WHAT'S NEW IN POLYOMINO PUZZLES AND THEIR DESIGN

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The idea of using sets of polyominoes as practical assembly puzzles can be traced back more than two and a half centuries. *Les Amusmens*, published in 1749, included a cross puzzle made of three Z-pentominoes and two L-polyominoes. The earliest rectangular puzzle formed from polyominoes that we found, named *The Jags and Hooks Puzzle*, was included in Catel's Catalogue of 1785 [1]. It consisted of four L-tetrominoes and four Z-pentominoes and the only solution is shown in Figure 1.

Figure 1. Jags and Hooks Puzzle, 1785.

Figure 2. Sectional Checkerboard Puzzle, 1880.

French and British versions of the same puzzle have been sold during the last century. A modern polyomino puzzle that has enjoyed great popularity over the years, requires the solver to arrange the complete set of twelve pentominoes inside a rectangular tray, or in the case of solid pentominoes inside a rectangular box. Closely related to these is the ubiquitous family of over 376 dissected checkerboard puzzles [2] which began in 1880 with the *Sectional Checkerboard Puzzle* (Fig. 2).

The earliest solid polyomino puzzle that we have found consists of six polyominoes and was shown in Professor Hoffmann's classic book, *Puzzles Old & New* [3]. *Cube Diabolique*, a French version of the puzzle shown in Figure 3, was made by Watillaux between 1874 and 1893. The most popular solid polyomino puzzle is the *SOMA Puzzle*, invented by Piet Hein in 1936 (Fig. 4). It consists of the seven pieces that three or four cubes can be arranged in, other than straight lines. Typically the pieces of polyomino puzzles are made up of wooden blocks joined together in different ways. By now you might expect that the design possibilities for new polyomino puzzles would have been pretty well exhausted. The purpose of this paper is to show that is not necessarily the case. Most of the new puzzle designs described here were created within the past year. We also hope to show that, as a design science, successful puzzle invention often has less to do with mathematical analysis and more to do with psychology, imagination, and craftsmanship. The *Few-Tiles Puzzle* (AP-ART design no. 133) and the *Trap Puzzle* (AP-ART design no. 153), shown in Figure 5, are examples of designs that uses psychology to make four-piece puzzles quite difficult. They cannot be solved when the pieces are placed, as expected by solvers, in the four corners.

Figure 3. *Cube Diabolique c. 1890.*

Figure 4. Piet Hein's *Soma Puzzle.*

Combinatorial problems that are of interest in the field of recreational math seldom translate directly into elegant assembly puzzles. One reason for this is the fundamental rule of simplicity. The most appealing

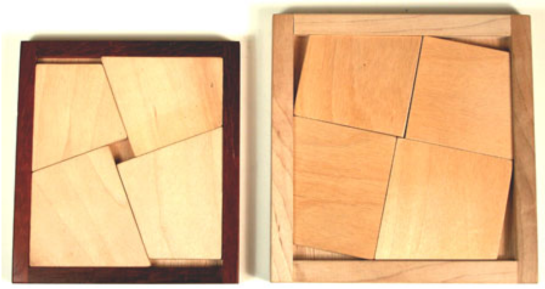

Figure 5. Coffin's *Few-Tiles Puzzle*, left; *Trap Puzzle*, right.

designs are those that achieve their diabolical or whimsical purpose with the fewest number of pieces. Four or five is a good number, with six or seven being the practical upper limit. The *Pyramid Puzzle*, with only *two* pieces, is amazingly difficult for most solvers (Figs. 6).

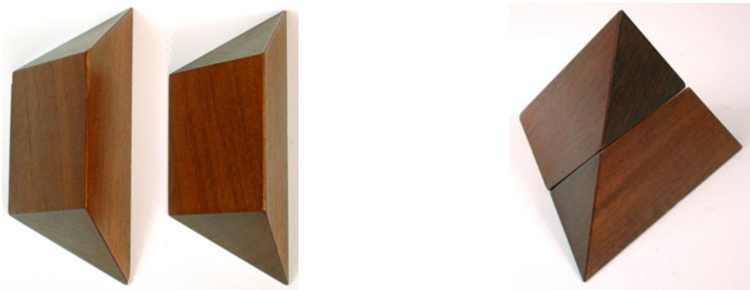

Figure 6. The *Pyramid Puzzle* pieces. The tetrahedron solved.

