions (Wunderlich, 1947). In this paper he introduced so-called higher cycloidal motions. Cycloidal motions are obtained if both centrodes of a planar motion are circles. The paths of points are cycloids (epi-cycloids, hypocycloids). The same motion can be obtained by the end-effector of a planar 2R-linkage when both revolute joints rotate with constant angular velocity. Wunderlich's generalization is now that he allows n systems. Therefore he obtains the one-parameter motion of an nR-chain where all links rotate with constant angular velocity (see Figure 2).

The paths of points of the final system are called *cycloids of nth stage*. The analytic representation of the paths is obtained easily when one uses complex numbers to describe planar motions:

$$z = x + Iy = \sum_{i=1}^{s} a_v e^{I\omega_v t}.$$

The description of planar motions with complex numbers requires a historical remark: Although Bereis (1951) is often cited to be the founder of this method, Wunderlich has used complex numbers (also the so-called isotropic coordinates) earlier. In private communications he claimed that he was the one who encouraged Bereis to develop a theory of planar kinematics using complex numbers. According to Wunderlich it was the Italian geometrician Bellavitis (1874) who underlined the vectorial interpretation of complex numbers using the symbol "ramun" (*radice di meno uno*) instead of I to describe the 90° rotation. The application of isotropic coordinates was introduced by Cayley (1868) and Laguerre (1870). The main result of Wunderlich concerning generalized cycloidal motions is the generalization of Euler's theorem: *Every cycloid of nth stage can be generated by a* $2(n-1)$*-chain of circles in* $n!$ *different ways*. Wunderlich shows that the moving and the fixed centrodes of the corresponding one-parameter higher cycloidal motions are higher cycloids and discusses a lot of geometric properties of the paths of points and the envelopes of lines. Wunderlich proves also another generalisation of Euler's theorem: *Every higher cycloid of nth stage can be generated in n different ways by rolling of two higher cycloids of stage* $n-1$. This theorem is visualized in Figure 3: a cycloidal motion is given by the equation:

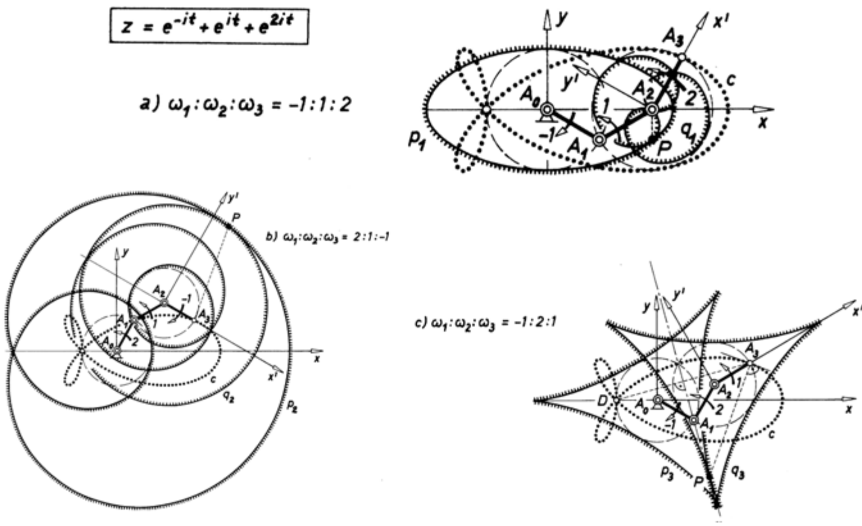

Fig. 3. Triple generation of a higher cycloidal curve of 3rd stage.

The path of A_3 in this motion is a rhodonea (rose-curve; the dotted curve in the three figures). The motion can be generated in three different ways in

(a) the fixed polhode is an ellipse and the moving polhode is Pascal's curve;
(b) an epi-cycloid rolls on a Pascal-curve; and
(c) the polhodes are two Steiner-cycloids.

Later on Wunderlich used higher cycloids for curve approximation in the plane (Wunderlich, 1950).

