

## 100 YEARS AGO

Our present knowledge of the theory of errors receives an interesting addition at the hands of M. Charles Lagrange in the form of a contribution to the Bulletin de l'Académie royale de Belgique (vol. xxxv. part 6). Without going into details of a purely mathematical nature, certain of M. Lagrange's conclusions appear to be sufficiently important to be worth noticing. In taking the arithmetic mean of a number of observations as the most probable value of the observed quantity, common sense suggests that any observations differing very widely from the rest should be left out of count as being purely accidental, and thus likely to vitiate the result. But as it is impossible to draw the line from theoretical considerations between values retained and values omitted, any such omission would necessarily be unjustifiable. This discrepancy between theory and common sense is, to a large extent, reconciled by M. Lagrange's "theory of recurring means." According to this theory, the weight to be assigned to any observation is inversely proportional to the square of the error of the observed value relative to the most probable value. ... The weighted mean is then taken as a second approximation to the most probable value. This mean determines a fresh series of weights to be assigned to the observations by which a new weighted mean ... is found, and so on ... These successive means are called by $M$. Lagrange "recurring means," and by their use the effects of sporadic errors are, to all practical purposes, eliminated, since the weight assigned to the corresponding observations soon becomes relatively small.
From Nature 15 September 1898.

## 50 YEARS AGO

In the possession of the Science
Museum, London, there are six beautifully engraved buttons, classified as diffraction gratings, which are still regarded as masterpieces. They were the work of Sir John Barton, deputy comptroller of the Royal Mint in the early part of the nineteenth century, about whom little is known personally, but who must have been an ingenious inventor and capable engineer, for in 1806 he invented a differential screw measuring instrument capable of measuring $10^{-5}$ inch. From Nature 18 September 1948.

Thesefindings are significant because of the growing interest in the mechanisms that underlie the formation of episodic memory. Episodic memory is memory for events ${ }^{6}$, each of which occurs in a unique setting of space and time. As such, it can be distinguished from semantic memory, which is memory for facts, or 'knowledge'. Episodic memory is disproportionately affected in some types of amnesia, such as that seen in Alzheimer's dementia, and it is thought to depend on thehippocampus ${ }^{7}$, an important structure for spatial memory in both birds and mammals. To storean episodic memory, some method is needed for temporally ordering the sequence of happenings that make up an event. Perhaps episodes can be temporally ordered in the episodic memory of non-humans, as well as humans. In this light, the finding that animals as different
from us as birds possess a mechanism for representing spatiotemporal events is enormously important, and could be a big step towards understanding how space, timeand events are represented and remembered in thevertebratebrain.
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## Mathematics <br> Magic squares cornered

## Martin Gardner

Dame Kathleen Ollerenshaw, one of England's national treasures, has solved a long-standing, extremely difficult problem involving the construction and enumeration of a certain type of magic square. The solution comes in a book* written with David Brée.

First, some background on magic squares, and their hierarchy of perfection. For many centuries, mathematicians especially those concerned with combinatorics - have been challenged by magic squares. These are arrangements of $n^{2}$ distinct integersin an $n \times n$ array such that each row, column and main diagonal hasthesame sum. The sum is called the magic constant, and $n$ is called the square'sorder. Traditional magic squares are made with consecutive integers starting with 0 or 1 . If it starts with 0 it can bechanged to a squarestarting with 1 simply by adding 1 to each cell.

No order-2 squareis possible. Theorder3 square(Fig. 1) barely exists. Why? Because there are just eight different triplets of distinct digits from 1 to 9 that add up to 15 , the square'sconstant. Each triplet appears asone of the square's eight straight lines of three numbers. Thepattern isunique- except for rotations and mirror reflections, which are only trivially different.

This little gem of combinatorial number theory was called the lo shu in ancient China, meaning 'Lo River writing'. Legend has it that in the 23 rd century BC , a mythical KingYu saw thepattern on theback of a sacred turtle in the River Lo. (M odern historians, however, find

[^0]no evidence that the pattern was known before the fourth or fifth century BC .) The Chinese identify it with their familiar yinyang circle. Theeven digits, hereshown shaded, arelinked to the dark yin; the Greek cross of odd digits is linked to the light yang. For centuries the lo shu has been used as a charm on jewellery and other objects. Today, large passenger ships often feature the lo shu on their main deck as a pattern for the game of shuffleboard.

At order 4, the number of magic squares jumpsto 880. Among them is a special subset of 48 squares called pandiagonal, which have three amazing properties. This is illustrated by theexamplein Fig. 2, whoseconstantis30.

First, each broken diagonal also adds up to 30 . Thesequences $0,3,15,12$ and $7,13,8,2$ are examples. This can be expressed in another way: imaginean endlessarray of this squareplaced side-by-sidein all directionsto make a wallpaper pattern. Then every $4 \times 4$ square drawn on the pattern will also be a pandiagonal magic square- in other words, every straight line of four numbers will add up to 30 . Second, every $2 \times 2$ square on the wall paper also adds up to 30 . Third, along every diagonal, any two cells separated by onecell add up to 15.