The previous puzzles also have another very important feature of great puzzles, the instructions such as "make a cube" or pyramid, "fit the pieces in the tray" or "assemble a checkerboard" are very simple, and in some cases obvious. Another consideration is in regard to complete sets. Many mathematical analysts like to work with logically defined complete sets of pieces. A typical problem might be to determine how they will or will not fit some given way. Many puzzle designers frequently go at it the opposite way, starting with the solution and then working backward seeking a set of pieces that will best achieve the intended purpose. It makes little difference whether or not the puzzle pieces constitute a complete set of anything. Few, if any, solvers will care.

One common misconception is that the best puzzles are those that are the most difficult. It is easy to design fiendishly difficult polyomino puzzles simply by increasing the number of pieces relative to the number of solutions, now easily accomplished with the aid of computer programs. For example, the colorful *Cornucopia Puzzle*, shown in Figure 7, (AP-ART design no. 76-A) popularized about fifteen years ago, has only one solution. The ten pieces are made of all different fancy woods, and the uniqueness of the solution assures that they will always be arranged in the most artistically pleasing pattern of mutually contrasting colors. It came with a printed solution under the assumption that few if any would ever have the patience to solve it by trial and error, rendering it more of a curiosity than a puzzle.

The puzzle designer must always bear in mind that the ultimate object of successful design is not to demonstrate the inventor's own mental prowess but rather to bring pleasure and satisfaction to the puzzle solver. Usually this can occur only after the puzzle has been successfully solved, which further supports the argument for puzzles that are not too difficult for the average person to solve.

Figure 7. Coffin's *Cornucopia Puzzle.*

It is taken for granted that a combinatorial puzzle ideally should have pieces that are all, or nearly all, dissimilar and preferably non-symmetrical. Contrarily the solution should be symmetrical or at least logical and aesthetically pleasing. Beyond that and the need for simplicity, setting down rules for successful design of combinatorial puzzles is probably futile, but here are a couple of suggestions.

First is the novelty test. We puzzle designers delight in being asked: "Wow, how did you ever come up with that idea?" Merely another routine checkerboard or $3 \times 3 \times 3$ cube dissection is not going to pass that test. And then there is the "Eureka" test. Upon discovering the solution, does the puzzle solver exclaim, "Eureka! Why didn't I think of that sooner?" Most combinatorial puzzles do not score highly on this test, but there are some exceptions. Original puzzles by their very nature defy rules for successful design. In fact, probably the best rule of all for the designer is to ignore all rules.

Our first example of a recent Pentomino design is the paradoxical *Vanishing Trunk Puzzle* (Fig. 8) (AP-ART design no. 181-B). The five puzzle pieces fit snugly into the tray to form (given a little imagination) a tree. There are two solutions. Now turn the tray over and assemble what is practically the same shape on the other side but with the tree trunk missing (Fig. 8). There is only one solution. This puzzle is based on a favorite device of puzzle designers - the idea of careless misdirection by force of habit or what seems to be the "obvious" approach. Incidentally this little set of puzzle pieces has many other possibilities for recreation. For example, they form a 5×5 square with a hole in the center or a 4×6 rectangle, and both solutions are unique.

Figure 8. Coffin's *Vanishing Trunk Puzzle* with the trunk, left and without, right.