In 1968 Wunderlich published his book on planar kinematics (*Ebene Kinematik*). This book contains all topics of planar kinematics treated from a

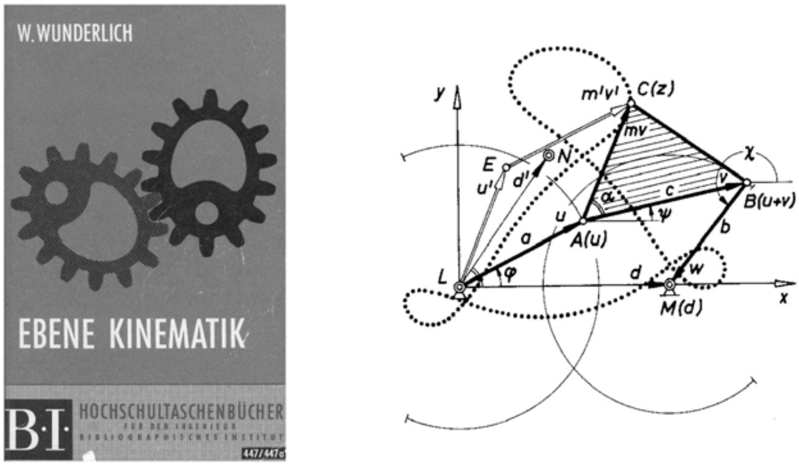

Fig. 4. Coupler curves.

geometric point of view, but it has many links to the applications and some unusual facets. It starts with the classical basics of planar kinematics but, a noteworthy point, complex numbers are used continuously to describe the motions and their properties analytically. The advantage of the method may be shown in Wunderlich's derivation of the equation of the coupler curve of a planar four-bar using isotropic coordinates.

Referring to the notation of Figure 4 he writes the bars LA, AB and BM of the four-bar in complex numbers. These complex numbers have constant absolute value (modulus) $|\mathbf{u}| = a$, $|\mathbf{v}| = c$, $|\mathbf{w}| = b$ but changing arguments (angles) and the constant sum

$$\mathbf{u} + \mathbf{v} + \mathbf{w} = d.$$

An arbitrary complex number **m** is then used to describe a point of the coupler system:

$$\mathbf{z} = \mathbf{u} + \mathbf{m}\mathbf{v}.$$

Using the relations:

$$\mathbf{u}\bar{\mathbf{u}} = \mathbf{a}^2, \quad \mathbf{v}\bar{\mathbf{v}} = \mathbf{c}^2, \quad \mathbf{w}\bar{\mathbf{w}} = \mathbf{b}^2$$

to eliminate **u**, **v**, **w** and their conjugates, he obtains a relation between **z** and $\bar{\mathbf{z}}$ which is the equation of the coupler curve in minimal (isotropic) coordinates:

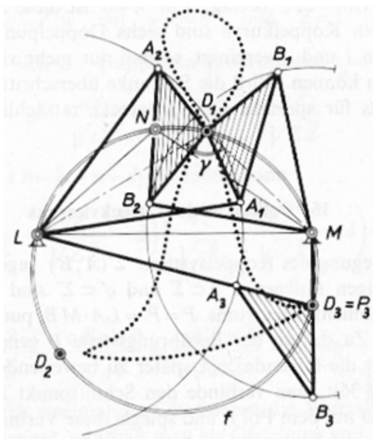

Fig. 5. Focal circle.

$$[\bar{\mathbf{n}}(\mathbf{z}-d)P - \bar{\mathbf{m}}\mathbf{z}Q][\mathbf{n}(\bar{\mathbf{z}}-d)P - \mathbf{m}\bar{\mathbf{z}}Q] + c^2R^2 = 0,$$

where P, Q, R are quadratic polynomials in $\mathbf{z}$, and $\bar{\mathbf{z}}$ and $\mathbf{n} = \mathbf{m} - 1$. This concept of minimal coordinates is used in the book consequently up to the third differential order to derive e.g. the equation of center-point curves and all the other well-known curves in planar kinematics.

