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100 YEARS AGO

Our present knowledge of the theory of errors receives an interesting addition at the hands of M. Charles Lagrange in the form of a contribution to the Bulletin de l'Académie royale de Belgique (vol. xxxv. part 6). Without going into details of a purely mathematical nature, certain of M. Lagrange's conclusions appear to be sufficiently important to be worth noticing. In taking the arithmetic mean of a number of observations as the most probable value of the observed quantity, common sense suggests that any observations differing very widely from the rest should be left out of count as being purely accidental, and thus likely to vitiate the result. But as it is impossible to draw the line from theoretical considerations between values retained and values omitted, any such omission would necessarily be unjustifiable. This discrepancy between theory and common sense is, to a large extent, reconciled by M. Lagrange's "theory of recurring means." According to this theory, the weight to be assigned to any observation is inversely proportional to the square of the error of the observed value relative to the most probable value. ... The weighted mean is then taken as a second approximation to the most probable value. This mean determines a fresh series of weights to be assigned to the observations by which a new weighted mean ... is found, and so on ... These successive means are called by M. Lagrange "recurring means," and by their use the effects of sporadic errors are, to all practical purposes, eliminated, since the weight assigned to the corresponding observations soon becomes relatively small.

From Nature 15 September 1898.

50 YEARS AGO

In the possession of the Science Museum, London, there are six beautifully engraved buttons, classified as diffraction gratings, which are still regarded as masterpieces. They were the work of Sir John Barton, deputy comptroller of the Royal Mint in the early part of the nineteenth century, about whom little is known personally, but who must have been an ingenious inventor and capable engineer, for in 1806 he invented a differential screw measuring instrument capable of measuring 10⁻⁵ inch. From *Nature* 18 September 1948.

These findings are significant because of the growing interest in the mechanisms that underlie the formation of episodic memory. Episodic memory is memory for events⁶, each of which occurs in a unique setting of space and time. As such, it can be distinguished from semantic memory, which is memory for facts, or 'knowledge'. Episodic memory is disproportionately affected in some types of amnesia, such as that seen in Alzheimer's dementia, and it is thought to depend on the hippocampus⁷, an important structure for spatial memory in both birds and mammals. To store an episodic memory, some method is needed for temporally ordering the sequence of happenings that make up an event. Perhaps episodes can be temporally ordered in the episodic memory of non-humans, as well as humans. In this light, the finding that animals as different

from us as birds possess a mechanism for representing spatiotemporal events is enormously important, and could be a big step towards understanding how space, time and events are represented and remembered in the vertebrate brain.

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Mathematics Magic squares cornered

Martin Gardner

ame Kathleen Ollerenshaw, one of England's national treasures, has solved a long-standing, extremely difficult problem involving the construction and enumeration of a certain type of magic square. The solution comes in a book* written with David Brée.

First, some background on magic squares, and their hierarchy of perfection. For many centuries, mathematicians — especially those concerned with combinatorics — have been challenged by magic squares. These are arrangements of n^2 distinct integers in an $n \times n$ array such that each row, column and main diagonal has the same sum. The sum is called the magic constant, and n is called the square's order. Traditional magic squares are made with consecutive integers starting with 0 or 1. If it starts with 0 it can be changed to a square starting with 1 simply by adding 1 to each cell.

No order-2 square is possible. The order-3 square (Fig. 1) barely exists. Why? Because there are just eight different triplets of distinct digits from 1 to 9 that add up to 15, the square's constant. Each triplet appears as one of the square's eight straight lines of three numbers. The pattern is unique — except for rotations and mirror reflections, which are only trivially different.

This little gem of combinatorial number theory was called the *lo shu* in ancient China, meaning 'Lo River writing'. Legend has it that in the 23rd century BC, a mythical King Yu saw the pattern on the back of a sacred turtle in the River Lo. (Modern historians, however, find no evidence that the pattern was known before the fourth or fifth century BC.) The Chinese identify it with their familiar yinyang circle. The even digits, here shown shaded, are linked to the dark yin; the Greek cross of odd digits is linked to the light yang. For centuries the *lo shu* has been used as a charm on jewellery and other objects. Today, large passenger ships often feature the *lo shu* on their main deck as a pattern for the game of shuffleboard.

At order 4, the number of magic squares jumps to 880. Among them is a special subset of 48 squares called pandiagonal, which have three amazing properties. This is illustrated by the example in Fig. 2, whose constant is 30.