In the previous example, note that a puzzle solving computer program might well be used to discover the solution, or at least as an aid in discovering it. In our next example, a computer was used as an aid in its design, yet paradoxically one objective of the design was to thwart solution by computer. Five pentominoes are to be assembled inside a square tray that measures 5.814×5.814. (AP-ART design no. 177-A) This design exploits the natural tendency to fit a square block into a square corner, or when that fails, to look for a solution with a 45-degree grid orientation, neither of which will lead to the one neat symmetrical solution. (Fig. 9)

Carrying this idea a step further, this next example, *Nice Try*, (Fig. 10) (AP-ART design no. 179) has, in addition to the confusion created by the diagonal grid, a void in the center for additional confusion. Successful puzzles of this sort are extremely difficult to design, this being a contrary example. The designer must always test them thoroughly for unintended false solutions of the type that we call "incongruous" solutions. No computer program that we are aware of is able to do this. After several copies of this design had been distributed to expert solvers, one of them stumbled upon an incongruous solution (Fig. 11), sending

Figure 9. S.T. Coffin Design No. 177-A.

it to the ever-growing trash bin labeled "nice try." By the way, not all of us lament the inability of computers to solve every design problem that comes along. On the contrary, consider it something to be thankful for. We puzzle designers look forward with considerable apprehension to the day, if it ever comes, when computers may be able to solve all problems, mathematical or otherwise.

Figure 10. Coffin's *Nice Try Puzzle.*

Figure 11. Incongruous solution for *Nice Try Puzzle.*

Many attempts have been made over the years by puzzle collectors and authors to classify puzzles. The first two examples shown here would probably fall within the general classification of combinatorial put together puzzles. Another popular type is the so-called sliding block sequential movement puzzle. Some of the more promising areas waiting to be explored are those combining two (or more) different puzzle classifications. Our next example, *Through the Looking Glass*, (AP-ART design no. 184-A) is one in which these two types are combined in a manner especially designed to confuse and confound anyone who attempts to solve it by computer, while at the same being not hopelessly difficult to solve using intuition, (Fig. 12). The six polyomino puzzle pieces are to be assembled inside a 5×5 square tray that has a two-unit opening on one edge through which the pieces must be inserted. All of the dimensions are exact, without any rounding of corners required. In the practical model shown here, the top of the tray is covered with Plexiglas, which has a large hole in the center to facilitate moving the pieces around using the eraser end of a pencil. There is only one solution and essentially only one possible order of assembly with minor variations. As you can probably guess, the solution involves simultaneous rotation of groups of pieces inside the tray in a manner that is far from obvious and probably beyond the ability of present computer programs to solve.

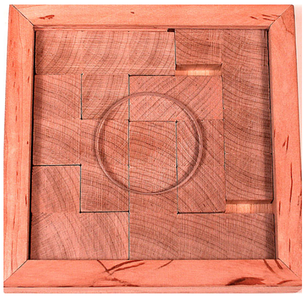

Figure 12. Coffin's *Through the Looking Glass Puzzle.*

Our final example is a novel box packing puzzle that involves maneuvering seven solid polyominoes to all fit inside a $3 \times 3 \times 3$ box (Fig. 13). The pieces must be inserted through a 1×2 slot in the transparent plastic top, hence the name *Slot Machine.* (AP-ART design no. 185). There is only one solution, which involves multiple shifting of the pieces in a manner perhaps best described as "interesting", and essentially one order of assembly.

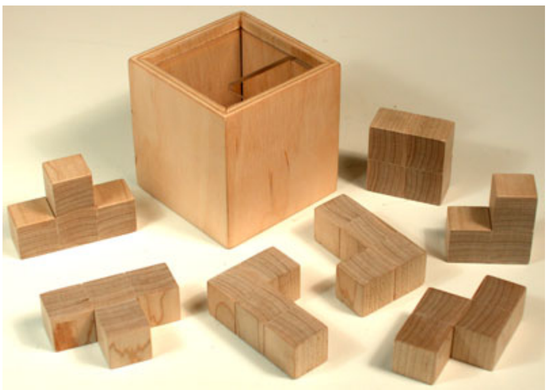

Figure 13. Coffin's *Slot Machine Puzzle.*

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