Figures 5 and 6 show typical examples from the book on planar kinematics. There is no other book in planar kinematics which visualizes the theoretical concepts in such a clear way. Figure 5 deals with the focal circle. At first Wunderlich shows, using the complex representation of the coupler-curve, that the three points L, M and N forming a triangle similar to the coupler triangle are exceptional foci of the coupler curve, because they are intersections of isotropic asymptotes of the coupler curve. A simple symmetry argument in the two triangles A_1DA_2 and B_1DB_2 yields that D must be seen from L and M under the angle γ, therefore D must be on the focal circle f. Because f and the coupler curve, which is of degree six, have 12 points of intersection, from which six are in the circle-points, one can conclude immediately that each coupler curve has three double points. Figure 5 now shows clearly Wunderlich's ability to visualize the whole complexity of this theorem: in one figure, he displays all the possibilities that can happen. The double points can be real (D_1), complex (D_2) or the double point can be a cusp (D_3). Figure 6 presents an example of a symmetrical four-bar

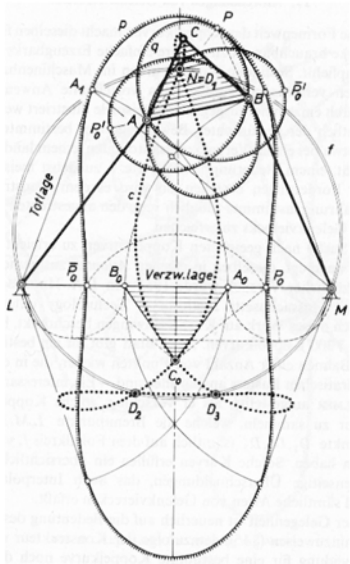

Fig. 6. Pole curves and coupler curves.

with coupler curve having three real double points and a moving and a fixed centrode. In a full chapter, applications and special layouts of four-bar mechanisms are discussed. Figure 7 shows how Wunderlich has linked in the book the geometric and kinematic theory with practical applications. On the left-hand side one can see the electro-mechanical device, namely a high voltage switch and on the right-hand side there is the kinematic analysis of the device containing the centrodes of the motion and the coupler curve of the point of interest P.

An important part of the book is devoted to multi-body mechanisms, focal mechanisms and singular planar multi-body systems. The investigation of singular frameworks is one of the main contributions of Walter Wunderlich. Most the time he provides simple, convincing geometric arguments for the exceptionality of a linkage. In Figures 8 and 9 two examples are shown. Both examples show nine-bar frameworks. In the first example two triangles ABC and LMN are linked by three bars AL, BM and CN and in the second we see a hexagon $ABCDEF$ with its three diagonals AD, BE and CF.

In both figures, from left to right, rigid, shaky (infinitesimal movable) and movable designs are shown. Without computations Wunderlich shows the shakiness in the first case by resorting to an old and well-known theorem

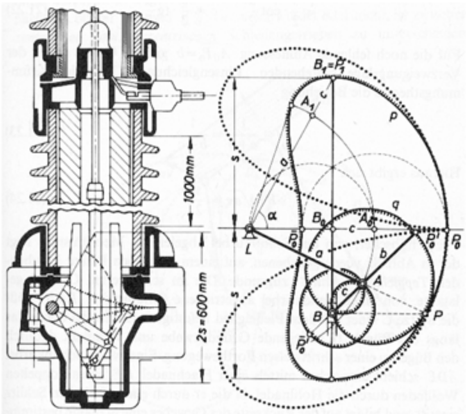

Fig. 7. High voltage switch.

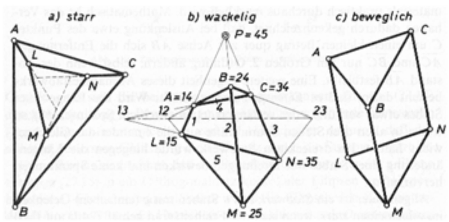

Fig. 8. Nine-bar framework, first layout.