First, each broken diagonal also adds up to 30. The sequences 0, 3, 15, 12 and 7, 13, 8, 2 are examples. This can be expressed in another way: imagine an endless array of this square placed side-by-side in all directions to make a wallpaper pattern. Then every 4×4 square drawn on the pattern will also be a pandiagonal magic square — in other words, every straight line of four numbers will add up to 30. Second, every 2×2 square on the wallpaper also adds up to 30. Third, along every diagonal, any two cells separated by one cell add up to 15.

In general, a magic square is called pandiagonal if all its broken diagonals add up to the magic constant. Such squares can be constructed of any odd order above three and of any order that is a multiple of four. If a pandiagonal square also has similar properties to the order-4 pandiagonals, it is called 'most perfect': for example, the most-perfect order-8 square in Fig. 3 has a magic constant of 252, and its 2×2 sub-squares add up to 126, and any two numbers that are n/2 = 4

^{*}Most-Perfect Pandiagonal Magic Squares: Their Construction and Enumeration. Due for publication on 1 October by The Institute of Mathematics and its Applications, Catherine Richards House, 16 Nelson Street, Southend-on-Sea, Essex SS1 1EF, UK.

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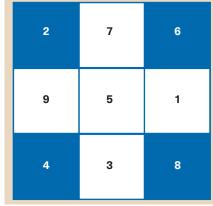


Figure 1 *Lo shu*, the only 3 × 3 magic square.

0	13	6	11	
7	10	1	12 2 5	
9	4	15		
14	3	8		

Figure 2 A 4×4 magic square that is pandiagonal — the broken diagonals also add up to its magic constant of 30.

cells apart add up to $n^2 - 1 = 63$.

Although all order-4 pandiagonals have been known to be most perfect for three centuries, little has been known about mostperfect squares of higher orders. There was no method of constructing them all, or even of determining the number of squares of a given order.

These questions are finally settled by Kathleen Ollerenshaw and David Brée in their new book. The authors have devised a method for constructing all the mostperfect squares of any order, and a way of calculating their number.

Unlike the ordinary pandiagonals, there are no most-perfect squares with odd order, so the only possible orders are multiples of four. At each leap in order, the number of essentially different most-perfect squares increases rapidly: from 48 squares of order four, to 368,640 of order eight, to 2.22953 $\times 10^{10}$ of order 12. When you reach order 36, the number is 2.76754×10^{44} — around a thousand times the number of pico-picoseconds since the Big Bang.

This solution of one of the most frustrating problems in magic-square theory is an achievement that would have been remarkable for a mathematician of any age. In Dame Kathleen's case it is even more remarkable,

0	62	2	60	11	53	9	55
15	49	13	51	4	58	6	56
16	46	18	44	27	37	25	39
31	33	29	35	20	42	22	40
52	10	54	8	63	1	61	3
59	5	57	7	48	14	50	12
36	26	38	24	47	17	45	19
43	21	41	23	32	30	34	28

Figure 3 A most-perfect square of order eight. Its rows, columns, diagonals and broken diagonals all add up to 252, and all 2×2 sub-squares add up to 126. Kathleen Ollerenshaw and David Brée have found a way to construct most-perfect pandiagonal magic squares of any order.

because she was 85 when she and Brée finally proved the conjectures she had earlier made. In her own words, "The manner in which each successive application of the properties of the binomial coefficients that characterize the Pascal triangle led to the solution will always remain one of the most magical mathematical revelations that I have been fortunate enough to experience. That this should have been afforded to someone who had, with a few exceptions, been out of active mathematics research for over 40 years will, I hope, encourage others. The delight of discovery is not a privilege reserved solely for the young."

Bacterial infection For whom the bell tolls

Craig Gerard

"Now this bell tolling softly for another, says to me, Thou must die."

he 1623 Meditation 17 by the English metaphysical poet John Donne was probably inspired by the church bells that tolled to announce death by the plague. Death in this and many other infectious diseases typically follows septic shock, often caused by so-called Gram-negative bacteria. The path that leads from these organisms to septic shock has been under investigation for over a century, and, on page 284 of this issue, Yang *et al.*¹ report that a cell-surface receptor, Toll-like receptor 2 (TLR2), may be part of the long-awaited solution to the puzzle.

In 1884, the Danish physician Christian

Gram discovered that the outer membranes of bacteria may be classified as Gram negative or Gram positive, based on the ability of acetone/alcohol to decolorize cells stained with Gram's iodine and crystal violet. Gramnegative organisms contain a complex glycolipid in their outer membranes, known as lipopolysaccharide (LPS) or endotoxin. In picogram quantities, this substance can induce mammalian white blood cells to secrete cytokines which, if left unchecked, can lead to fever, coagulation defects, lung dysfunction, kidney failure and circulatory collapse.

How are trace quantities of this glycolipid recognized, and the signals necessary for the production of toxic cytokines transduced?