In general, a magic square is called pandiagonal if all its broken diagonals add up to themagic constant. Such squares can beconstructed of any odd order abovethree and of any order that is a multiple of four. If a pandiagonal squarealso hassimilar propertiesto the order-4 pandiagonals, it is called 'most perfect': for example, the most-perfect order- 8 squarein Fig. 3 has a magic constant of 252 , and its $2 \times 2$ sub-squares add up to 126 , and any two numbers that are $n / 2=4$


Figure 1 Lo shu, the only $\mathbf{3} \times \mathbf{3}$ magic square.

| 0 | 13 | 6 | 11 |
| :---: | :---: | :---: | :---: |
| 7 | 10 | 1 | 12 |
| 9 | 4 | 15 | 2 |
| 14 | 3 | 8 | 5 |

Figure $2 \mathrm{~A} 4 \times 4$ magic square that is pandiagonal - the broken diagonals also add up to its magic constant of 30 .
cells sapart add up to $n^{2}-1=63$.
Although all order-4 pandiagonals have been known to be most perfect for three centuries, little has been known about mostperfect squares of higher orders. There was no method of constructing them all, or even of determining the number of squares of a given order.

These questions are finally settled by Kathleen Ollerenshaw and David Brée in their new book. The authors have devised a method for constructing all the mostperfect squares of any order, and a way of calculatingtheir number.

Unlike the ordinary pandiagonals, there are no most-perfect squares with odd order, so the only possible orders are multiples of four. At each leap in order, the number of essentially different most-perfect squares increases rapidly: from 48 squares of order four, to 368,640 of order eight, to 2.22953 $\times 10^{10}$ of order 12 . When you reach order 36, the number is $2.76754 \times 10^{44}$ - around a thousand times the number of pico-picosecondssincetheBigBang.

This solution of one of the most frustrating problems in magic-square theory is an achievement that would have been remarkablefor a mathematician of any age. In Dame Kathleen's case it is even more remarkable,

| 0 | 62 | 2 | 60 | 11 | 53 | 9 | 55 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 15 | 49 | 13 | 51 | 4 | 58 | 6 | 56 |
| 16 | 46 | 18 | 44 | 27 | 37 | 25 | 39 |
| 31 | 33 | 29 | 35 | 20 | 42 | 22 | 40 |
| 52 | 10 | 54 | 8 | 63 | 1 | 61 | 3 |
| 59 | 5 | 57 | 7 | 48 | 14 | 50 | 12 |
| 36 | 26 | 38 | 24 | 47 | 17 | 45 | 19 |
| 43 | 21 | 41 | 23 | 32 | 30 | 34 | 28 |

Figure 3 A most-perfect square of order eight. Its rows, columns, diagonals and broken diagonals all add up to 252 , and all $2 \times 2$ sub-squares add up to 126 . Kathleen Ollerenshaw and David Brée have found a way to construct most-perfect pandiagonal magic squares of any order.
because she was 85 when she and Bréefinally proved the conjectures she had earlier made. In her own words, "Themanner in which each successive application of the properties of the binomial coefficients that characterize the Pascal triangle led to the solution will always remain oneof themost magical mathematical revelationsthat I havebeen fortunateenough
to experience. That this should have been afforded to someone who had, with a few exceptions, been out of active mathematics research for over 40 years will, I hope, encourage others. The delight of discovery is not a privilegereserved solely for theyoung." M artin Gardner is at 3001 Chestnut Road,
Hendersonville, North Carolina 28792, USA.

## Bacterialinfection

## For w hom the bell tolls

## Craig Gerard

"N ow this bell tolling softly for another, saysto me, Thou must die."

The 1623 M editation 17 by the English metaphysical poet John Donne was probably inspired by the church bells that tolled to announce death by the plague. Death in this and many other infectious diseases typically follows septic shock, often caused by so-called Gram-negative bacteria. The path that leads from these organisms to septicshock hasbeen under investigation for over a century, and, on page 284 of thisissue, Yangetal. ${ }^{1}$ reportthat a cell-surfacereceptor, Toll-like receptor 2 (TLR2), may be part of thelong-awaited solution to thepuzzle.

In 1884, the Danish physician Christian

Gram discovered that the outer membranes of bacteria may be classified as Gram negativeor Gram positive, based on the ability of acetone/alcohol to decolorize cells stained with Gram'siodineand crystal violet. Gramnegative organisms contain a complex glycolipid in their outer membranes, known aslipopolysaccharide(LPS) or endotoxin. In picogram quantities, this substance can induce mammalian white blood cells to secrete cytokines which, if left unchecked, can lead to fever, coagulation defects, lung dysfunction, kidney failure and circulatory collapse.

How aretracequantities of this glycolipid recognized, and the signals necessary for the production of toxic cytokines transduced?


[^0]:    *M ost-Perfect Pandiagonal M agic Squares: Their Construction and Enumeration. Due for publication on 10 ctober by The Institute of M athematics and its Applications, Catherine Richards H ouse, 16 Nelson Street, Southend-on-Sea, Essex SS1 1EF, UK