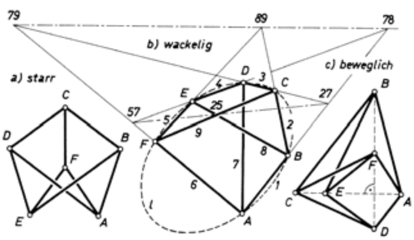

Fig. 9. Nine-bar framework.

in geometry, namely that the two triangles have to be in a Desarguian configuration and the movability yields the parallel bar mechanism. It should be noted that this mechanism became famous almost thirty years after Wunderlich published his book as the planar analogue of the Stewart–Gough platform, namely the 3-RPR planar parallel manipulator. The second framework is in its movable realisation known as Dixon's mechanism. But the condition for shakiness is interesting. Wunderlich explains the condition again by resorting to the old theorem of Pappus–Pascal, which yields that the points A, B, C, D, E, F have to be on a conic section, when the framework is shaky. Wunderlich's early investigations on shakiness culminate in the first proof of the projective invariance of shaky structures. An earlier proof had been given by H. Liebmann, but with the restriction that the framework has to contain at least one triangle. Wunderlich shows that this assumption is not necessary and gives a relatively simple proof for the theorem (Wunderlich, 1980).

The book *Ebene Kinematik* also has a full chapter on the geometric theory of gears and cams. This is due to the fact that Wunderlich made significant contributions to this theory. Starting with the geometric theory of constructing gear profiles for constant transmission ratio, he develops the classical theory of involute gears due to F. Reuleaux, the theory of cycloidal gears due to Ch.E.L. Camus based on the earlier work of Ph. de la Hire. Remarkable is the paragraph on the geometry and kinematics of the Wankel motor.

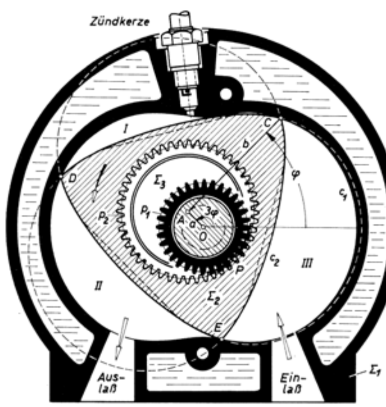

Fig. 10. Wankel motor.

Here one can see clearly Wunderlich's approach to difficult kinematic problems. One of the main problems of this motor type is the geometric form of the moving system (piston, Σ_2 in Figure 9). Wunderlich shows that this problem can be solved as a gear profile problem. Moreover he uses the so-called cyclographic mapping which maps circles into points of a three-

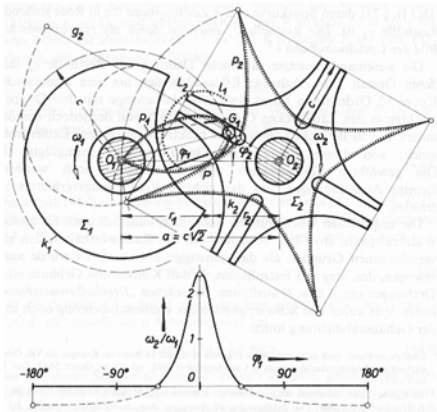

Fig. 11. Geneva wheel.

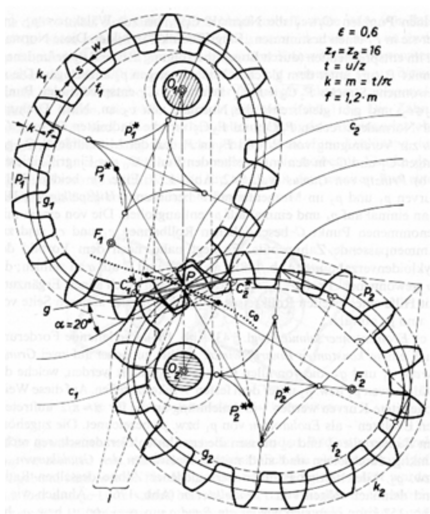

Fig. 12. Non-circular gears.

dimensional parameter space to investigate the geometric properties of the design curve of the piston.

In the book he discusses also in detail motion transmission with changing transmission ratio (non-circular gearing). As examples we take the Geneva wheel (Figure 11) and the involute gears with ellipses as base curves (Figure 12). Whereas the Geneva wheel is treated in many of the English text-

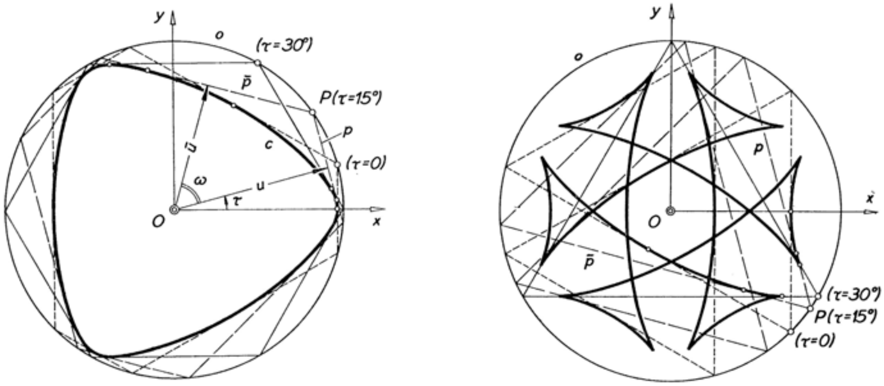

Fig. 13. Curves having an isoptic circle.

books on planar kinematics, the involute gear problem with non-circular base curves can be found (to the author's best knowledge) in none of the popular English textbooks. The elliptic gearing problem is also treated in Wunderlich and Zenow (1975).

Walter Wunderlich is best known to the community of mechanical engineers for his significant contributions to the theory of cams. Therefore it is not surprising that a full chapter of *Ebene Kinematik* is devoted to this problem. Although two papers (Wunderlich, 1971a, 1984) are published in English, the important papers dealing with the geometric basics of cams are published in German. In these papers he shows that the problem of designing single cams, that steer a flat-faced follower pair, is closely related to the construction of curves having an isoptic circle.[4] In Wunderlich (1971) he gives a complete solution to the problem which had been posed before (Green, 1950) but incompletely solved, and additionally he shows algebraic examples of such curves.

To finish the review of Wunderlich's book *Ebene Kinematik*, one has to mention the chapter on curvature theory of planar motions. He gives the first treatment of curvature theory in isotropic coordinates. But he does not limit to the mathematical description of all the well-known properties. The inflection circle (w in Figure 14), the circle point curve (k), the center-point curve (k^*) and Ball's point (U) are visualized for different types of mechanisms.

[4] An isoptic curve is the locus of points from which two tangents to the curve subtend a constant angle ω.

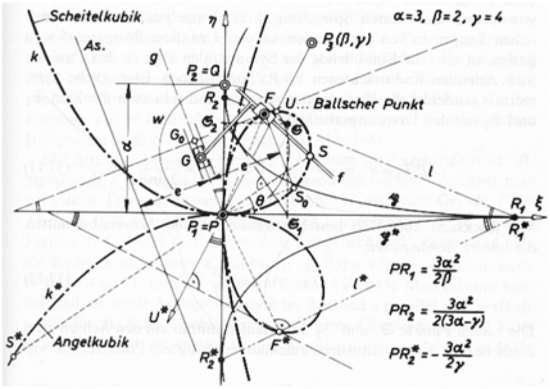

Fig. 14. Center-point curve, circle-point curve and Ball's point.

The most important kinematic topic of Wunderlich's later work are his contributions to overconstrained and shaky (infinitesimal movable) structures. He shows many remarkable examples but also develops methods that can be applied in today's research in singularity theory of nR-chains and parallel manipulators. Moreover he exhibits the close relation of shaky and pathologically movable structures to singularity problems in geodesy (see Wunderlich 1977, 1978). Already his first paper on "Rigid, snapping, shaky and movable octahedra", where he gives geometric conditions for all phenomena mentioned in the title, can be viewed as a complete singularity analysis of 3-3 Stewart–Gough manipulators (see Merlet, 2000). In the paper "Starre, kippende, wackelige und bewegliche Gelenkvierecke im Raum" one can find a complete singularity analysis of closed serial $4R$-chains up to the conditions for movability (Bennett-mechanism). This analysis is of course mathematically identical to the inverse kinematics problem of serial $4R$-manipulators.

In the papers "Fokalkurvenpaare in orthogonalen Ebenen und bewegliche Stabwerke" and "Bewegliche Stabwerke vom Bricardschen Typus" he corrects and improves an old result of Bricard concerning frameworks of rods connected by spherical joints: *Are* $m \geq 4, 5, 6$ *points* P_i *located on an ellipse and* $n \geq 6, 5, 4$ *points* Q_i *located on the corresponding focal hyperbola and all points* P_i *are connected to points* Q_i *by rigid rods, then this framework is movable even when all rods are connected by spherical joints*. Bricard claims that this framework has two degrees of freedom (dof) but Wunderlich proves that this is not correct: He shows that the correct dof is 3. Moreover he shows that one can take any pair of focal curves (focal curves of arbitrary degree,

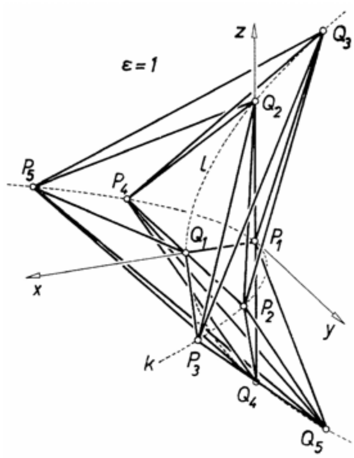

Fig. 15. Bricard framework.

not only ellipses and hyperbolas) and that the corresponding framework for arbitrary focal curves has 2 dofs.

In a whole series of papers he discusses the geometric conditions for infinitesimal movability or snapping[5] of polyhedra (icosahedra, dodecahedra, pyramids, prisms) and gives examples for each of the different cases. It must be mentioned that polyhedra having more then three edges in one face are considered as panel structures having revolute joints in the edges. Due to the basic theorem of Cauchy, all of these polyhedra have to be non-convex to allow infinitesimal movability. Therefore for each of such polyhedra there exists more then one assembly mode.

Figure 15 shows such a three-degrees-of-freedom framework. The knots are distributed on two focal parabolas.

Four of Wunderlich's papers on shakiness deserve special attention. In Wunderlich (1980d, 1982a) he proofs the projective invariance of shakiness of spatial frameworks. Projective invariance means: if a framework is shaky, then any linear transformation of its design will not resolve the shakiness. This includes of course any kind of projections. In his last paper (Wunderlich, 1990), written at the age of more than 80 years, and in Wunderlich (1982c) he gives examples of shaky polyhedra with genus > 0 (Figure 16). He provides

[5] This means that two assembly solutions are very close to each other and can be transformed into each other because of tolerances in the joints.

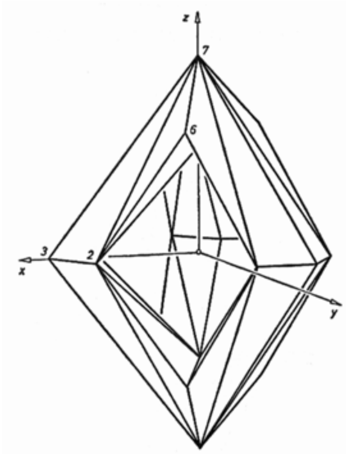

Fig. 16. Shaky structure of genus 3.

a geometric algorithm to generate shaky polyhedra of arbitrary genus. The existence of such structures had been doubted by all scientists working in the field.

Short Review of Wunderlich's Contribution to Other Fields

To show Wunderlich's versatility, a selection of other significant contributions will be presented. Through his whole scientific life he was investigating the geometry of curves and surfaces. This interest was awakened at the beginning of his scientific career when he worked on the descriptive geometry of non-Euclidean screw surfaces and spiral surfaces. The theory of spiral motion and the curves and surfaces generated by this motion is treated extensively in his book *Descriptive Geometry II*. The spiral motion is a generalization of a screw motion where instead of a rotation a planar spiral motion is concatenated with a translation along the spiral axis.

The generation of special surfaces with special motions is a traditional topic in Austrian geometry. Wunderlich especially contributed to this topic kinematic generations of J. Steiner's famous Roman surface. But there are also papers on kinematic generation of cubic ruled surfaces where a parabola is moved to describe the surface and a developable Möbius strip.

Another main area of Wunderlich's work is on surfaces with special fall lines (curves of steepest slope on a surface). Here he investigates surfaces with planar fall lines, Roman surfaces with planar fall lines or surfaces with conic sections as fall lines.

Wunderlich's contributions to the geometry of special curves would fill another overview paper. But a short enumeration of some topics where Wunderlich has made excellent contributions is essential to understand the breadth of his interests: Pseudo-geodesic lines of cylinders and cones, D-curves on quadrics, principal tangent curves of special surfaces, loxodromic curves on different surfaces, irregular curves and functional equations, auto-involute curves, curves with constant global curvature, Zindler-curves, Bertrand curves, spherical curves and auto-polar curves.

Concerning Wunderlich's versatile interest in geometry one has to mention his papers "On the statics of the rope ladder", or "Geometric considerations on an apple skin" or "On the geometry of bird eggs".

Modern Interpretation of Main Contributions to Mechanism Design

Walter Wunderlich's work still exerts significant influence in kinematic research. Algebraic manipulation systems allow answering some of the questions he left open. As a recent example, we mention his work on Dixon's mechanisms. Dixon showed in 1899 that a linkage consisting of nine bars connected by revolute joints is paradoxically mobile when certain geometric conditions are fulfilled. There are two layouts that yield mobile linkages.

Wunderlich gives an elegant proof for the mobility, but for the number of assembly modes of the nine-bar linkage he could only conjecture that there should be eight. Only recently Walter and Husty (2007) proved that he was right. Moreover, in the same paper it was proven that the two linkage layouts already known to Dixon (Figures 17 and 18) are the only possible mobile layouts. Further generalisations of Wunderlich's papers are given by Stachel. In [Stachel 1997] he shows that Dixon's mechanisms can be transferred to spherical kinematics and, using the principle of transference, one obtains an overconstrained spatial mechanism. This mechanism is a two-degree-of-freedom paradoxical linkage.

Shakiness of polyhedral structures has become an important subject in the field of combinatorial geometry. Wunderlich's papers on rigidity have been reviewed and extended. The recently published second edition of the *Hand-*

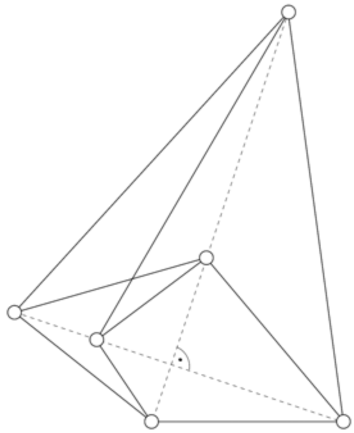

Fig. 17. Dixon Mechanism Type I.

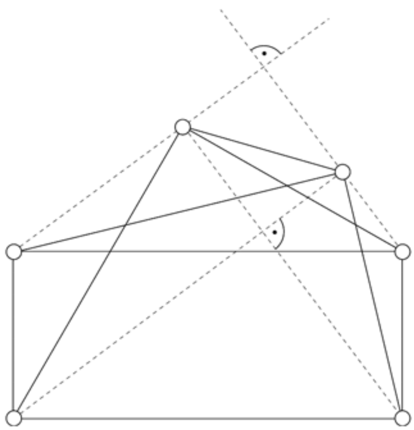

Fig. 18. Dixon Mechanism type II.

book of Discrete and Computational Geometry mentions 19 of Wunderlich's papers.

People working in the field of geodesy have applied Wunderlich's results in the theory of GPS navigation systems.

There is virtually no recent Austrian geometrician who has not used one of Wunderlich's results for his own scientific work. But of course Wunderlich himself was personally known to most of the now active scientists in kinematics and geometry, either as their teacher or reviewer of their theses and papers. Unfortunately, because of language problems the dissemination of Wunderlich's results in the Anglophone world is rather small. It is hoped that this paper may help to advertise his results within this community.

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